



**R. C. Patel College of Engineering and Polytechnic,
Shirpur**

Class: TYEE (EE5K)

Course:

A.C. MACHINES PERFORMANCE (ACM)

(Course Code: 315333)

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Unit - I

Three phase induction motors

(20 Marks)

Course Outcome:

**CO1 - Test the performance of
three phase induction motor.**

* Construction of 3 ϕ Induction Motor:

- Q. Explain the Working of 3 ϕ Induction motor. state two applications (4M)
(Summer-2012)
- Q. Enlist any Four parts of 3 ϕ Induction motor and state the function of each listed. (4M) (Summer-2013).

Ans: There are mainly two parts of a 3 ϕ Induction motor:

1. Stator
2. Rotor.

1. Stator:

Stator is stationary part of the motor. This part is made of silicon steel stampings (i.e. thin sheets). These stampings are slotted.

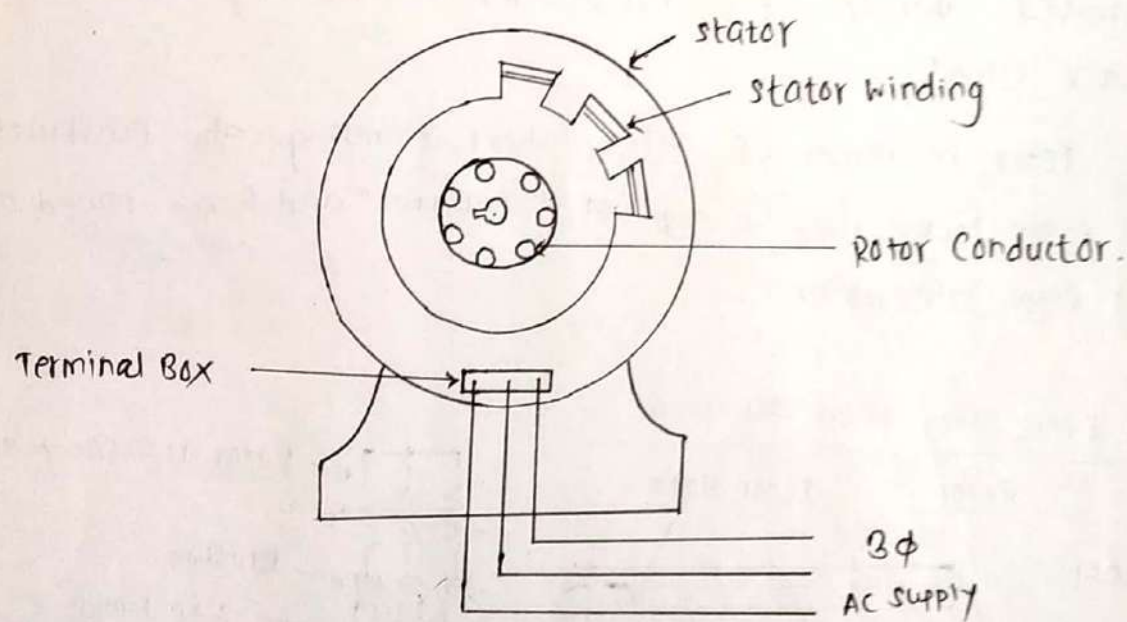


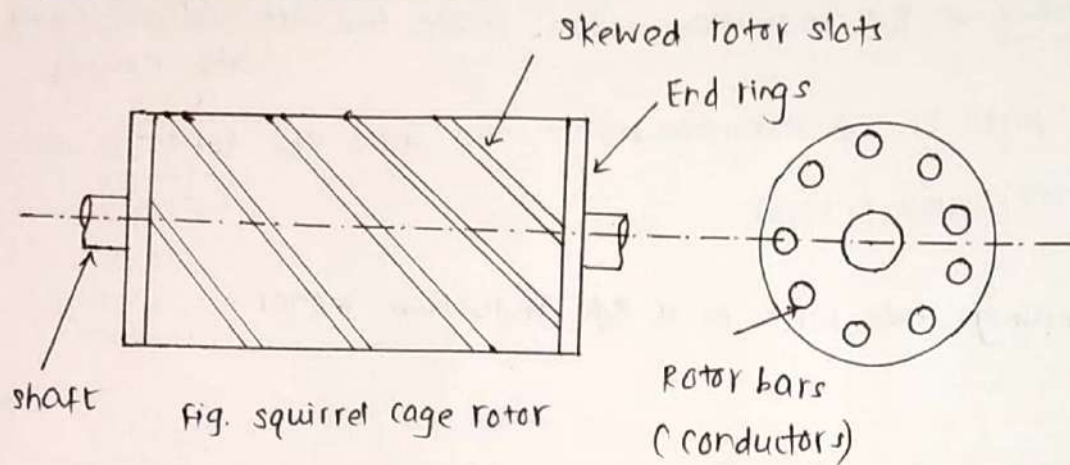
Fig: Simple diagram of 3 ϕ Induction motor.

There are slots on the inner side of the stator. The slots may be open type, semi-open type or closed type. In these slots, a 3 ϕ winding is accommodated (placed). This winding is made of super enameled copper. This winding may be star or delta connected. The three ends of this winding are brought out into the terminal box where 3 ϕ AC supply can be connected.

2. Rotor: There are two types of induction motor depending upon the construction of the rotor.

- a) Squirrel cage type b) Wound rotor type.

a) Squirrel cage type rotor:



squirrel cage type rotor is simple and most rugged construction. The rotor consists of a cylindrical laminated core with skewed rotor slots. Thick copper bars are placed in these slots and are brazed or welded to end rings. Thus, the rotor conductors are permanently short-circuited. Therefore it is not possible to add any external resistance in the rotor circuit.

Rotor is made of silicon steel stampings. The construction of this rotor looks like a cage of "squirrel" and hence called as "squirrel cage type motor."

b) Wound type rotor with slip rings:-

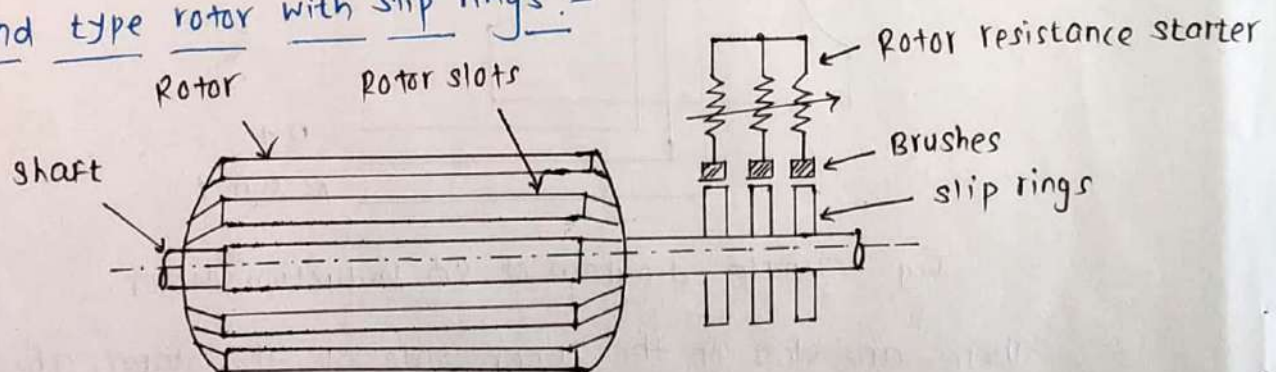
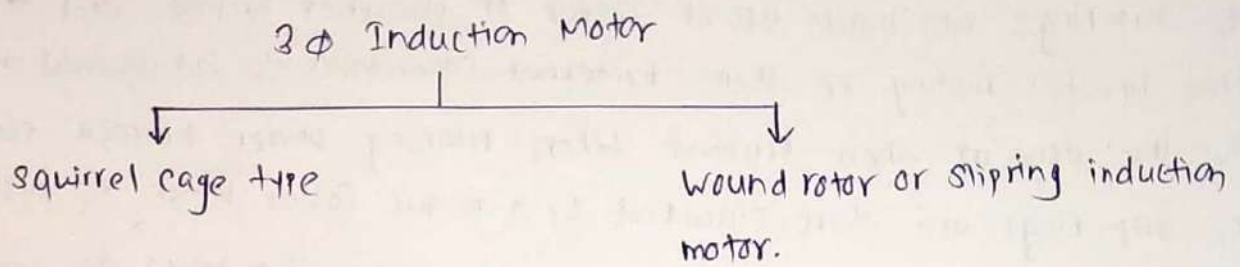


Fig: slip ring type rotor (wound type rotor).

In this type of induction motor, the rotor is wound for the same number of poles as that on the stator. The rotor is made up of laminations with slots on the outer periphery in which a ϕ rotor winding is placed.

* Types of 3 ϕ Induction motor:

Depending on type of rotor there are two types:



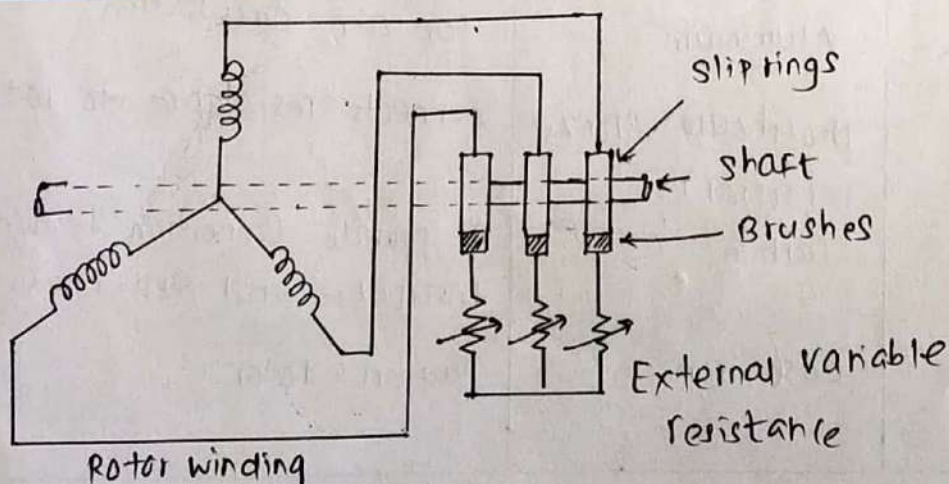
* Squirrel Cage Motor:

(Note: refer fig. of squirrel cage rotor)

When higher starting torque is not the requirement, the rotor is constructed in a very simple manner. Fig. shows that the rotor drum is provided ~~in the rotor drum of the motor~~ with a number of circular holes, parallel to the shaft are provided in the rotor drum of the motor.

Solid copper or aluminium bars are placed in these holes and all these bars are welded to two end rings to make contact between them. The bars and the end rings together look like a cage and hence such a rotor is called a cage rotor or squirrel cage rotor. The rotor bars are short circuited at end rings therefore no external connection can be made to the cage type rotor, therefore it is not possible to connect any external resistance to the rotor.

* Wound Rotor or Slip Ring Induction Motor:



The three phases are connected in star connection internally. The remaining three terminals are brought out and connected to the slip-rings mounted on the shaft.

The slip rings are made up of copper or phosphor bronze and there are three brushes resting on them. External connections to additional resistances are done at these brushes. When running under normal conditions, the slip-rings are short-circuited by a metal collar which is pushed along the shaft and the brushes are lifted from the slip-rings to reduce the frictional losses and wear.

As 3 ϕ winding is wound on rotor, this type of rotor is called "phase wound rotor". It is also called as "slip ring type" because slip rings are used.

