



R.C.Patel College Of Engineering & Polytechnic, Shirpur

Department of Civil Engineering



Course Title- Theory of Structure
Programme Name -Civil Engineering

Course Code -315313
Semester-Fifth

Unit	Title	COs	Learning hours	R Level	U Level	A Level	Total Marks
IV	Moment Distribution Method	CO4	16	2	4	12	16

* 4.1 Introduction :- Moment distribution method is an interactive method for analysis of statically indeterminate beams and frames.

- Also known as Hardy cross method.
- Based on distribution of unbalanced moment of joints according to their stiffness and carry over portion of distributed moment to far end.

* 4.2 Sign convention :-

- For moment distribution at joint
 - ↶ Anticlockwise moment at a joint (-ve)
- ↷ clockwise moment at a joint (+ve)

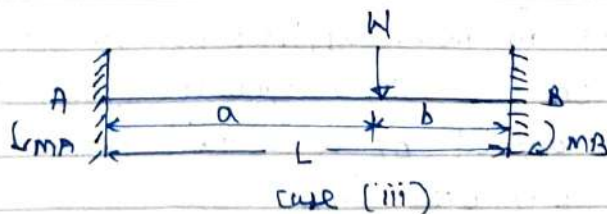
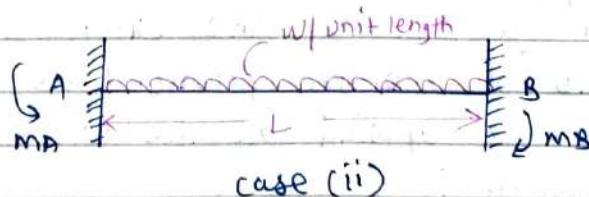
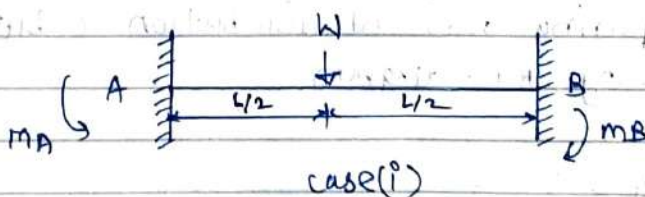
use the same convention throughout the calculation

* For BMD

sagging moment = +ve

hogging moment = -ve

let us consider a fixed beam subjected loadings as shown in fig.



Usual sign convention	Hardy cross sign convention
case i) $M_A = M_B = \frac{WL}{8}$ (Hogging) $= -\frac{WL}{8}$	case i) $M_A = -\frac{WL}{8}$ $M_B = +\frac{WL}{8}$
case ii) $M_A = M_B = \frac{WL^2}{12}$ (Hogging) $= -\frac{WL^2}{12}$	case ii) $M_A = -\frac{WL^2}{12}$ $M_B = +\frac{WL^2}{12}$
case iii) $M_A = \cancel{M_B} = +\frac{wab^2}{L^2}$ (Hogging) $= -\frac{wab^2}{L^2}$ $M_B = \frac{wba^2}{L^2}$ (Hogging) $= -\frac{wba^2}{L^2}$	case iii) $M_A = -\frac{WL^2}{12}$ $M_B = \frac{WL^2}{12}$

* Note → Hardy cross sign convention has to be used only for preparing moment distribution table and not for drawing B.M. diagram.

4.3 * Basic concept :-

* carry over factor :- The carry over factor is the ratio of moment produced as a joint to the moment applied at the other joint without displacing it.

- The effect of moment applied at one joint can be determined on the other opposite joint by multiplying the moment applied by a certain factor which is termed as carry over factor.

* 4.2.1 carry over factor for a Beam having one end fixed and other end simply supported or hinged

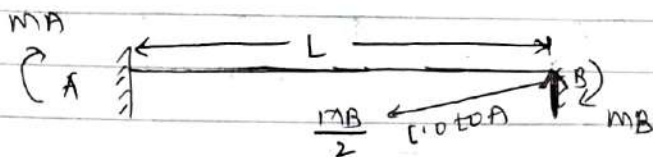


fig. ① Propped cantilever

- In this case a hogging moment ' M_B ' applied at simply supported beam end B will induce a sagging moment ' M_A ' at a fixed end A such that $M_A = \frac{M_B}{2} \therefore \frac{M_A}{M_B} = \frac{1}{2}$

Hence for this case carry over factor $\frac{1}{2}$ and $\frac{1}{2}$

M_B carry over to the fixed end A as shown in fig. ①

* 4.3.2 carry over factor for a Beam simply supported or Hinged at both ends

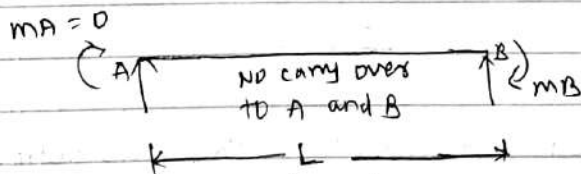


fig. ② Beam with simple support

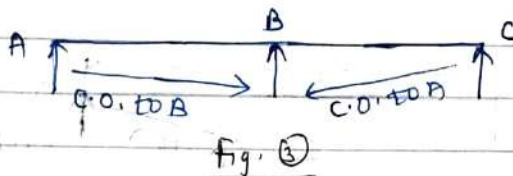
- in such case, no moment will be induced at A because A end is simply supported or Hinged.

$$\therefore M_A = 0, \quad \frac{M_A}{M_B} = 0$$

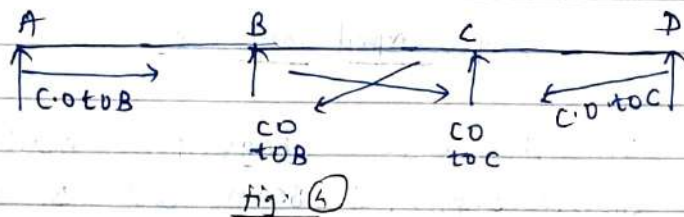
Hence for this case carry over factor is zero and no moment carry over to the S.S. end A or B as shown in fig. ②

4.3.3 carry over (C.O) in some of the cases as follows

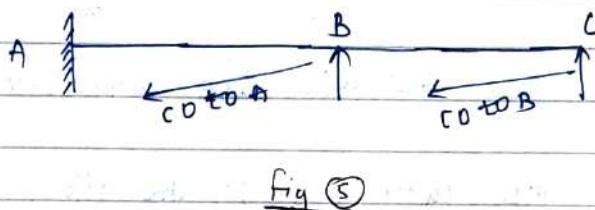
case I



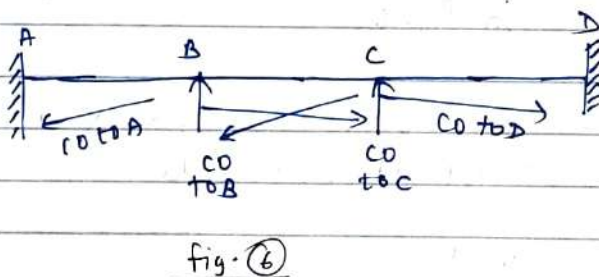
case II



case III



case IV



*4.3.4 carry over moment: The moment induced at the fixed end of a beam by the action of the moment applied at the other simply supported or hinged end is called as carry over moment.

*4.4.0 stiffness factor k :- The moment required at the simply supported end of a beam so as to produce unit rotation at that end without translation of the other end is called as stiffness factor.