* Table of parts and their functions:

Q. state the function of following parts of an induction motor :

1. slip-rings 2. Fan 3. Brushes and 4. Frame (Winter-15, 4M)

Sr.No.	part	material	function
1.	stator frame	Cast iron	supports the core, protects inner parts
2.	stator core	Silicon steel	Houses stator winding
3.	stator winding	Copper & insulated	To produce rotating magnetic field.
4.	rotor core	Silicon steel	Houses rotor winding
5.	rotor winding	Copper & insulated	To produce rotor current
6.	Air gap	-	To provide gap in rotor & stator
7.	Air inlet/outlet	-	for air circulation
8.	Cooling fan	Aluminium	for air circulation
9.	slip rings	phosphorous bronze (or copper)	connects resistance to rotor ckt.
10.	brushes	carbon	To provide connection between resistance and slip-rings.
11.	shaft	cast iron	supports rotor.

Sr. No.	Squirrel cage rotor	Wound or slipring rotor
1.	Rotor is in Form of bars which are shorted at the ends with the help of end rings	Rotor is in Form of 3 ϕ winding.
2.	No sliprings or brushes are used	sliprings and brushes are used.
3.	Simple construction	construction is complicated.
4.	External resistances can not be connected	It is possible to connect the external resistance to the rotor.
5.	Small or moderate starting torque.	High starting torque can be obtained.
6.	Starting torque can not be adjusted	Starting torque can be adjusted by adjusting the external resistance.
7.	Frequent maintenance is not required due to absence of sliprings and brushes	Frequent maintenance is necessary due to the use of sliprings and brushes.
8.	Rotor resistance starter can not be used.	Rotor resistance starter can be used.
9.	Speed control using rotor resistance control can not be used	speed control using rotor resistance control is possible to achieve.
10.	Less rotor copper loss.	High rotor copper loss
11.	High efficiency	Low efficiency
12.	Applications: Lathes, Fans, Water pumps, blowers etc.	Applications: cranes, elevators, Compressors, lifts etc.

* Direction of RMF:

The direction of RMF depends on the phase sequence of the ac supply being connected to the stator winding.

As we have proved, the RMF rotates in the clockwise direction if the phase sequence is R, Y, B i.e. the normal sequence as shown in fig. (a).

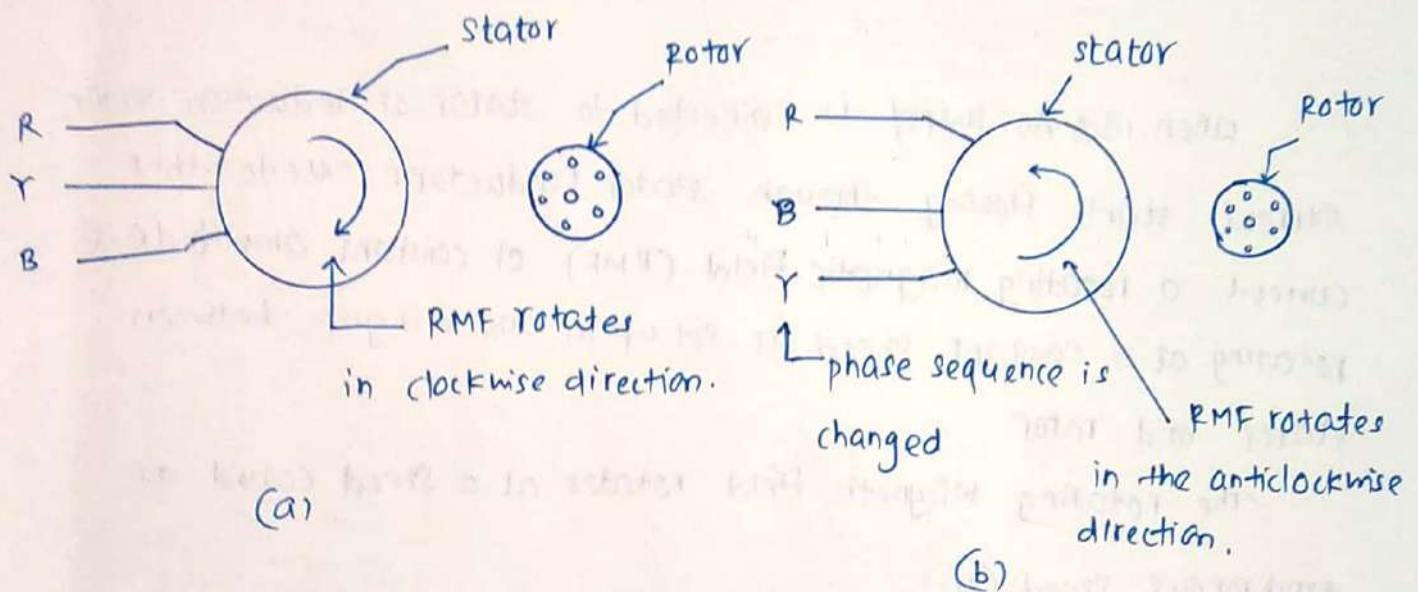


Fig: Direction of RMF

But if we interchange any two phases of the ac supply to get a new phase sequence say R, B, Y then the direction of RMF will reverse.

It will start rotating in anticlockwise direction as shown in fig. (b).

* principle of operation: (S-15)

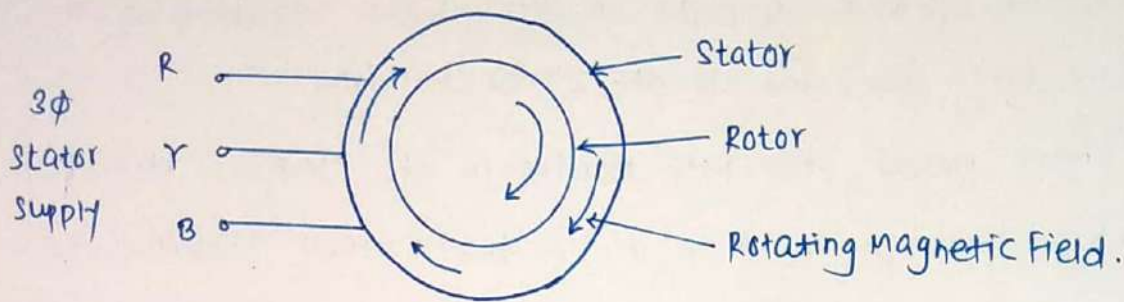


Fig: Principle of operation of induction motor.

When 3 ϕ Ac supply is connected to stator of induction motor, current starts flowing through stator conductors. Due to this current a rotating magnetic field (RMF) of constant amplitude & rotating at a constant speed is set up in the airgap between stator and rotor.

The rotating magnetic field rotates at a speed called as synchronous speed (N_s).

$$\text{The Synchronous speed } N_s = \frac{120F}{P} \quad \text{Where } F = \text{Frequency of supply} \\ P = \text{No. of poles of motor}$$

The rotor winding is stationary at starting. So the rotating magnetic field cuts the stationary rotor conductors and induces an emf in the rotor winding. according to Faraday's law of electromagnetic induction. This induced emf gives rise to rotor current. (As the rotor is closed circuited, current starts flows through rotor). The direction of the rotor current is such that it will oppose the very cause that produce this current. And the cause behind producing the rotor current is the "relative velocity" between the rotating field and the rotor.

So, the rotor current will flow in such a direction that the rotor will experience a force that accelerate it in the same direction as that of the rotating magnetic field as shown in fig.

Ideally, at no load, the rotor should rotate at the same speed as that of the rotating magnetic field i.e. N_s . But practically it rotates at slightly less speed than N_s due to friction and windage.

Motor on load:

When the induction motor is loaded mechanically its speed (N) decreases to produce the required amount of torque. The reduction in the motor speed (N) will stop as soon as the torque produced by the motor (T) is exactly equal to the load torque (T_L). The percentage difference between the synchronous speed (N_s) and the actual speed (N) is known as slip (s).

$$s = \frac{N_s - N}{N}$$

Generation of torque:

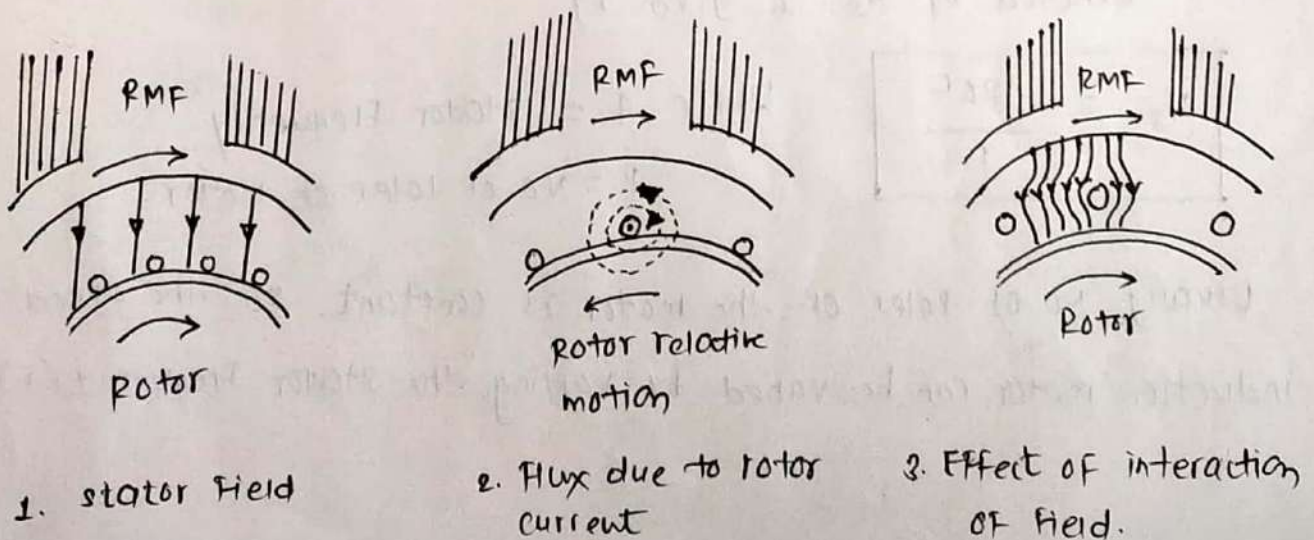


Fig: Generation of torque.

* Can the Actual speed N be Equal to N_s ? (W-14)

At no load, ideally we can say that rotor speed should be equal to the synchronous speed if the mechanical load is zero. But practically, actual speed (N) is never equal to synchronous speed (N_s). Practically, N is always less than N_s . ($N < N_s$).

If N becomes equal to N_s (ie. $N = N_s$), the relative velocity between RMF and rotor becomes zero. Hence the induced current in the rotor conductor becomes zero. For production of induced current relative motion is important. So as induced current becomes zero, torque produced by the motor will be zero, & the motor slows down. This happens everytime when N tends to become equal to N_s . Therefore the rotor can not rotate at synchronous speed.

* Concept of Synchronous speed and slip: (W-13)

The speed at which the rotating magnetic field rotates is known as synchronous speed.

It is denoted by N_s . & given by

$$N_s = \frac{120F}{P}$$

Where F = stator frequency

P = No. of poles of motor.

Usually, no. of poles of the motor is constant, so the speed of induction motor can be varied by varying the stator frequency (F).

* Slip (s):

When the mechanical load on motor increased, the induction motor slows down to produce required torque.

The difference between the synchronous speed (N_s) and the actual speed (N) is known as slip "s".

The % slip is given as,

$$\% S = \frac{N_s - N}{N_s} \times 100 \%$$

The actual speed N can be given by

$$N = N_s (1 - s)$$

The value of slip will vary between 0 and 1 for the motoring operation. At stand still (stationary) condition $N = 0$, hence $s = 1$

Whereas at $N = N_s$, $s = 0$.

The slip decreases as the motor speed increases.