Note that $\theta = 1$ radian
stiffness factor is generally denoted by 'k'

4.4.1 stiffness factor for beam fixed at one end and simply supported or hinged at the other end

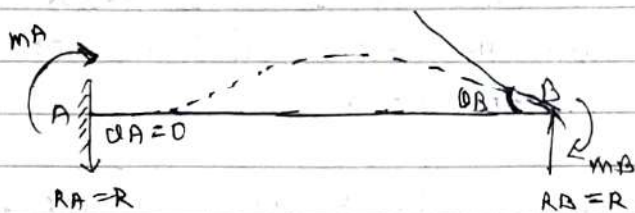


fig. 4.4.1

- For better understanding refer fig. 5.4.1
- In this case, slope at A is zero i.e. $\theta_A = 0$ and slope exists at B i.e. θ_B .

$$\therefore \theta_B = \frac{M_B L}{4EI} \quad \therefore M_B = \frac{4EI \cdot \theta_B}{L}$$

But, $\theta_B = 1$ (unit rotation)

$$\therefore \boxed{M_B = \frac{4EI}{L}}$$

Hence stiffness factor for this beam is $\frac{4EI}{L}$

4.4.2 stiffness factor for a beam simply supported at both the ends

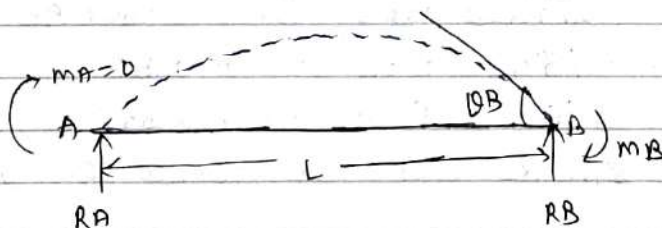


fig 4.4.1

In this case, slope at A and B exists but $M_A = 0$ at A

$$\text{slope at B i.e. } \theta_B = \frac{M_B L}{3EI}$$

$$\therefore M_B = \frac{3EI \theta_B}{L}$$

But $\theta_B = 1$ (unit rotation) $\therefore M_B = \frac{3EI}{L}$

Hence stiffness factor for this beam is $\frac{3EI}{L}$

4.5 Distribution factor

- The distribution factor for a member at a joint is the ratio of stiffness factor for that member and the total stiffness of all the members meeting at a joint.
- This concept of distribution factor can be illustrated in a better manner by following example
- consider a continuous beam ABC subjected to moment m_B applied at the rigid joint B as shown in fig.

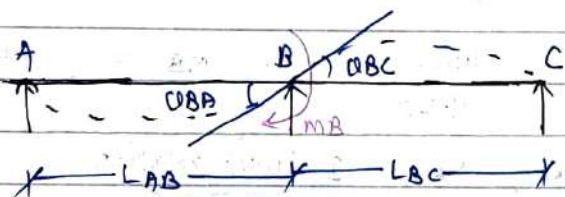


Fig. 4.5

- Note that stiffness factor at A and C is zero due to simply supported ends. In a continuous beam ABC, AB, BA, BC and CB are the members and one joint B as shown in fig 4.5

K_{AB} = stiffness factor for member AB at A = 0

K_{CB} = stiffness factor for member CB at C = 0

K_{BA} = stiffness factor for member BA at joint B

K_{BC} = stiffness factor for member BC at joint B

from defn of distribution factor (D.F.) we get the distribution factor for joint B as follows

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{K_{BA}}{\Sigma K} \quad (\because \Sigma K = K_{BA} + K_{BC})$$

$$\text{and } DF_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{K_{BC}}{\Sigma K}$$

Note $\rightarrow$ moment m_B applied at the joint B is distributed amongst the members BA and BC in proportion to their distribution factors.

4.6 Procedure to find the support moments by MDM :-

- * Step 1 :- Assume all the supports are to be fixed and find fixed end moments (FEM) acting at the joints by using Hardy-cross sign convention.
- * Step 2 :- calculate the stiffness factors 'k' and Distribution Factors (DF) for all members by considering the intermediate support as a joint.
- * Step 3 :- prepare moment Distribution table as follows

* specimen table format for two span continuous beam ABC as shown in fig. 1

Joint	A	B	C
member	AB	BA BC	CB
Distribution factor (DF)			
Fixed end moment (FEM)			
Release support A and C and carry over to B			
Initial moments			
1 st distribution (Balance the support B)			
Final moments			

- * Step 4 :- Release the joints by applying an equal and opposite moment.
 - In case of overhanging end span, balance the overhanging support by applying an equal and opposite joints.
 - Now consider the intermediate supports at joints and balance the unbalanced moment by multiplying the distribution factor at these joints and then distribute these moments among two connecting members in proportion. Results are entered into moment distribution table of step 3.

* Step 5 :- Then take carry over so as to evaluate the effect of distributed moments on the opposite joints in moment distributing table of step 3.

* Step 6 :- The method of balancing at the joints and carry over to the opposite joints should be done till it gives the required degree of accuracy of final moments. The final moments are obtained by the algebraic sum of end moments of all members mentioned in table.

* Note → i) The operations or process can be stopped when any distribution is completed and the distribution members or balanced moments are small as compared to the initial fixed end moments (FEM).

ii) In this method, the final moments on either side of a support are equal in magnitude and opposite in sign, but while writing the answer of support moment, hogging moment are considered to be -ve and that of sagging moment to be +ve as usual.

Note that, i) when the end supports are fixed then the moments distribution method is stopped by simply taking carry over moments to the fixed ends.

ii) When the end supports are simple, the moment distribution method should end with the balancing at the joint and not with a carry over.

iii) Four cycles of moment distribution are sufficient to obtain the support moment because the unbalanced moments decreases to zero.

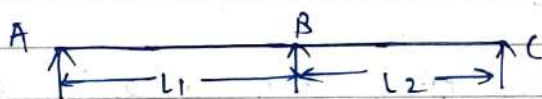
* Advantages of moment Distribution method

- Though it is an approximate method, it gives quick result. Lots of calculations are not needed hence it is less tedious.
- At every joint unbalanced moment can be known and further it can be balanced with the help of distribution factor.
- Distribution of moment over the support can be understood.

* Important points :-

1) To find stiffness factor 'k'

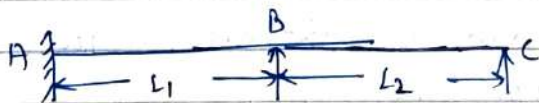
- * Case I : $\rightarrow$ continuous beam with both the ends simply supported and one intermediate support or hinged.



* For joint B

$$K_{BA} = \frac{3EI}{L_1}, \quad K_{BC} = \frac{3EI}{L_2}$$

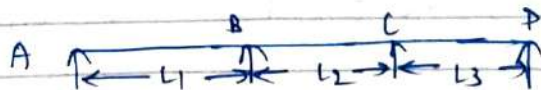
- * Case II $\rightarrow$ Cont. beam with end fixed and other end simply supported and one intermediate support.



For joint B :

$$K_{BA} = \frac{4EI}{L_1}, \quad K_{BC} = \frac{3EI}{L_2}$$

- * Case III $\rightarrow$ cont. beam with both ends simply supported and two intermediate supports.