Numericals on slip & synchronous speed:

① A 4-pole and 3 ϕ Induction motor operates from a supply whose frequency is 50 Hz. Calculate:

1. The speed at which magnetic field of the stator is rotating.

2. Determine the percentage slip if rotor is rotating at 1200 rpm.

Soln:

Given:

$$p = 4,$$

$$F = 50 \text{ Hz}$$

$$N = 1200 \text{ rpm}$$

1. Synchronous speed:

$$N_s = \frac{120F}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

2. Percentage slip:

$$\% s = \frac{N_s - N}{N_s} \times 100$$

$$= \frac{1500 - 1200}{1500} \times 100$$

$$\boxed{\% s = 20\%}$$

② A 6 pole, 3 phase induction motor operates from a supply whose frequency is 50 Hz. Calculate:

1. Synchronous speed of the motor
2. The speed of the rotor when the slip is ~~0.04~~ 0.04.

Soln:

Given: $p = 6$

$F = 50 \text{ Hz}$

$s = 0.04$

1. Synchronous speed:

$$N_s = \frac{120F}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

2. Rotor speed:

$$s = \frac{N_s - N}{N_s}$$

$$\therefore 0.04 = \frac{1000 - N}{1000}$$

$$\therefore N = 1000 - 40$$

$$\boxed{N = 960 \text{ rpm}}$$

③ A particular load is to be driven at about 700 rpm. What should be the number of poles for a three phase induction motor when,

1. $F = 60 \text{ Hz}$ 2. $F = 50 \text{ Hz}$.

Calculate the actual speed in each case if the rated slip is 4%.

Soln:

Given: $N_s = 700 \text{ rpm}$
 $s = 0.04$ (i.e. 4%)

Find: 1. $p = ?$ if $F = 60 \text{ Hz}$, $N = ?$

2. $p = ?$ if $F = 50 \text{ Hz}$, $N = ?$

1. $p = ?$ if $F = 60 \text{ Hz}$, $N = ?$

$$N_s = \frac{120F}{p}$$

$$\therefore p = \frac{120F}{N_s} = \frac{120 \times 60}{700} = 10.28$$

$$\therefore p = 10.28 \approx \underline{10}$$

2. Actual speed $N = ?$

$$N_s = \frac{120F}{p} = \frac{120 \times 60}{10} = 720 \text{ rpm}$$

$$\therefore N = (1 - s) N_s$$

$$= (1 - 0.04) \times 720$$

$$\boxed{N = 691.2 \text{ rpm}}$$

2. $p = ?$ if $F = 50 \text{ Hz}$, $N = ?$

$$p = \frac{120F}{N_s} = \frac{120 \times 50}{700} = 8.57 \approx 8$$

$$\therefore N_s = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

Actual speed $N = ?$

$$N = (1-s) N_s \\ = (1-0.04) \times 750$$

$$\boxed{N = 720 \text{ rpm}}$$

- ⑨ A three phase 50 Hz induction motor runs at 960 rpm on full load. Find number of poles and percentage slip. state methods for measurement of slip. (4M, S-12)

Soln:

Given: $F = 50 \text{ Hz}$
 $N = 960 \text{ rpm}$

Find: p and $\%$ s

1. NO. OF poles $P :=$

Here N_s is not given, so Assume $N_s = 1000 \text{ rpm}$

$$\therefore N_s = \frac{120F}{P}$$

$$\therefore 1000 = \frac{120 \times 50}{P}$$

$$\therefore P = \frac{120 \times 50}{1000}$$

$$\therefore \boxed{P = 6}$$

2. $\%$ slip:

$$\% \text{ slip } (s) = \frac{N_s - N}{N_s} \times 100 = \frac{1000 - 960}{1000} \times 100 = 4 \%$$

* Equation of rotor induced emf, frequency, current, reactance, impedance and rotor emf under standstill and running condition:

* Equation of rotor induced emf Frequency: (f_r) (slip frequency)

A three phase voltage is induced in the rotor winding, due to the relative velocity between rotor and rotating magnetic field (RMF). The frequency of this voltage is dependent on slip s .

$$\boxed{f_r = s F}$$

where f_r = rotor frequency.

F = stator frequency.

$f_r = F$ when $s = 1$ i.e. when $N = 0$, i.e. at standstill condition (stationary condition)

$f_r = 0$ when $s = 0$ when $N = N_s$.

As the frequency of the rotor induced voltage is proportional to slip, it is called as slip frequency.

* Induced emf in the rotor:

(a) At standstill ($N = 0$):

The rotor induced emf per phase at standstill (stationary) is given by,

$$\boxed{E_2 = \frac{N_2}{N_1} E_1}$$

where N_2 = No. of rotor turns per phase

N_1 = No. of stator turns per phase

E_1 = stator voltage per phase.

This equation is similar to that of transformer where the secondary voltage is proportional to the turns ratio of the windings. Hence, induction motor is therefore known as the rotating transformer.

(b) Induced voltage under running conditions (E_{2r}):

When the rotor accelerates, the relative velocity between the PMF, and the rotor decreases. This will reduce the induced voltage in the rotor winding. The rotor induced voltage per phase under running condition is given by,

E_{2r} = Rotor induced voltage per phase in running condition.

But $E_{2r} \propto (N_s - N)$ i.e. the relative velocity &

$E_2 \propto N_s$ i.e. synchronous speed.

$$\therefore \frac{E_{2r}}{E_2} = \frac{(N_s - N)}{N_s}$$

$$\therefore \frac{E_{2r}}{E_2} = s$$

$$\therefore \boxed{E_{2r} = s E_2}$$

Where, E_2 = Induced rotor voltage per phase at standstill.

* Rotor Resistance:

Let R_2 be the rotor resistance per phase at standstill and

let f_r be the frequency of rotor emf.

There is no effect of rotor frequency on the rotor resistance.

Hence the rotor resistance remains constant at R_2 Ω /phase irrespective of the speed of the I.M.

* Rotor Reactance:

Let X_2 be the rotor reactance per phase at standstill. The rotor frequency at standstill is $F_r = F$.

$$\therefore X_2 = 2\pi F L_2 \text{ } \Omega/\text{phase} \quad \text{Where } F = \text{stator frequency.}$$

In the running condition the frequency of rotor voltage is $F_r = sF$. Hence rotor reactance in the running condition X_{2r} is given by,

$$X_{2r} = 2\pi F_r L_2$$

$$= 2\pi sF L_2$$

$$= s (2\pi F L_2)$$

$$= s X_2$$

$$\boxed{\therefore X_{2r} = s X_2}$$

This indicates that the rotor reactance is directly proportional to slip s . It is maximum at standstill ($s=1$) and reduces with increase in speed (due to reduction in s).

* Rotor Impedance:

(a) At standstill:

The rotor impedance per phase at standstill is given by,

$$Z_2 = \sqrt{(R_2^2 + X_2^2)} \text{ } \Omega/\text{phase}$$

Where, R_2 = Rotor resistance per phase and

X_2 = Leakage reactance of the rotor winding per phase at standstill.

(b) Under running conditions:

The rotor impedance under running condition is given by,

$$Z_{2r} = \sqrt{(R_2^2 + s^2 X_2^2)} \text{ 2-phase}$$

As the frequency of rotor induced voltage (E_r) is dependent on slip s ; the leakage reactance of the rotor is also depend on slip. Therefore, the rotor impedance will change with change in speed.

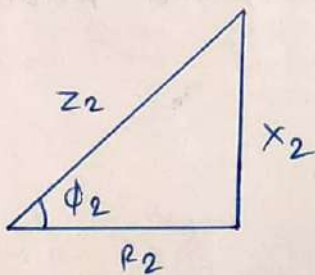
* Rotor power factor:

(a) At standstill:

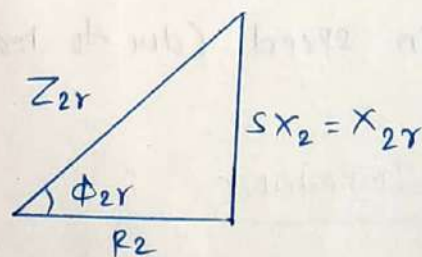
The rotor power factor at stand is given by,

$$\cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

The impedance triangle at standstill is shown in fig.



(a) For rotor at standstill



(b) Under running condition.

(b) P.F. under running condition:

The rotor P.F. under running condition is given by,

$$\cos \phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

* Rotor current:

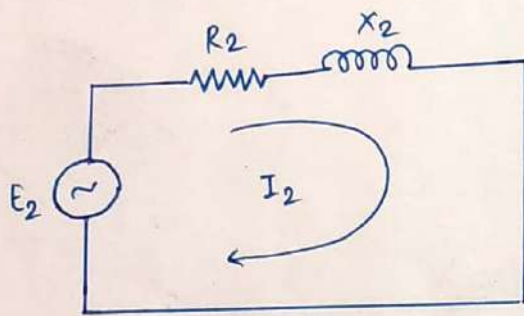
The rotor current magnitude is equal to the ratio of induced rotor voltage per phase and the rotor impedance.

(a) Rotor current at standstill:

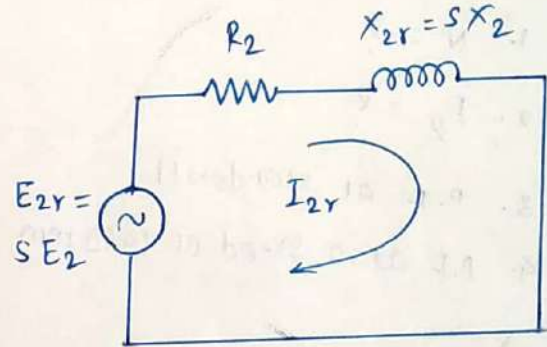
The rotor current at standstill is given by the following equation,

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

The equivalent circuit of the rotor at standstill & running condition is as shown in fig:



(a) At standstill



(b) Under running condition.

Fig: Rotor equivalent circuits.

(b) Rotor current under running conditions:

The rotor current under running conditions is given by

$$I_{2r} = \frac{E_{2r}}{Z_{2r}} = \frac{sE_2}{\sqrt{R_2^2 + s^2 X_2^2}}$$

or

$$I_{2r} = \frac{E_2}{[(R_2^2/s^2) + X_2^2]^{1/2}}$$

* Numerical:

① A 3 ϕ 50Hz 4 pole induction motor has a slip of 4%.

(calculate: 1. Speed of motor

2. Frequency of rotor emf: if the rotor has a resistance of 1Ω and standstill reactance of 4Ω , calculate the rotor p.f. at standstill & at a speed of 1440 rpm.

~~3. At~~

Soln:

Given:

$$P = 4$$

$$X_2 = 4\Omega$$

$$F = 50\text{Hz}$$

$$R_2 = 1\Omega$$

$$S = 4\% = 0.04$$

Find:

1. $N = ?$

2. $F_2 = ?$

3. P.F. at standstill

4. P.F. at a speed of 1440 rpm.

1. $N = ?$

$$\text{Synchronous speed } N_s = \frac{120F}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

2. Speed of motor (N) = $N_s - S N_s = 1500 - (0.04 \times 1500) = 1440 \text{ rpm}$

2. Frequency of rotor emf:

$$F_2 = S F_1 = 0.04 \times 50 = 2 \text{ Hz}$$

3. P.F. i) At standstill

$$\cos \phi_2 = \frac{R_2}{\sqrt{R_2^2 + X_2^2}} = \frac{1\Omega}{5\Omega} = 0.2 \text{ (lag)}$$

ii) Under running cond:

$$N = 1440 \text{ rpm} \therefore S = 0.04$$

$$X_{2r} = S X_2 = 0.04 \times 4 = 0.16\Omega$$

$$\begin{aligned} \therefore \cos \phi_{2r} &= \frac{R_2}{\sqrt{R_2^2 + X_{2r}^2}} \\ &= \frac{1}{\sqrt{(1)^2 + (0.16)^2}} \\ \cos \phi_{2r} &= 0.9874 \text{ (lag)} \end{aligned}$$

② A 3 ϕ Induction motor is wound for 4 poles and is supplied

From 50 Hz supply. calculate:

1. The synchronous speed
2. Speed of rotor at 4% slip
3. The rotor frequency when rotor runs at 1000 rpm.

Soln:

Given: $P = 4$

$f_s = 50 \text{ Hz}$

$S = 4\% = 0.04$

Find: 1) $N_s = ?$ 2) $N = ?$ at $S = 4\%$ 3) $F_{2r} = ?$ when $N = 1000 \text{ rpm}$.