For joint B : $K_{BA} = \frac{3EI}{L_1}, \quad K_{BC} = \frac{4EI}{L_2}$

For joint C : $K_{CB} = \frac{4EI}{L_2}, \quad K_{CD} = \frac{3EI}{L_3}$

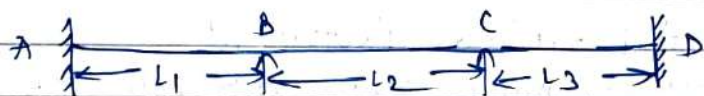
- * Case IV :- Cont. beam with both the ends fixed and one intermediate support



For joint B :-

$$K_{BA} = \frac{4EI}{L_1}, \quad K_{BC} = \frac{4EI}{L_2}$$

- * Case V :- Cont. beam with both the ends fixed and two intermediate support



joint B :- $K_{BA} = \frac{4EI}{L_1}, \quad K_{BC} = \frac{4EI}{L_2}$

joint C :- $K_{CB} = \frac{4EI}{L_2}, \quad K_{CD} = \frac{4EI}{L_3}$

- * 2> In case of overhanging end span the support near overhanging end is considered to be simply supported for calculating for stiffness and distribution factors for the span adjoining to the overhanging span.
- * 3> D.F. for the overhanging span is zero and for the span adjoining to the overhanging span D.F. is one.
- * 4> The sum of Distribution factor at a joint is always one.
- * 5> moments are not carried over from the opposite joint to the last simple support or last simple support with overhanging ends.
- * 6> Fixed support do not give or carry over any moment to the continuous support but absorb the moment from the adjoining support.

* Portal frames

- A frame in which a beam rests on two columns is called as portal frame.
- The junction of the beam and column behaves like a rigid or fixed joint.

* Types of portal frames

- types $\rightarrow$
- $\Rightarrow$ symmetrical portal frame (non-sway type portal frame)
 - $\Rightarrow$ unsymmetrical portal frame or (sway-type portal frame)

+ 1) symmetrical Portal (Non-sway type) :-

- The Portal frame which is having two identical columns, same length, same end conditions, same moment of inertia, same modulus of elasticity and which is only subjected to symmetrical loading. Then such portal frame is called as symmetrical portal frame.

- Note that a symmetrical portal frame do not sway to any side.

- fig. 4.8.1 show a single bay, single storey symmetrical portal frame subjected to u.d.l. which can be analysed by opening it out and treating exactly like a three span continuous beam as shown in fig. 4.8.2

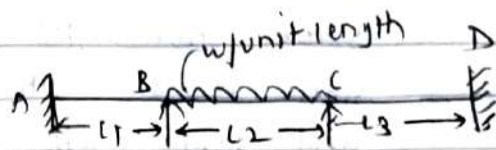
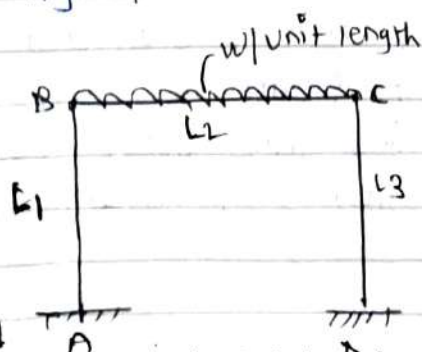


fig. 4.8.2 is opened out like 3 span cont. beam

4.8.1

fig. symmetrical portal frame

fig. 4.8.3 shows a single bay, single storey symmetrical portal frame subjected to a central point load w which can be analysed by opening it out and treating exactly like a three span cont. beam as shown in fig. 4.8.4

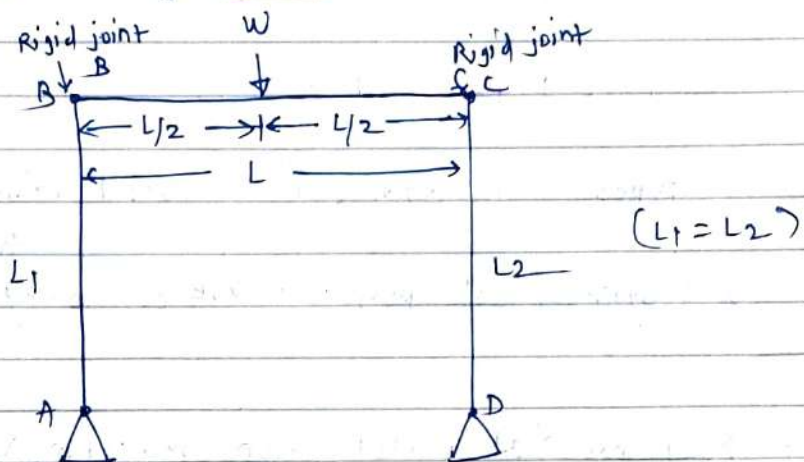


fig. 4.8.3 Symmetrical portal frame S.S. hinged at the base

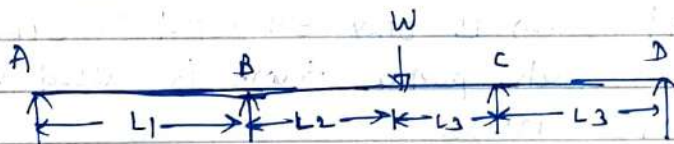


fig. 4.8.4 is opened out like a three span cont. beam.

- * 2) Un-symmetrical Portal frame (sway-type) :-
 when the frame in which two columns are not symmetrical or the frame not symmetrical loaded, then it is called as un-symmetrical portal frame. It is also called as sway-type portal frame.

due to unsymmetry, portal frame is subjected to some horizontal movement (called as sway) to one side or the other and rigid joints between column and beam will have motion of translation.

Fig. 4.8.5 a shows un-symmetrical portal-frame

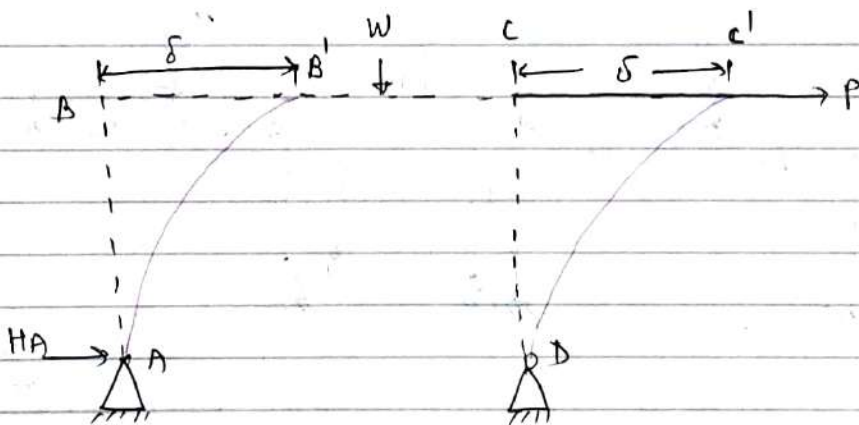
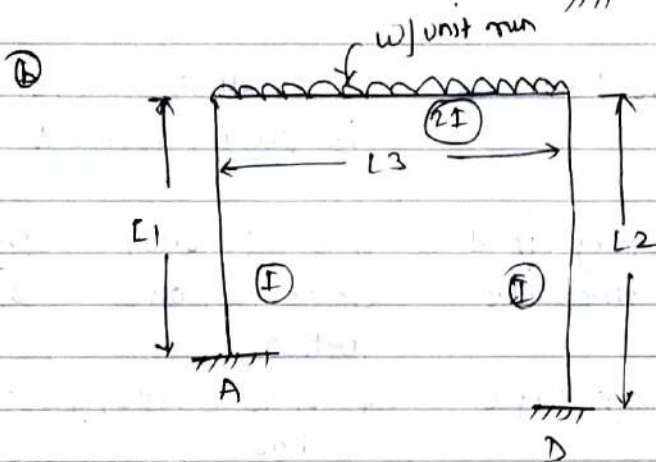
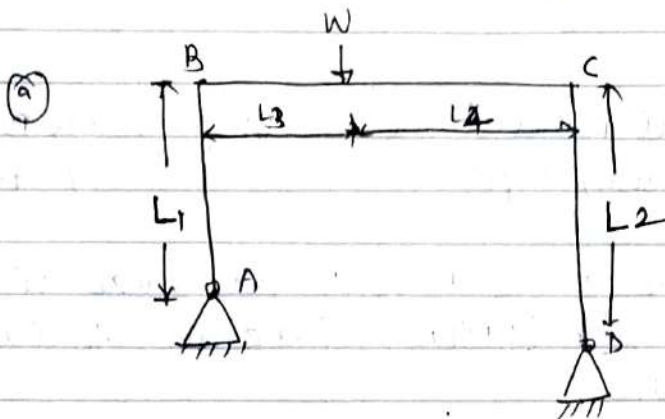
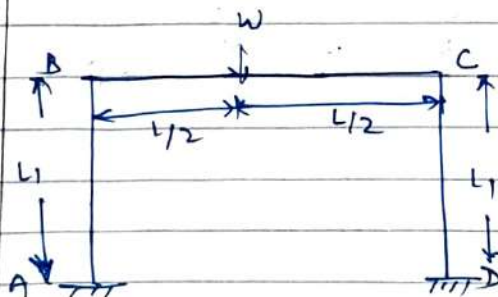
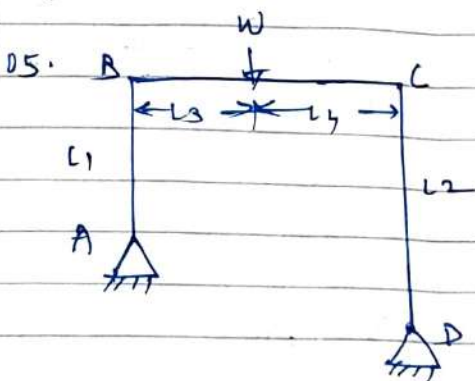