1. Synchronous speed:

$$N_s = \frac{120f_s}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

2. Speed at $S = 4\%$:

$$N = (1 - S)N_s = (1 - 0.04) \times 1500 \\ = 1440 \text{ rpm}$$

3. Rotor frequency at $N = 1000 \text{ rpm}$:

$$S = \frac{N_s - N}{N_s} = \frac{1500 - 1000}{1500} = 0.33$$

$\therefore$ Rotor frequency $F_{2r} = SF = 0.33 \times 50$

$$\boxed{F_{2r} = 16.67 \text{ Hz}}$$

* starting and running torque equation of squirrel cage & slip ring Induction Motor:

* Torque Equation of Induction Motor: (W-14) (4M)

The torque produced by an induction motor depends on the following three factors:

1. The rotor power factor under the running condition.
2. The rotor current under the running condition
3. The part of rotating magnetic field which induces emf in the rotor winding.

So, we can express the torque mathematically as,

$$T \propto \phi I_{2r} \cos \phi_{2r} \quad \text{--- (1)}$$

where ϕ = Flux that induces emf in the rotor

I_{2r} = Rotor current under running condition

$\cos \phi_{2r}$ = Rotor power factor under running condition.

But ϕ is proportional to stator voltage E_1 ,

$$\boxed{\therefore \phi \propto E_1}$$

But $\frac{E_2}{E_1} = K = \text{Transformation ratio}$

$\therefore E_2 \propto E_1$ and hence $E_2 \propto \phi$

Hence ϕ in eqn (1) can be replaced by E_2 ,

We have seen that

$$I_{2r} = \frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} \quad \text{--- (2)}$$

(14)

$$\text{and, } \cos \phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}} \quad \text{--- (3)}$$

substituting eqn (2) and (3) into eqn (1), we get

$$T \propto E_2 \times \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \cdot \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$T \propto \frac{s \cdot E_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N.m}$$

this is called as torque equation of the induction motor.
Using the constant of proportionality K we can write the torque equation of an induction motor as follows:

$$T = \frac{K s E_2^2 R_2}{R_2^2 + (sX_2)^2} \quad \text{--- (4)}$$

* Full load torque:

If s corresponds to the value of full load slip, then eqn (4) represents the full load torque.

It can be shown that the value of K is

$$K = \frac{3}{2\pi N_s'} \quad \text{Where } N_s' = \text{Synchronous speed in Rps} \\ = \frac{N_s}{60}$$

Substituting this value of K , the torque equation can be obtained as follows:

$$T = \frac{3}{2\pi N_s'} \cdot \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ N.m} \quad \text{--- (5)}$$

* Starting torque:

starting torque is defined as the torque produced by the induction motor when the motor is being started.

starting torque is denoted by T_{st} and corresponds to $N=0$ or $s=1$.
So by substituting $s=1$ in eqn (5), we get the expression for starting torque as,

$$T_{st} = \frac{3}{2\pi Ns'} \cdot \frac{E_2^2 R_2}{R_2^2 + (X_2)^2} \quad \text{--- (5)}$$

This expression indicates that the starting torque is directly proportional to the rotor resistance R_2 .

* Condition For maximum Torque: (S-15)(4M)

The expression for the torque is,

$$T = \frac{KsE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

This shows that the value of torque depends on the value of slip 's' under the running condition.

The maximum torque can be obtained by differentiating the expression for torque with respect to 's' and then equating the derivative to zero.

$$\text{i.e. } \frac{dT}{ds} = 0$$

Differentiating the equation for torque T with respect to 's'

we get,

$$\therefore \frac{dT}{ds} = \frac{(R_2^2 + s^2 X_2^2) K E_2^2 R_2 - s K E_2^2 R_2 (2s X_2^2)}{(R_2^2 + s^2 X_2^2)^2} = 0$$

$$\therefore R_2^2 + s^2 X_2^2 = S(2S X_2^2) = 2S^2 X_2^2$$

$$\therefore R_2^2 - S^2 X_2^2 = 0$$

$$\therefore S^2 = \frac{R_2^2}{X_2^2}$$

$$\therefore S = \left(\frac{R_2}{X_2} \right)$$

This is the value of slip which corresponds to maximum torque T_m . Hence it is denoted by S_m .

$$\boxed{S_m = \frac{R_2}{X_2}}$$

* Expression for Maximum Torque (T_m):

The expression for maximum torque can be obtained by substituting the expression for S_m into the expression for load torque equation below:

$$T = \frac{K S E_2^2 R_2}{R_2^2 + (S X_2)^2}$$

$$\therefore T_m = \frac{K S_m E_2^2 R_2}{R_2^2 + (S X_2)^2}$$

$$\text{but } S_m = \frac{R_2}{X_2}$$

Substituting this value of slip in the above expression, we get,

$$T_m = \frac{K (R_2 / X_2) E_2^2 R_2}{R_2^2 + (R_2 / X_2)^2 X_2^2} = \frac{K E_2^2}{2 X_2}$$

$$\boxed{T_m = \frac{K E_2^2}{2 X_2}} \quad \text{--- (1)}$$

Equation ① indicates that the maximum torque T_m does not depend on the value of rotor resistance R_2 .

Maximum torque is directly proportional to the square of rotor voltage E_2^2 .

T_m is inversely proportional to the rotor reactance at standstill i.e. X_2 . The maximum torque T_m is independent of R_2 , but the value of slip s_m at which T_m occurs is directly proportional to the rotor resistance R_2 .

* Torque slip characteristics of Induction motor (W-15).

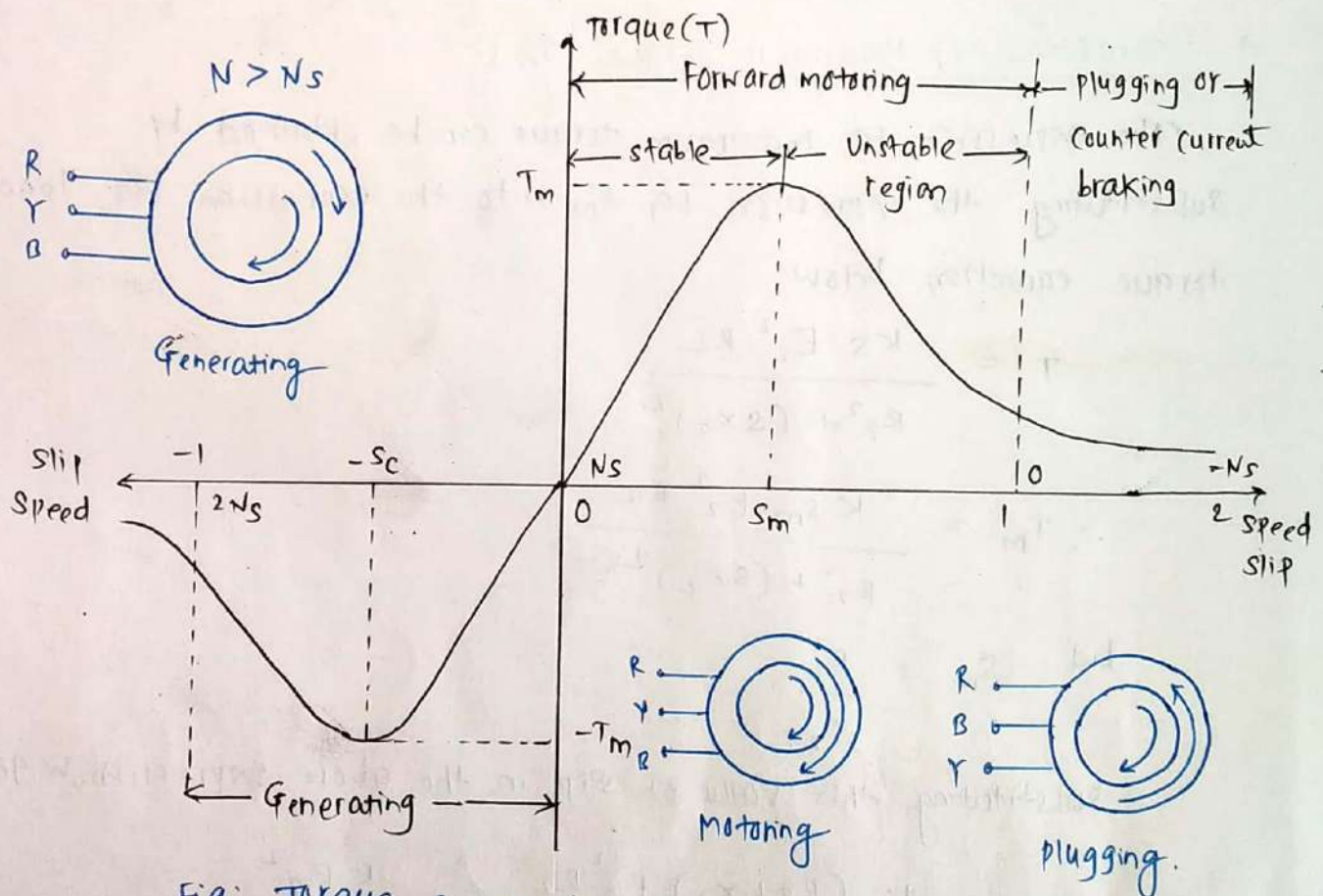


Fig: Torque-speed characteristics of induction motor.

The torque slip characteristic of an induction motor can be divided into three sections:

1. Forward motoring
2. plugging
3. Regeneration

1. Forward motoring:

The forward motoring region corresponds to the values of slip betn 0 and 1. In forward motoring region of the characteristics shown in fig. the motor rotates in the same direction as that of rotating magnetic field (RMF).

The torque produced by the motor is zero at synchronous speed or for $s=0$. This is because the induced voltage in rotor is zero where $N=N_s$.

The torque increases as the slip increases while the air gap flux remains constant. Once the torque reaches its maximum value T_{max} at the critical slip $s = s_m$, the torque decreases with increase in slip due to reduction in air gap flux.

Forward motoring can be divided into two parts:

1. stable region extending from $s=0$ to $s=s_m$
2. Unstable region extending from $s=s_m$ to $s=1$.

1. stable region ($s=0$ to $s=s_m$):

The general expression for the motor torque is as follows:

$$T \propto \frac{s R_2}{R_2^2 + (s X_2)^2}$$

In the stable region, the value of slip is small. Hence this region is also called as the low slip region.

As 's' is small, $(s X_2)^2$ becomes negligible as compared to R_2 .

Hence the expression for torque gets modified as,

$$T \propto \frac{s R_2}{R_2^2} \quad \therefore \boxed{T \propto \frac{s}{R_2}}$$

But R_2 is constant. Hence

$$\boxed{T \propto s} \quad \dots \dots \text{For the low slip region.}$$

thus in the low slip region, the torque is directly proportional to the slip 's'. Hence as load increase, speed decreases, slip increases which in turn increases the torque in a linear manner. Therefore in the low slip region the torque increases linearly with increase in slip. At $s=0$, $T=0$.