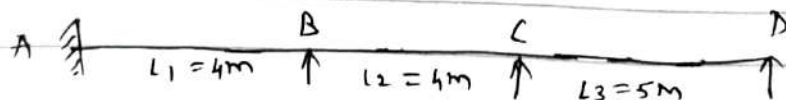
Fig. 4.8.6 un-symmetrical portal frame sway to one side.

* Difference between symmetrical and unsymmetrical portal frame

Sr.No.	symmetrical Portal Frame (Non-sway type)	Unsymmetrical Portal frame (sway-type)
01	Two columns of the frame are identical with identical end conditions.	01. Two columns of the frame are not identical
02	Loading on the frame is symmetrical	02. Loading on the frame is unsymmetrical
03	due to symmetry, it does not sway to one side	03. Due to unsymmetry, it sway to one side or other.
04	It is non-sway type portal	04. It is sway-type Portal frame.
05		

* Problems on continuous beam based with simply supported ends ~~based with~~ to find distribution factors (D.F)

Ex: 1) Determine distribution factor at continuity for a continuous beam A-B-C-D which is fixed at A and supported over B, C and D. Take $AB = BC = 4\text{m}$ and $CD = 5\text{m}$. Assume same MI for all spans.



stiffness factors

* joint B $K_{BA} = \frac{4EI}{L_1} = \frac{4EI}{4} = EI$

$$K_{BC} = \frac{4EI}{L_2} = \frac{4EI}{4} = EI$$

$$\Sigma K = K_{BA} + K_{BC}$$

$$= EI + EI$$

$$\boxed{\Sigma K = 2EI}$$

* joint C $K_{CB} = \frac{4EI}{L_2} = \frac{4EI}{4} = EI$

$$K_{CD} = \frac{3EI}{L_3} = \frac{3EI}{5} = 0.6EI$$

$$\Sigma K = K_{CB} + K_{CD}$$

$$= EI + 0.6EI$$

$$\boxed{\Sigma K = 1.6EI}$$

* Distribution Factor :

* joint B $(DF)_{BA} = \frac{K_{BA}}{\Sigma K} = \frac{EI}{2EI} = 0.5$

$$(DF)_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{EI}{2EI} = 0.5$$

$$\text{check: } (0.5 + 0.5 = 1)$$

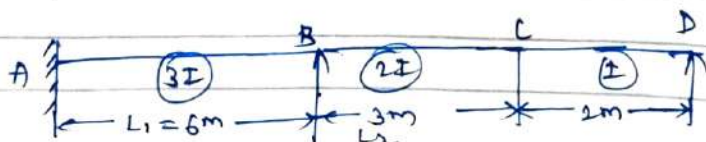
* joint C $(DF)_{CB} = \frac{K_{CB}}{\Sigma K} = \frac{EI}{1.6EI} = 0.625$

$$(DF)_{CD} = \frac{K_{CD}}{\Sigma K} = \frac{0.6EI}{1.6EI} = 0.375$$

$$\text{check: } (0.625 + 0.375 = 1)$$

ex: 2 Det. D.F. at continuity for a continuous beam ABCD which is fixed at A and simply supported at B, C and D. Take AB = 6m, BC = 3m, and CD = 2m if m.i. for span is $I_{AB} = 3I$, $I_{BC} = 2I$ and $I_{CD} = I$

Given: $L_1 = \text{span AB} = 6m$, $L_2 = \text{span BC} = 3m$
 $L_3 = \text{span CD} = 2m$.



step 1: stiffness factor k for the joint B and C

For joint B

$$K_{BA} = \frac{4EI_1}{L_1} = \frac{4E \times 3I}{6} = 2EI$$

$$K_{BC} = \frac{4EI_2}{L_2} = \frac{4E \times 2I}{3} = 2.67EI$$

$$\Sigma K = 2EI + 2.67EI = \underline{4.67EI}$$

For joint C :

$$K_{CB} = \frac{4EI_2}{L_2} = \frac{4E \times 2I}{3} = 2.67EI$$

$$K_{CD} = \frac{3EI_3}{L_3} = \frac{3E \times I}{2} = 1.5EI$$

$$\Sigma K = 2.67EI + 1.5EI = \underline{4.17EI}$$

* step 2: distribution factor DF for joint B and C

* for joint B :

$$(DF)_{BA} = \frac{K_{BA}}{\Sigma K} = \frac{2EI}{4.67EI} = \underline{0.428} = 0.43$$

$$(DF)_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{2.67EI}{4.67EI} = \underline{0.57}$$

$$(\text{check : } 0.43 + 0.57 = 1)$$

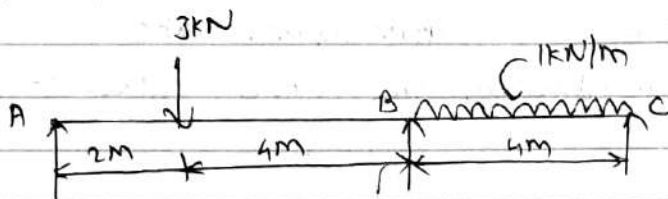
* for joint C :

$$(DF)_{CB} = \frac{K_{CB}}{\Sigma K} = \frac{2.67EI}{4.17EI} = \underline{0.64}$$

$$(DF)_{CD} = \frac{K_{CD}}{\Sigma K} = \frac{1.5EI}{4.17EI} = 0.359 = \underline{0.36}$$

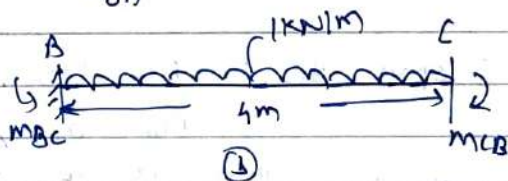
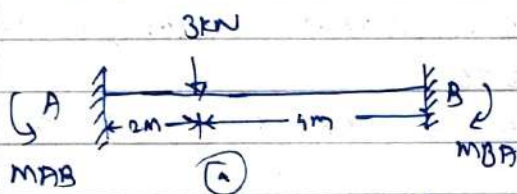
$$\underline{\text{check!}} : (0.64 + 0.36 = 1)$$

Ex: 1) A cont. beam ABC 10m long rests on three supports A, B, C at the same level and loaded as shown in fig



→ Given: $L_{AB} = 5m$, $L_{BC} = 4m$, $W = 3kN$, $w = 1kN/m$

step 1:- Assume span AB and BC as a fixed beam and fixed end moment (FEM)



* For span AB: $L = 5m$, $W = 3kN$, $a = 2m$, $b = 4m$

$$M_{AB} = -\frac{Wab^2}{L^2} = -\frac{3 \times 2 \times 4^2}{5^2} = -2.07 \text{ kN}\cdot\text{m} \quad (1)$$

$$M_{BA} = +\frac{Wba^2}{L^2} = \frac{3 \times 4 \times 2^2}{5^2} = +1.33 \text{ kN}\cdot\text{m} \quad (2)$$

* For span BC: $L = 4m$, $w = 1kN/m$

$$M_{BC} = -\frac{wL^2}{12} = -\frac{1 \times 4^2}{12} = -1.33 \text{ kN}\cdot\text{m} \quad (3)$$

$$M_{CB} = +\frac{wL^2}{12} = +\frac{1 \times 4^2}{12} = +1.33 \text{ kN}\cdot\text{m} \quad (4)$$

* step 2 To find the stiffness factor at the joint B

For joint B $K_{BA} = \frac{3EI}{L_{AB}} = \frac{3EI}{5} = 0.5EI$

$$K_{BC} = \frac{3EI}{L_{BC}} = \frac{3EI}{4} = 0.75EI$$

$$\Sigma K = 0.5EI + 0.75EI = 1.25EI$$

step 3: To find the distribution factor (DF) at the joint B

for joint B

$$(DF)_{BA} = \frac{K_{BA}}{\Sigma K} = \frac{0.5EI}{1.25EI} = 0.4$$

$$(DF)_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{0.75EI}{1.25EI} = 0.6$$

check $0.4 + 0.6 = \underline{1}$ OK

* step 4 To prepare the moment distribution table to find support moment.

joint	A	B	C
member	AB	BA BC	CB
Distribution factor DF		0.4 0.6	
Fixed end moment (FEM)	-2.67	1.33 -1.33	1.33
Release support A and C, and then carry over from A to B and from C to B.	+2.67		-1.33
Initial moment	0	1.335 -1.995	0
1st distribution (Balance B)		-0.268 -0.402	
Final moments	0	2.397 -2.397	0

* method to balance the moment at joint B

$$2.665 + (-1.995) = 0.67$$

$$\text{Take} = \underline{-0.67}$$

$$(-0.67 \times DF_{BA})$$

$$= -0.67 \times 0.4$$

$$= \underline{-0.268}$$

$$-0.67 \times DF_{BC}$$

$$= -0.67 \times 0.6$$

$$= \underline{-0.402}$$

$M_A = 0$ Support moment at A

$M_B = -2.397$ (Hogging) support moment at B

$M_C = 0$ support moment at C

Draw M' -diagram the above values of support moments.

step ⑤ Assume span AB and BC to be simply supported and find free B.M.

* for span AB : $L = 6m$, $a = 2m$, $b = 4m$, $W = 3kN$

$$M_A = 0, M_B = 0$$

$$M_D = \frac{Wab}{L} = \frac{3 \times 2 \times 4}{6} = 4 kNm$$

* for span BC : $L = 4m$, $W = 1kN/m$

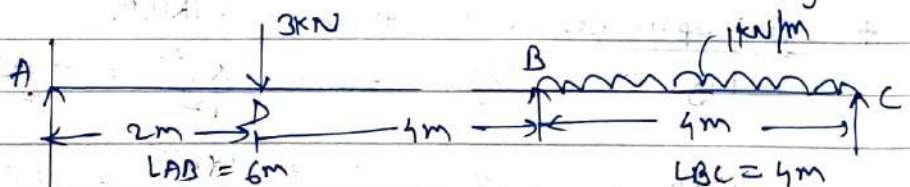
$$M_B = 0, M_C = 0$$

$M_{Bmax} =$ B.M. at the centre of span BC

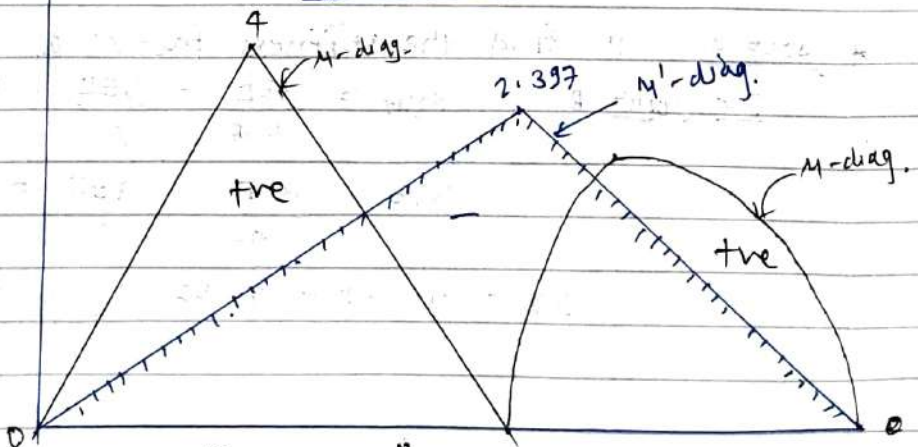
$$\frac{WL^2}{8} = \frac{1 \times 4^2}{8} = 2 kNm$$

Draw M -diagram from above free B.M.

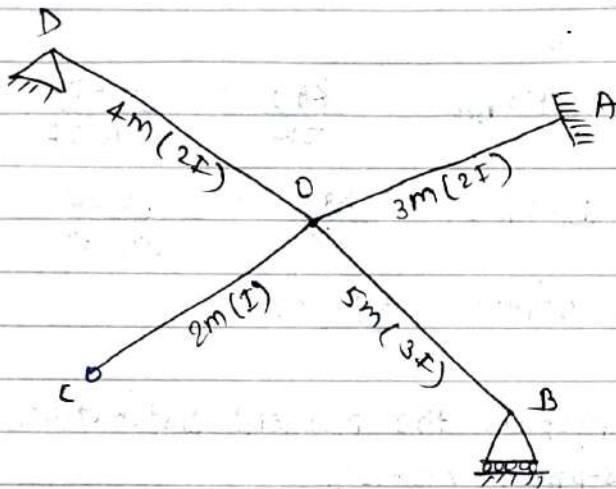
step ⑥ Superimpose the M' -diagram over the M -diagram so as to obtain net B.M.D. as shown in fig.



d) cont beam.



Ex:3 calculate distribution factor at joint 'O' shown in fig.