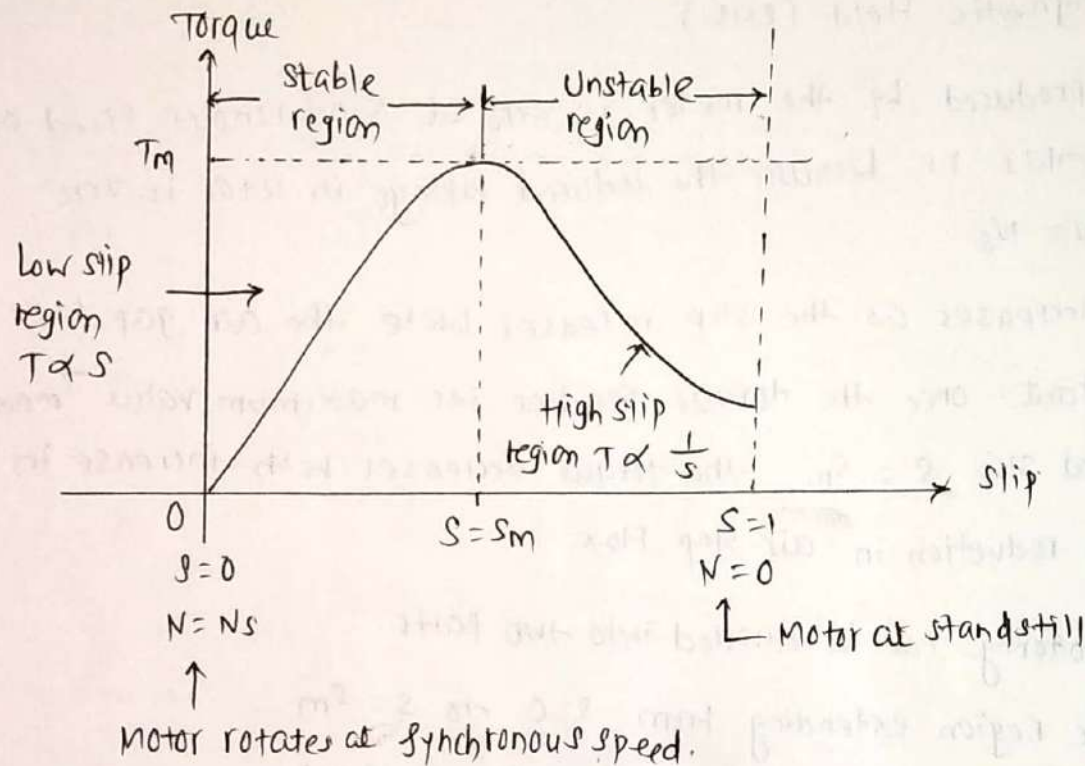


Fig: Forward motoring region of Torque-slip characteristics.

2. Unstable region: ($s_m < s \leq 1$):

In the unstable region extending from $s=s_m$ to $s=1$, the value of 's' is large so this region is also called as the high slip region. As 's' is large, $(sX_2)^2$ is much larger than R_2^2 . Hence we can neglect R_2^2 to get modified expression for torque as follows:

$$T \propto \frac{s R_2}{(s X_2)^2} \propto \frac{R_2}{s X_2^2}$$

but $\left(\frac{R_2}{X_2^2}\right)$ is a constant. Hence,

$$\boxed{T \propto \frac{1}{s}}$$

Thus in the high slip region the motor torque is inversely proportional to slip ' s '. Hence torque decreases with increase in slip as shown in above fig. and the reduction in torque is non-linear.

* Pull out Torque or Breakdown Torque:

The maximum torque T_m obtained at $s = s_m$ is called as the pull out torque or breakdown torque.

If the load torque increases beyond the pull out torque then the induction motor will be pushed into the unstable region and will finally come to a standstill i.e. stop rotating.

* Generating region:

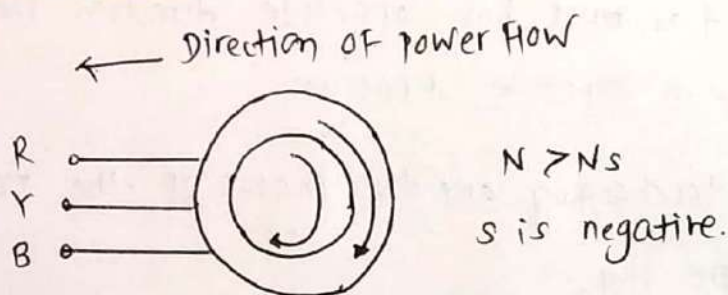


Fig: Generating mode

For the generating region, the slip needs to be negative and between 0 and -1 as shown in torque slip characteristics.

The slip will be negative if and only if the rotor speed N is higher than the synchronous speed N_s . However rotor and rotating magnetic field both rotate in the same direction.

The torque produced is in the opposite direction to that of motoring mode so it is shown to be negative.

In this region, the motor acts as a generator and returns the power back to the ac source as shown in fig. Maximum torque in the generating mode is obtained at slip $s = -s_m$.

→ Sometimes the ratio $\left(\frac{R_2}{X_2}\right)$ is substituted as 'a' instead of s_m .

Hence the ratio is given by,

$$\boxed{\frac{T_{st}}{T_m} = \frac{2a}{1+a^2}}$$

* Ratio of Full load Torque and Maximum Torque: (s-1s)

Taking ratio of full load torque and maximum torque, we get,

$$\begin{aligned}\frac{T_{FL}}{T_m} &= \frac{K s E_2^2 R_2}{R_2^2 + (s X_2)^2} \\ &= \frac{K s E_2^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{2 X_2}{K E_2^2} \\ &= \frac{2 s R_2 X_2}{R_2^2 + (s X_2)^2}\end{aligned}$$

Divide numerator and denominator by X_2^2 to get,

$$\frac{T_{FL}}{T_m} = \frac{2s \left(\frac{R_2}{X_2}\right)}{\left(\frac{R_2}{X_2}\right)^2 + s^2}$$

But $(R_2/X_2) = s_m$

$$\boxed{\frac{T_{FL}}{T_m} = \frac{(2 s_m s)}{s_m^2 + s^2}}$$

Sometimes (R_2/X_2) is substituted as 'a' instead of s_m , then

$$\boxed{\frac{T_{FL}}{T_m} = \frac{2 a s}{a^2 + s^2}}$$

→ When motor develops maximum torque, the slip is s_m equal to (R_2/X_2) and the torque given as, (18)

$$\therefore T_m = \frac{KE_2^2}{2X_2}$$

→ At any general working point such as full load, if s is the slip then,

$$T = \frac{KsE_2^2R_2}{R_2^2 + s^2X_2^2}$$

* starting Torque to maximum Torque Ratio:

Substituting the values of T_{st} and T_m , we get,

$$\frac{T_{st}}{T_m} = \frac{KE_2^2R_2}{R_2^2 + X_2^2} \div \frac{KE_2^2}{2X_2}$$

$$= \frac{KE_2^2R_2}{R_2^2 + X_2^2} \times \frac{2X_2}{KE_2^2}$$

$$= \frac{R_2}{R_2^2 + X_2^2} \cdot 2X_2$$

Dividing both numerator and denominator by X_2^2 .

$$\frac{T_{st}}{T_m} = \frac{2R_2/X_2}{\left[\frac{R_2}{X_2}\right]^2 + 1} = \frac{2s_m}{1 + (s_m)^2}$$

$$\therefore \boxed{\frac{T_{st}}{T_m} = \frac{2s_m}{1 + s_m^2}}$$

$$\text{Where } s_m = \frac{R_2}{X_2}$$

→ Sometimes the ratio $\left(\frac{R_2}{X_2}\right)$ is substituted as 'a' instead of s_m .

Hence the ratio is given by,

$$\boxed{\frac{T_{st}}{T_m} = \frac{2a}{1+a^2}}$$

* Ratio of Full load Torque and maximum Torque: (s-1s)

Taking ratio of Full load torque and maximum torque, we get,

$$\begin{aligned}\frac{T_{FL}}{T_m} &= \frac{K s E_2^2 R_2}{R_2^2 + (s X_2)^2} \\ &= \frac{K s E_2^2 R_2}{R_2^2 + (s X_2)^2} \times \frac{2 X_2}{K E_2^2} \\ &= \frac{2 s R_2 X_2}{R_2^2 + (s X_2)^2}\end{aligned}$$

Divide numerator and denominator by X_2^2 to get,

$$\frac{T_{FL}}{T_m} = \frac{2 s \left(\frac{R_2}{X_2}\right)}{\left(\frac{R_2}{X_2}\right)^2 + s^2}$$

But $(R_2/X_2) = s_m$

$$\boxed{\frac{T_{FL}}{T_m} = \frac{(2 s_m s)}{s_m^2 + s^2}}$$

Sometimes (R_2/X_2) is substituted as 'a' instead of s_m , then

$$\boxed{\frac{T_{FL}}{T_m} = \frac{2 a s}{a^2 + s^2}}$$

* Numericals:

- ① A 400V, 40 Hz, 3 ϕ slip ring I.M. has a rotor resistance of 0.1 Ω and stand still reactance of 0.5 Ω per phase with the rotor circuit star connected. The full load slip is 4%. Calculate the extra resistance to be connected in the rotor circuit to achieve maximum torque at 50% slip.

Soln:

Given: $f_1 = 40 \text{ Hz},$
 $R_2 = 0.1 \Omega$
 $X_2 = 0.5 \Omega$
 $s_{FL} = 4\% = 0.04$

Find: External rotor resistance R to achieve maximum torque at 50% slip.

It is given that $s_m = 50\%$ or 0.5

$$s_m = \frac{(R_2 + R)}{X_2} \quad \text{Where } R = \text{External rotor resistance.}$$

$$\therefore 0.5 = \frac{0.1 + R}{0.5}$$

$$\therefore R = 0.15 \Omega$$

- ② A 12 pole, 50 Hz, 3 ϕ I.M. has rotor resistance of 0.15 Ω and standstill reactance of 0.25 Ω per phase. On full load it is running at a speed of 480 rpm. The rotor induced emf per phase at standstill is observed to be 32V. Calculate:
1. Starting torque
 2. F.L. Torque
 3. Max. torque.
 4. Speed at maximum torque.

(W-14, 4M)

Soln: Given: $P = 12, \quad R_2 = 0.15 \Omega$
 $f = 50 \text{ Hz}, \quad X_2 = 0.25 \Omega$

$$N_{FL} = 480 \text{ rpm}$$

$$E_2 = 32 \text{ V}$$

Find: i) $T_{st} = ?$ ii) $T_{FL} = ?$ iii) $T_m = ?$ iv) $N = ?$ at T_m .