⇒ joint 'O'

stiffness factors: $K_{OA} = \frac{4EI_{OA}}{L_{OA}} = \frac{4E \times 2I}{3}$

$$= 2.66EI = 2.67EI \quad (\because I_{OA} = 2I)$$

$$K_{OB} = \frac{3EI_{OB}}{L_{OB}} = \frac{3E \times 3I}{5} = 1.8EI \quad (I_{OB} = 3I)$$

$$K_{OC} = 0 \quad (\text{As end C is free})$$

$$K_{OD} = \frac{3EI_{OD}}{L_{OD}} = \frac{3E \times 2I}{4} = 1.5EI \quad (\because I_{OD} = 2I)$$

$$\Sigma K = K_{OA} + K_{OB} + K_{OC} + K_{OD}$$

$$= 2.67EI + 1.8EI + 0 + 1.5EI$$

$$= \underline{5.97EI}$$

* Distribution factor:

$$(DF)_{OA} = \frac{K_{OA}}{\Sigma K} = \frac{2.67EI}{5.97EI} = \underline{0.447} \text{ Ans.}$$

$$(DF)_{OB} = \frac{K_{OB}}{\Sigma K} = \frac{1.8EI}{5.97EI} = \underline{0.30}$$

$$(DF)_{OC} = 0$$

$$(DF)_{OD} = \frac{K_{OD}}{\Sigma K} = \frac{1.5EI}{5.97EI} = 0.25$$

* check: $(0.447 + 0.30 + 0.25 = 0.997 \approx \underline{1})$

Ex: 2) calculate support moments and draw B.M.D. for a beam as shown in fig i) by using moment distribution method.

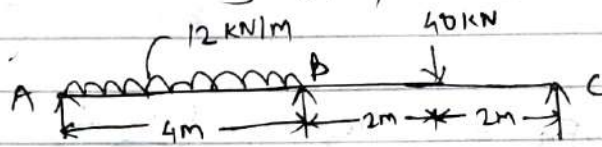


fig. 2.1 i) conti beam.

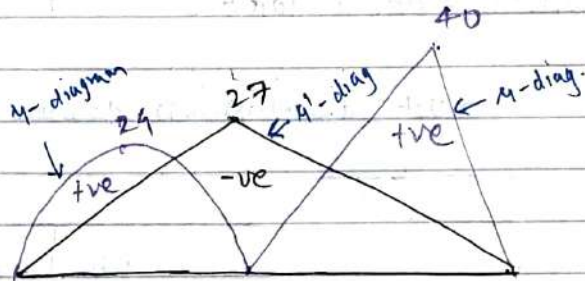
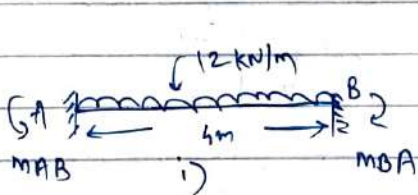


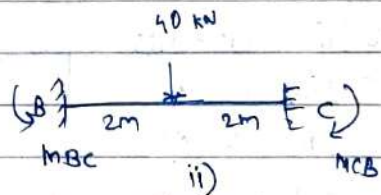
fig. 2.2 ii) BMD in kN.m.

Step 1 Find Fixed end moments



$$M_{AB} = -\frac{WL^2}{12} = -\frac{12 \times 4^2}{12}$$

$$= -16 \text{ kN.m}$$



$$M_{BA} = +\frac{WL^2}{12} = \frac{12 \times 4^2}{12}$$

$$= +16 \text{ kN.m}$$

$$M_{BC} = -\frac{WL^2}{8} = -\frac{40 \times 4}{8}$$

$$= -20 \text{ kN.m}$$

$$M_{CB} = +\frac{WL^2}{8} = +\frac{40 \times 4}{8}$$

$$= +20 \text{ kN.m}$$

* step II) To find stiffness factors

$$\text{joint B} = k_{BA} = \frac{3EI}{L_1} = \frac{3EI}{4} = 0.75EI$$

$$k_{BC} = \frac{3EI}{L_2} = \frac{3EI}{4} = 0.75EI$$

$$\Sigma k = k_{BA} + k_{BC} = 0.75EI + 0.75EI = 1.5EI$$

step 3) find out the distribution factor at joint B
 joint B $= DF_{BA} = \frac{K_{BA}}{\Sigma K} = \frac{0.75 EI}{1.5 EI} = 0.5$

$$DF_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{0.75 EI}{1.5 EI} = 0.5$$

(check :- ΣDF at joint B = $0.5 + 0.5 = 1$)

step 4) moment distribution table

JOINT	A	B	C
member	AB	BA	BC CB
DF		0.5	0.5
FEM	-16	16	-20
Release A & C carry over to B	+16	+8	-10
Initial moments	0	24	-30
1st distribution (balance B)		3	3
final moments	0	27	-27

step 5) support moments

$m_A = 0$, $m_B = -27 \text{ kN}\cdot\text{m}$, $m_C = 0$
 draw M' -diagram the above values of support moments

step 6) Free BMD

For M -diagram

max. B.M. at centre of AB

$$= \frac{WL^2}{8} = \frac{12 \times 4^2}{8} = 24 \text{ kN}\cdot\text{m}$$

max. B.M. at centre of BC

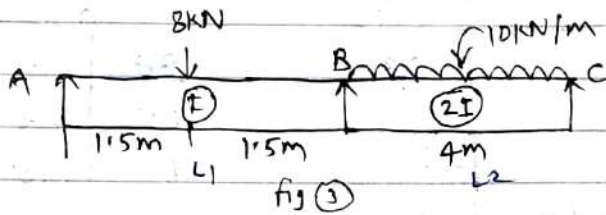
$$= \frac{WL^2}{4} = \frac{40 \times 4}{4} = 40 \text{ kN}\cdot\text{m}$$

draw M -diagram the above values of free moment.

step 7) super imposed the M' -diagram over the M -diag.
 so as to obtain net BMD as shown in fig 2.2

* continuous beam having varying M.I.

Ex-3 Determine support moment by MDM for a given cont. beam as shown in fig. (3)



Given data $L_1 = 3\text{m}$, $L_2 = 4\text{m}$, $I_1 = I$, $I_2 = 2I$
To find M_A , M_B , M_C

Known moments: $M_A = M_C = 0$

step 1) To find fixed end moments

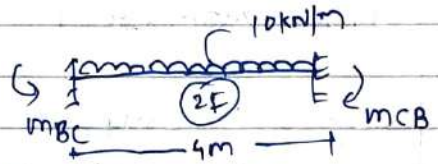
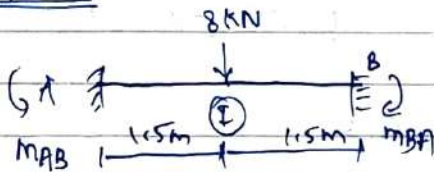


fig. (3.1)

for span AB

$$M_{AB} = -\frac{WL_1}{8} = -\frac{8 \times 3}{8}$$

$$M_{AB} = -3 \text{ kN}\cdot\text{m}$$

$$M_{BA} = +\frac{WL_1}{8} = +\frac{8 \times 3}{8}$$

$$M_{BA} = +3 \text{ kN}\cdot\text{m}$$

for span BC

$$M_{BC} = -\frac{WL_2^2}{12} = -\frac{10 \times 4^2}{12}$$

$$= -13.33 \text{ kN}\cdot\text{m}$$

$$M_{CB} = +\frac{WL_2^2}{12} = \frac{10 \times 4^2}{12}$$

$$= +13.33 \text{ kN}\cdot\text{m}$$

step 2) To determine stiffness factors:

joint B $K_{BA} = \frac{3EI_1}{L_1} = \frac{3EI}{3} = EI$

$$K_{BC} = \frac{3EI_2}{L_2} = \frac{3E(2I)}{4} = 1.5EI$$

$$\Sigma K = K_{BA} + K_{BC} = EI + 1.5EI = 2.5EI$$

step 3) To det. distribution factors.

joint B $DF_{BA} = \frac{K_{BA}}{\Sigma K} = \frac{EI}{2.5EI} = 0.4$

$$DF_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{1.5EI}{2.5EI} = 0.6$$

step 4. Moment Distribution table

joint	A		B		C
Member	AB	BA	BC	CB	
DF		0.4	0.6		
FEM	-3	+3	-13.33	+13.33	
Release A and C carry over from A to B and C to B	+3	1.5	-6.665	-13.33	
Initial moments	0	4.5	-14.935		0
Balance B		6.148	4.297		
Final moments	0	10.648	-10.648		0

step 5 : support moments

$$M_A = M_C = 0$$

$$M_B = -10.648 \text{ kNm Hogging.}$$

* continuous beams with one end fixed and others simply supported :-

Ex: 4) calculate support moment by using moment distribution method for a beam as shown in fig.

W-23
6

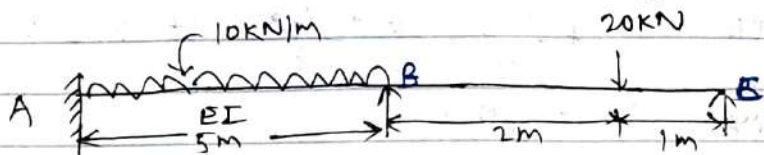


fig. (4)

→ Given data $L_1 = 5\text{m}$, $a = 2$, $b = 1$

Known moment = $M_C = 0$

To find M_A and M_B

step 1: To find fixed end moments

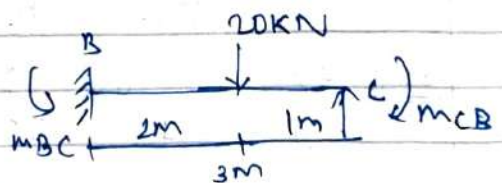
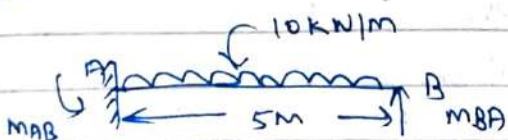


fig. (4.1)

$$+ M_{AB} = - \frac{wL_1^2}{12}$$

$$= - \frac{10 \times 5^2}{12}$$

$$M_{AB} = -20.83 \text{ kN}\cdot\text{m}$$

$$+ M_{BA} = + \frac{wL_1^2}{12}$$

$$= + \frac{10 \times 5^2}{12}$$

$$= +20.83 \text{ kN}\cdot\text{m}$$

$$+ M_{BC} = - \frac{wL_2^2}{12}$$

$$= - \frac{20 \times 2^2}{12}$$

$$= -4.44 \text{ kN}\cdot\text{m}$$

$$+ M_{CB} = + \frac{wL_2^2}{12}$$

$$= + \frac{20 \times 2^2}{12}$$

$$= +8.89 \text{ kN}\cdot\text{m}$$

* step-2 stiffness factors:

$$\text{joint B: } K_{BA} = \frac{4EI}{L_1} = \frac{4EI}{5} = 0.8EI$$

$$K_{BC} = \frac{3EI}{L_2} = \frac{3EI}{3} = EI$$

$$\Sigma K = 0.8EI + EI = 1.8EI$$

* step-3: distribution factors:

$$\text{joint B: } DF_{BA} = \frac{K_{BA}}{\Sigma K} = \frac{0.8EI}{1.8EI} = 0.44$$

$$DF_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{EI}{1.8EI} = 0.56$$

$$(\text{check: } \Sigma DF = 0.44 + 0.56 = 1)$$

* step-4: moment distribution table:

joint	A	B	C
Member	AB	BA BC	CA
DF		0.44 0.56	
FEM Release C	-20.83	20.83 -4.44	+8.89
Carry over to B			
Initial moments	-20.83	20.83 -8.89	0
1st distribution		-5.25 -6.69	
(Balance B) C.O. to A	-2.625		
Final moments	-23.455	15.58 -15.58	0

Step 5 Support moments

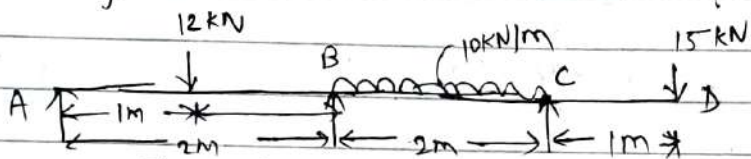
$$M_A = -23.455 \text{ kN}\cdot\text{m} \quad \text{Hogging}$$

$$M_B = -15.58 \text{ kN}\cdot\text{m} \quad \text{Hogging}$$

$$M_C = 0$$

* Continuous Beams with one end simply supported other end span overhanging - having constant M.I.

Ex: find the support moments for the continuous beam shown in fig.



step 1) To find fixed end moments for span AB

$$M_{AB} = -\frac{WL_1}{8} = -\frac{12 \times 2}{8} = -3 \text{ kN}\cdot\text{m}$$

$$M_{BA} = \frac{WL_1}{8} = \frac{12 \times 2}{8} = +3 \text{ kN}\cdot\text{m}$$

for span BC

$$M_{BC} = -\frac{WL_2^2}{12} = -\frac{10 \times 2^2}{12} = -3.33 \text{ kN}\cdot\text{m}$$

$$M_{CB} = \frac{WL_2^2}{12} = \frac{10 \times 2^2}{12} = +3.33 \text{ kN}\cdot\text{m}$$

* for cantilever portion CD

$$M_{CD} = -15 \times 1 = -15 \text{ kN}\cdot\text{m}, \quad M_{DC} = 0$$

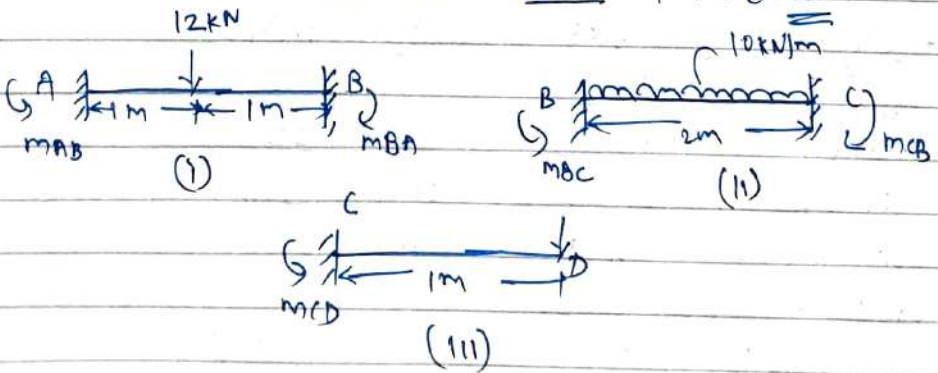


fig.

step 2 stiffness factors

joint B: $K_{BA} = \frac{3EI}{l_1} = \frac{3EI}{2} = 1.5EI$

$K_{BC} = \frac{3EI}{l_2} = \frac{3EI}{2} = 1.5EI$

$\Sigma K = K_{BA} + K_{BC}$
 $= 1.5EI + 1.5EI$
 $= \underline{3EI}$

joint C $K_{CB} = \frac{4EI}{l_2} = \frac{4EI}{2} = 2EI$

$K_{CD} = 0$ (D free end)