→ Synchronous speed:

$$N_s = \frac{120F}{P} = \frac{120 \times 50}{12} = 500 \text{ rpm.}$$

$$\therefore N_s' = \frac{N_s}{60} = \frac{500}{60} = 8.33 \text{ rps}$$

→ starting torque:

$$T_{st} = \frac{3}{2\pi N_s'} \cdot \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$
$$= \frac{3}{2\pi \times 8.33} \times \frac{(32)^2 \times 0.15}{(0.15)^2 + (0.25)^2}$$

$$\boxed{T_{st} = 103.53 \text{ N-m}}$$

→ Full load torque:

$$T_{FL} = \frac{3}{2\pi N_s'} \cdot \frac{s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

$$s = \frac{N_s - N}{N_s} = \frac{500 - 480}{500} = 0.04$$

$$\therefore T_{FL} = \frac{3}{2\pi \times 8.33} \times \frac{(0.04) \times (32)^2 \times 0.15}{(0.15)^2 + (0.04 \times 0.25)^2}$$

$$\boxed{T_{FL} = 15.58 \text{ N-m}}$$

→ Maximum torque:

$$(M.D., P.T. \& W) \quad T_m = \frac{3}{2\pi N_s'} \times \frac{E_2^2}{2X_2}$$
$$= \frac{3}{2\pi \times 8.33} \times \frac{(32)^2}{2 \times 0.25} = 117.38 \text{ N-m.}$$

→ Speed at maximum torque:

slip at maximum torque

$$s_m = \frac{R_2}{X_2} = \frac{0.15}{0.25} = 0.6$$

$$\therefore N = (1 - s_m) N_s = (1 - 0.6) \times 500$$

$$\therefore N = 200 \text{ rpm}$$

* Effect of change in supply voltage on torque-speed characteristics:

(S-15, W-15.4M)

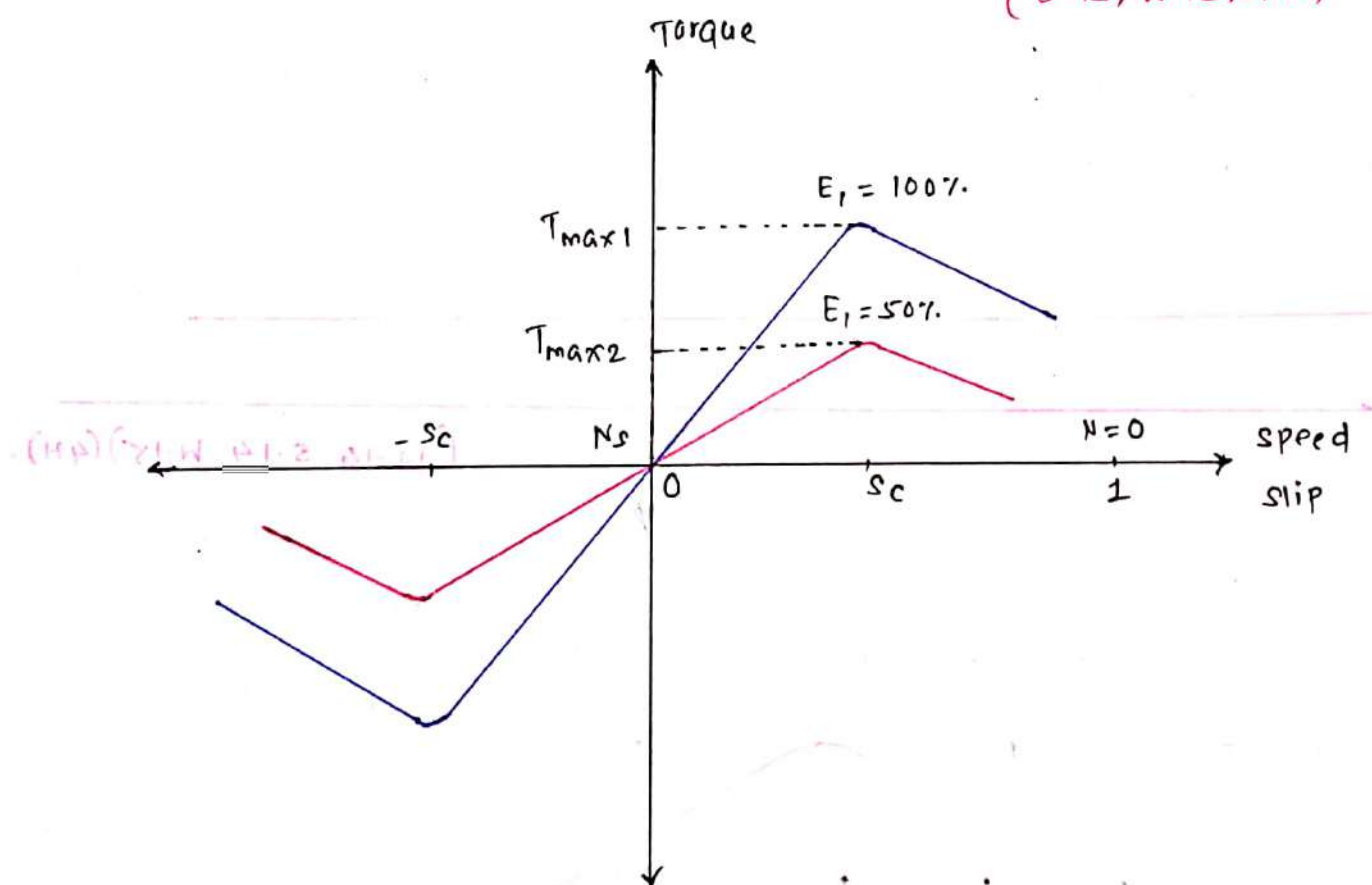


Fig. Effect of change in supply voltage on the torque speed characteristics.

The torque equation of induction motor is given by

$$T = \frac{K s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

In this expression E_2 the rotor induced emf is proportional to E_1 i.e. the stator supply voltage.

$$\therefore T \propto s E_1^2$$

Thus torque at any speed is proportional to square of the stator supply voltage.

The change in stator supply voltage will have an effect on the running torque, and maximum torque. This effect is shown in the torque speed characteristics shown in above fig. It shows that if we keep slip s constant and vary only the stator voltage E_1 , then the torque-slip characteristics gets shifted as shown.

The principle of change in stator supply voltage is used in the speed control of induction motors.

* Effect of change in Rotor Resistance on torque developed: or

Effect of change in Rotor Resistance on torque speed characteristics:

(W-14, S-14, W-15) (4M).

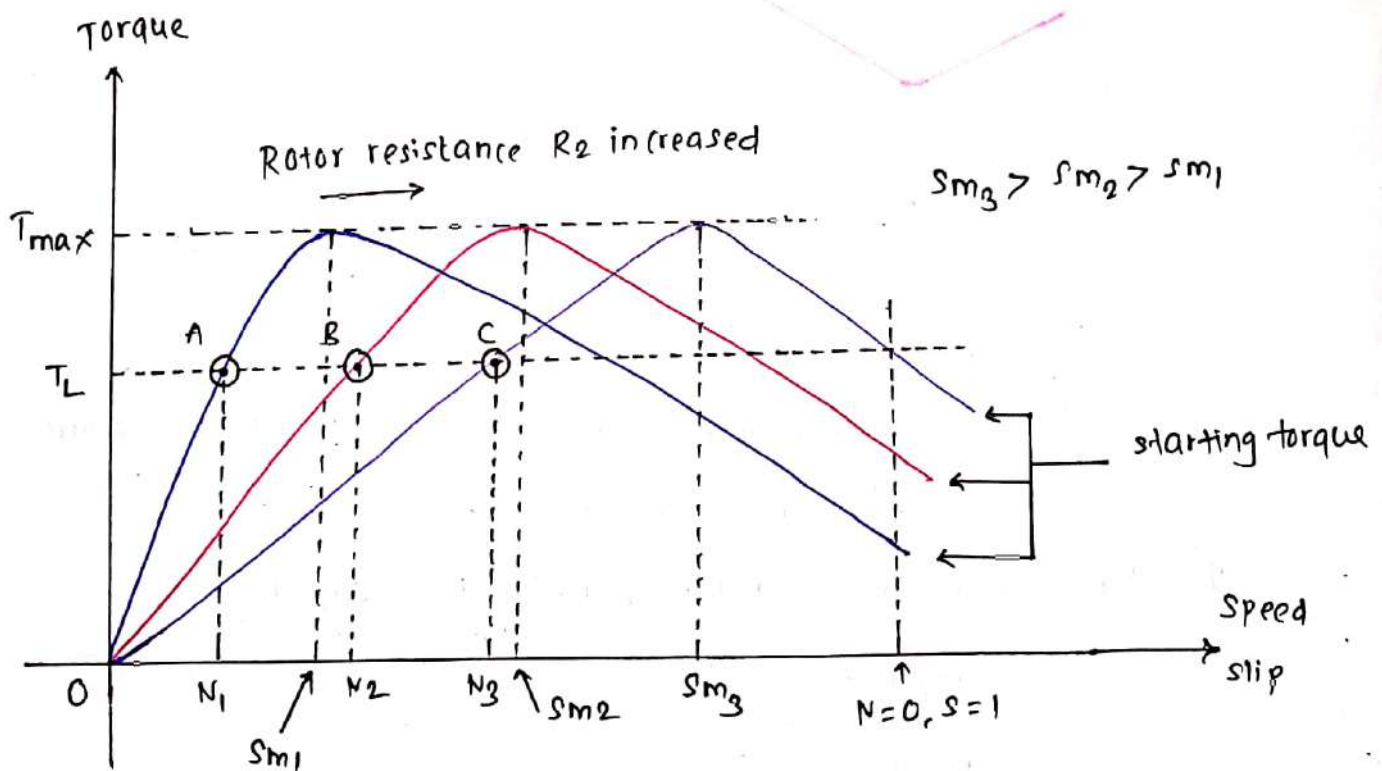


Fig: Torque speed characteristics with variable rotor resistance.

The effect of change in rotor resistance on torque speed char is as shown in fig.

As we know, the value of s_m i.e. the slip at which the motor produces maximum torque is proportional to the rotor resistance. Because $s_m = R_2 / X_2$, its value increases with increase in R_2 . Thus with increase in R_2 , $s_{m3} > s_{m2} > s_{m1}$, as shown in above fig.

The maximum torque however remains unaffected. Therefore for a constant load torque T_L , the motor operating point shifts from A to B to C with change in rotor resistance.

The torque speed characteristics shifts towards right with increase in rotor resistance R_2 as shown in fig.

The starting torque is given by,

$$T_{st} = \frac{K E_2^2 R_2}{R_2^2 + (X_2)^2}$$

so, it is directly proportional to R_2 . The starting torque increases with increase in rotor resistance R_2 as shown in fig. If we want T_{st} to be equal to T_m (maximum torque), then R_2 should be adjusted to be equal to X_2 .

* Measurement of slip:

There are following methods to measure the slip of an induction motor:

1. Actual measurement of speed (Tachometer method)
2. By Comparing rotor frequency and stator frequency.
3. Stroboscopic method.

* Tachometer method: (Actual speed measurement):

In this method, First we calculate Synchronous Speed N_s as,

$$N_s = \frac{120F}{P}$$

Then, we measure actual speed N with the help of a tachometer. calculate the percentage slip using following expression:

$$\% \text{ Slip } s = \frac{N_s - N}{N_s} \times 100$$

* By comparing Rotor Frequency and stator Frequency:

We know that the stator Frequency F_1 is 50 Hz. Whereas the frequency of rotor induced emf F_2 depends on slip as given by the following expression:

$$F_2 = sF_1, \quad \dots \quad s = \text{slip}$$

$$F_1 = 50 \text{ Hz} \dots \text{stator supply Frequency.}$$

Therefore by measuring F_2 , we can calculate the slip 's'.

Here, the value of F_2 is generally so small, that we can count the no. of cycles of rotor induced emf by using a d.c. moving coil millivoltmeter with a central zero.

→ Measurement of F_2 For a slip ring Induction motor:

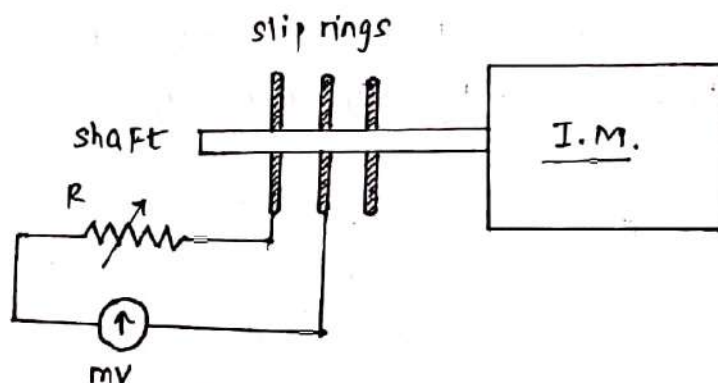


Fig: Measurement of F_2 For a slip ring motor.
(a)

In this case of slip ring I.M., the leads (contacts) of millivoltmeter are lightly pressed (connected) against the adjacent slip rings as they revolve (rotate).