$\Sigma K = K_{CB} + K_{CD} = 2EI + 0 = \underline{2EI}$

* step 3 distribution factors

joint B $= DF_{BA} = \frac{K_{BA}}{\Sigma K} = \frac{1.5EI}{3EI} = 0.5$

$DF_{BC} = \frac{K_{BC}}{\Sigma K} = \frac{1.5EI}{3EI} = 0.5$

joint C $= DF_{CB} = \frac{K_{CB}}{\Sigma K} = \frac{2EI}{2EI} = 1$

$DF_{CD} = \frac{K_{CD}}{\Sigma K} = \frac{0}{2EI} = 0$

check $= \Sigma DF = 1$

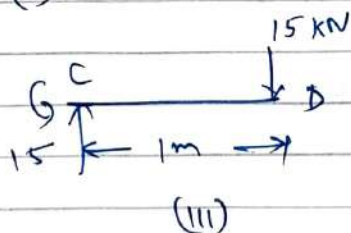
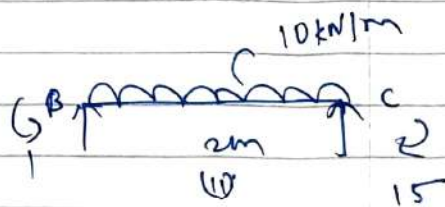
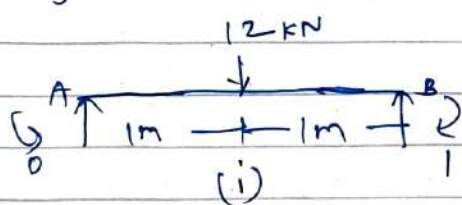
At joint B, $\Sigma DF = 0.5 + 0.5 = 1$

At joint C, $\Sigma DF = 1 + 0 = 1$

step 4 moment distribution table

joint	A	B	C	D
member	AB	BA, BC	CB, CD	DC
DF		0.5, 0.5	1, 0	
FEM	-3	3, -3.33	3.33, -15	0
Release A & balance C CD to B	+3		11.67	+0
Initial moments 1st distribution	0	4.5, 2.505	15, -15	0
(balance B)	0	-3.50	1.5, -1.5	0
final moment	0	1, -1	15, -15	0

step 5 supports moments : The supports moments acting on different spans of a cont. beam are as shown in fig.



from above Fig i, ii, iii the supports moments are

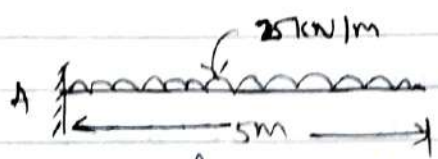
$$M_A = 0, \quad M_B = 1 \text{ kNm (Hogging)}$$

$$M_C = 15 \text{ kNm (Hogging)}$$

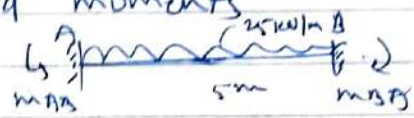
$$M_D = 0$$

* examples on propped cantilever

Ex: Using moment distribution method determine the moments at fixed end of a propped cantilever of span 5m carrying a udl of 25 kN/m over entire span



step 1) calculate fixed end moments



$$m_{AB} = -\frac{wL^2}{12}$$

$$= -\frac{25 \times 5^2}{12} = -52.08 \text{ kN}\cdot\text{m}$$

$$m_{BA} = +\frac{wL^2}{12} = \frac{25 \times 5^2}{12}$$

$$= 52.08 \text{ kN}\cdot\text{m}$$

* step 2 stiffness factor : at

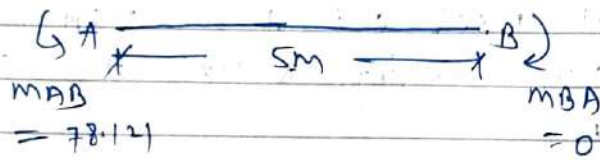
As there is no continuation at joint B and joint A is fixed there is no relative stiffness and there will not be any distribution factors.

* step 3 Distribution factors : NO distribution factors

* step 4 moment distribution table

joint	A	B
member	AB	BA
DF	-	-
FEM	-52.08	+52.08
Balance B		+52.08
C.O. to B	-26.04	
Final moments	-78.12	D

Step 5 Support moments



fixed-end moment at A

$$M_A = M_{AB} = -78.12 \text{ kN}\cdot\text{m}$$

(-ve sign indicates Hogging moment)

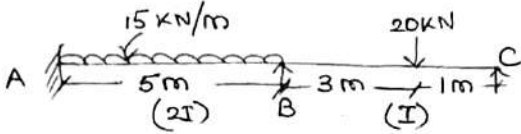
$$= 78.12 \text{ kN}\cdot\text{m} \text{ (Hogging)}$$

fixed-end moment at B

$$M_B = M_{BA} = \underline{\underline{0}}$$

Unit-4 Moment Distribution Method (Question Bank)

Que. No.	2 marks Questions
1	Define carry over moment and stiffness factor.
2	Draw a sketch of portal frame symmetrical and having one bay and three storey
3	Define stiffness factor and distribution factor.
4	Define perfect frame with neat sketch.
5	Differentiate between perfect and imperfect frame on any two point
6	Define stiffness factor.
7	Write the value of stiffness factor for beam – i) Simply supported at both ends ii) Fixed at one end, simply supported at other end.
8	Define distribution factor
9	Define carry over factor and distribution factor
10	Define carry over moment and carry over factor
Que. No.	4 marks Questions
1	Determine distribution factors for a simply supported beam continuous over support B and C and simply supported at A and D. $AB = BC = CD = 6\text{ m}$ and $I_{AB} = I$, $I_{BC} = 2I$ and $I_{CD} = 1.5 I$.
2	A continuous beam ABC 10 m long rests on three supports A, B and C at same level and loaded as shown in Figure No. 2. Determine support moments and draw BMD. Use moment distribution method.
	Fig. No. 2
3	Explain the procedure of moment distribution method.
4	Calculate distribution factors for the members OP, OQ and OR for the joint O as shown in Figure No. 4.
	Fig. No. 4
5	Differentiate between symmetrical portal frame and unsymmetrical portal frame with neat sketcher. (Any four points)
6	Using the moment distribution method, calculate support moments and Draw BMD for the beam as shown in Figure No. 6 EI is constant.
	Fig. No. 6
7	A continuous Beam of uniform is fixed at A and simply supported at B and C. Span $AB = 6\text{ m}$ and carries Udl of 40 kN/m and $BC = 8\text{ m}$, carries 80 kN point load at it's mid span. Using moment distribution method calculate the support moment and Draw BMD.
8	Using moment distribution method calculate the support moment of a beam as shown in Fig. No. 6.
	Fig. No. 6
9	Calculate support moment by using moment distribution method for a beam as shown in Fig. No. 10.
	Fig. No. 10

10	A beam ABC is simply supported at A, B and C span AB and BC are of lengths 3 m and 4 m respectively. AB carries udl of 15kN/m over entire span and BC carries a central point load of 30 kN. Find support moment using MDM.
11	A continuous beam of uniform rigidity of fixed at A and simply supported at B and C. AB = 6 m and carries udl of 40 kN/m. Span BC = 8 m and carries 80 kN point load at its mid-span. Using MDM calculate the support moment and draw BMD.
12	Draw BMD by using moment distribution method for a beam as shown in Figure No. 5  <u>Fig. No. 5</u>