The current starts flowing through the millivoltmeter. This current follows the variations in the instantaneous value of the rotor current. Due to this low frequency rotor current, the pointer of millivoltmeter oscillates about its central zero position as shown in fig. (B)

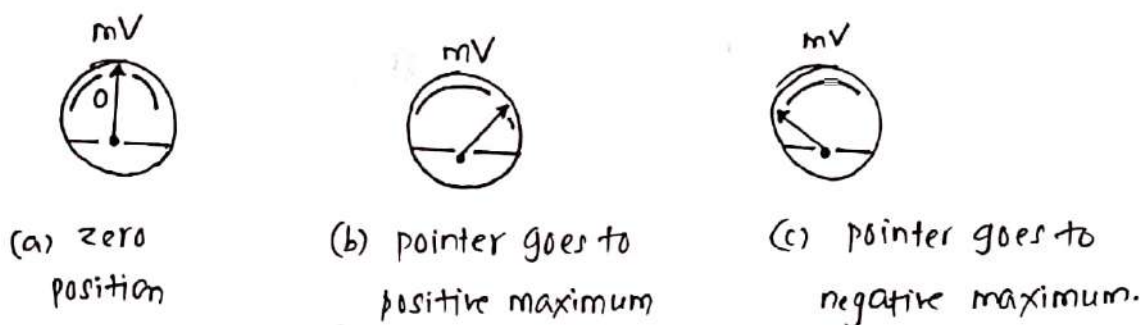


Fig: (B) pointer movement of the millivoltmeter.

Initially, the pointer is at center i.e. zero position. Then it goes to positive maximum, returns back to zero, goes to negative maximum & return back to zero against to complete the cycle.

Count the no. of cycles of pointer oscillations per second. This will be equal to the rotor frequency F_2 .

Calculate slip using the following expression

$$S = \frac{F_2}{F_1}$$

→ Measurement of F_2 For squirrel cage motor:

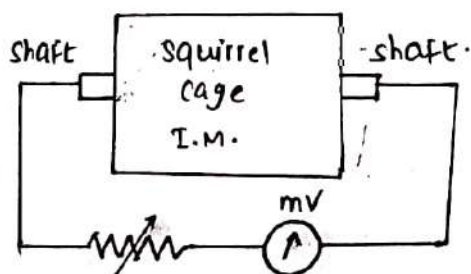


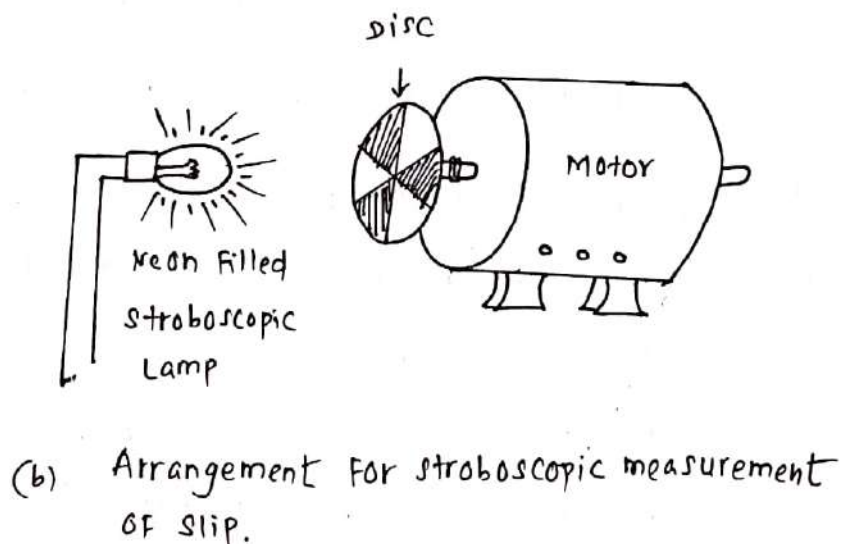
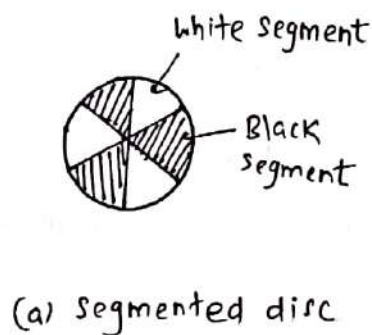
Fig: Measurement of F_2 For a squirrel cage I.M.

In squirrel cage I.M. slip rings are absent. so it is not possible to connect the millivoltmeter directly as we did for slip ring I.M. but it is sometimes possible to pick up some rotor induced voltage if we connect the millivoltmeter across the ends of the motor shaft as shown in above fig.

Then measure the Frequency of rotor EMF F_2 as in previous method and calculate s as

$$s = \frac{F_2}{F_1}$$

* stroboscopic method:



In this method, we take a circular disc and paint it with alternate black and white segments as shown in fig. (a)

The total no. of segments (white plus black) should be equal to the no. of poles (P) of the motor. For a 4 pole motor the total no. of segments will be 4 (2 black and 2 white) as shown in Fig. (a).

Then, we have to mount this disc on the shaft of the motor as shown in fig. (b). So the disc will rotate at the same speed of the motor. This rotating disc is then illuminated by a neon filled stroboscopic lamp from one side as shown in fig. (b).

We can connect this lamp either to a combined ac plus dc supply as shown in fig. (c) or only to an ac supply as shown in fig. (d).

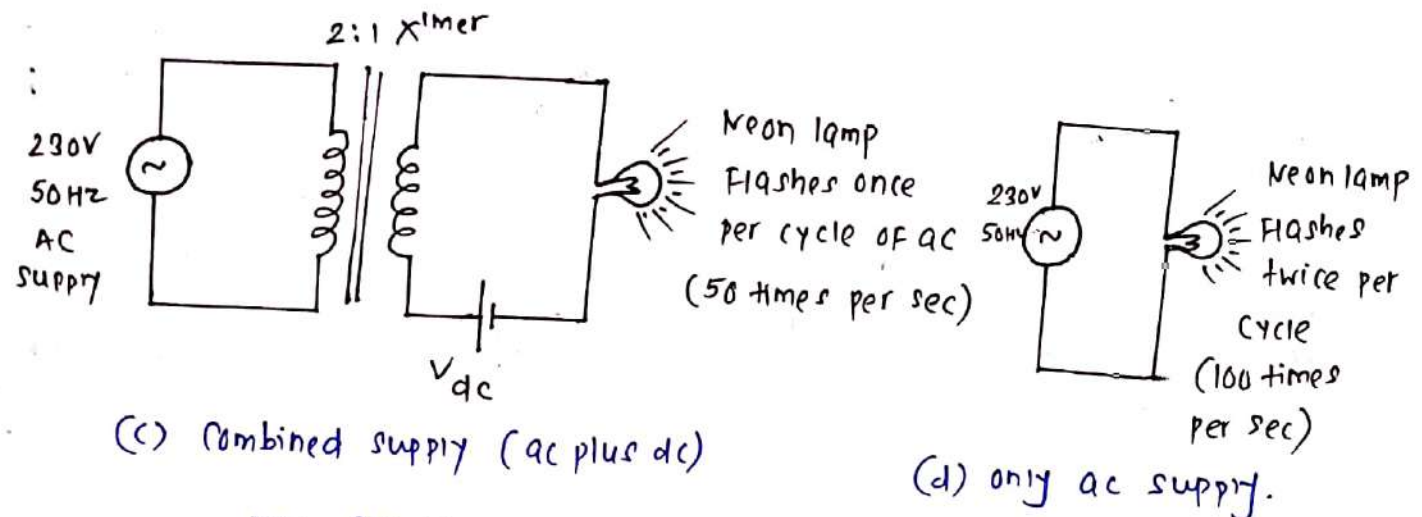


Fig: Supply connections for neon stroboscopic Lamp.

The lamp will flash only once per cycle of AC (i.e. 50 times per second) if the combined supply is used. But it will flash twice per cycle (i.e. 100 times per second) if only AC supply is used. The flashes of neon lamp will fall on the rotating disc. If the speed of the disc is equal to synchronous speed N_s then the disc would appear stationary. But if the speed is less than N_s then the disc appears to rotate slowly backwards.

Count the no. of white segments passing a fixed point per minute and divide it by the no. of lines seen. (2 in case of 4 pole motor). Find the apparent backward speed in rpm and calculate slip as follows:

$$N_s = \frac{120F}{P} \quad \% \text{ Slip } S = \frac{N_s - N}{N_s} \times 100$$

* Losses in Induction motor:

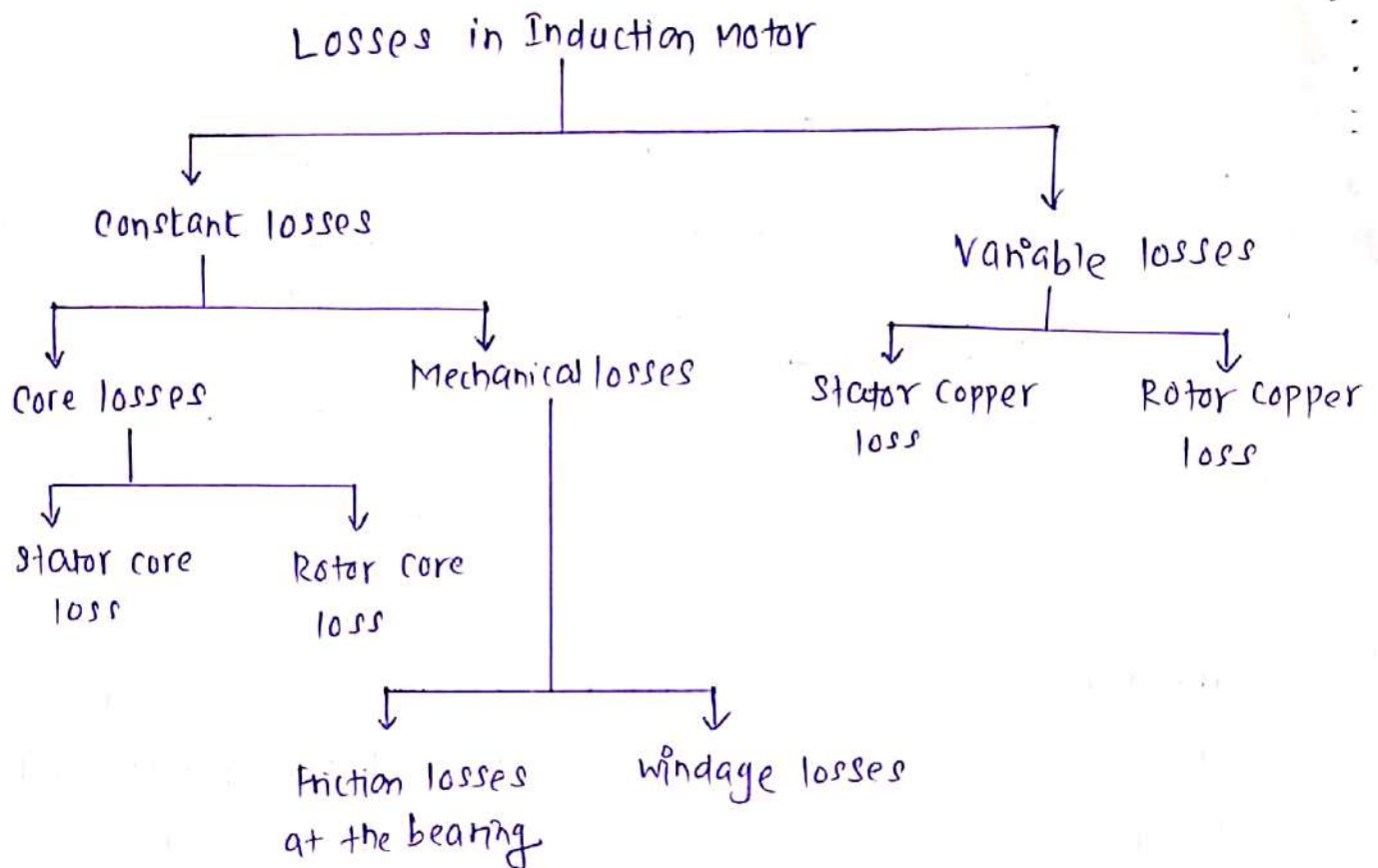


Fig: Classification of losses in induction motor.

* Variable Losses:

Variable losses are basically the copper losses which include the stator copper loss and rotor copper loss. The stator copper loss is proportional to the square of stator current respectively and rotor copper loss is proportional to the square of rotor current.

As these currents are dependent on the load, the copper losses vary with change in load. Hence they are called as variable losses.

* Constant losses: i) Core losses:

- The core losses are also called as the iron losses and they include the stator core loss and the rotor core loss.
- The core losses are made of hysteresis loss and eddy current losses.
- We can reduce the hysteresis losses by selecting a high permeability material for the core. The eddy current losses can be reduced by using the laminated cores instead of solid cores.

(ii) Mechanical losses:

They are constant losses which include the Friction and windage losses.

* power stages of 3 ϕ Induction motor:

The power Flow diagram for a three phase induction motor is as follows:

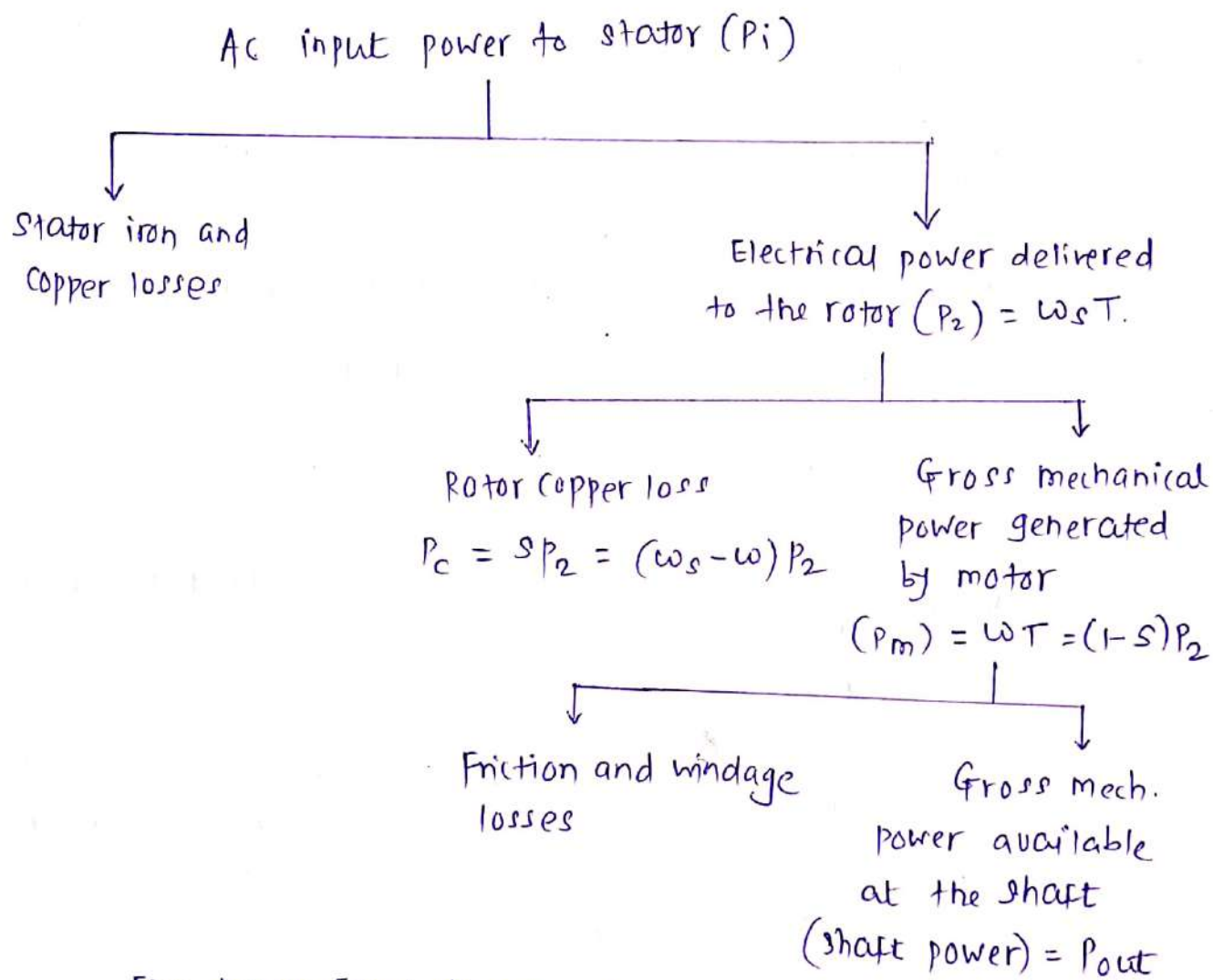


Fig: power Flow diagram For induction motor.

From the above fig. performance parameters of a three phase motor can be determined as follows:

The total power supplied to the stator winding of the motor is,

$$P_i = 3 E_1 I_1 \cos \phi_1$$

①

Where ϕ_1 is the angle between E_1 and I_1 . The input power is also the sum of power delivered to motor rotor (P_2), Core loss (P_{core}) and the stator copper loss (P_{cu}).

$$\therefore P_i = P_2 + (P_{core} + P_{cu}) \quad \text{--- (2)}$$

The electrical power delivered to the rotor is also known as the gap power (power passing from stator to rotor through the air gap). It is given by

$$P_{ag} = P_2 = \omega_s T = \frac{2\pi N_s}{60} \cdot T \quad \text{--- (3)}$$

Where ω_s = Angular synchronous speed rad/sec &

T = Torque produced by the motor.

$$\boxed{P_2 = P_m + P_c} \quad \text{--- (4)}$$

Where P_m = Gross mechanical power output of the motor.

P_c = rotor copper loss.

$$P_m = \omega T = \frac{2\pi N}{60} T \quad \text{--- (5)} \quad \text{Where } \omega = \text{Actual angular speed rad/sec.}$$

Substituting values of P_m and P_2 from Eqn (4) & (5), the rotor copper loss is given by, $P_c = P_2 - P_m$.

$$\therefore \text{Rotor Copper loss, } P_c = \omega_s T - \omega T \\ = (\omega_s - \omega) T \quad \text{--- (6)}$$

But slip $s = \frac{\omega_s - \omega}{\omega_s}$

$$\therefore (\omega_s - \omega) = s \omega_s,$$

substituting in eqn (6), we get,

(25)

$$P_c = s \omega_s T = \frac{s 2 \pi N_s}{60} T \quad \text{--- (7)}$$

The gross mechanical power at the output of rotor is

$$P_m = P_2 - P_c = \omega_s T - s \omega_s T$$

$$P_m = (1-s) \omega_s T \quad \text{--- (8)}$$

$$= (1-s) P_2 \quad \text{--- (9)}$$

The rotor efficiency is given by,

$$\eta_R = \frac{P_m}{P_2} = \frac{\text{Rotor output}}{\text{Rotor input}} \quad \text{--- (10)}$$

$$= \frac{(1-s) P_2}{P_2}$$

$$\boxed{\eta_R = (1-s)} \quad \text{--- (11)}$$

This equation shows that the rotor efficiency of induction motor varies with the value of slip 's' and hence with the speed of the motor.

$$\eta_R = 1 \text{ or } 100\% \text{ for } s = 0 \text{ i.e. } N = N_s.$$

$$\text{and } \eta_R = 0 \text{ for } s = 1 \text{ i.e. } N = 0 \text{ i.e. standstill.}$$

The net motor efficiency is given by,

$$\eta_M = \frac{\text{net output at shaft}}{\text{net electrical input to motor}} = \frac{P_{out}}{P_i}$$

The power flow in induction motor can be shown in the block diagram form as below:

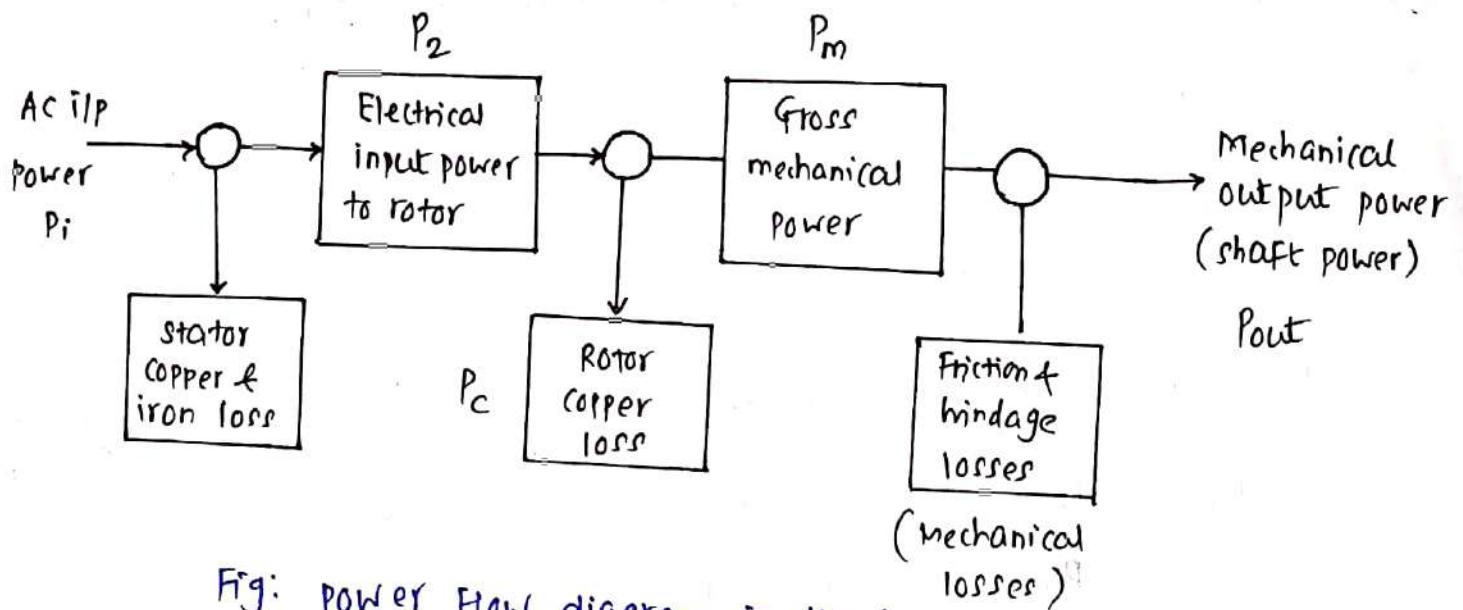


Fig: power flow diagram in the block diagram form.

Gross mechanical rotor output power

$$P_m = (1-s) P_2$$

$$\text{Rotor Copper loss } P_c = s P_2$$

$$\therefore \frac{\text{Rotor gross output}}{\text{Rotor Copper loss}} = \frac{(1-s) P_2}{s P_2} = \frac{1-s}{s}$$

* Summary of analysis of power flow diagram is as follows:

1. Electrical input to motor $P_i = 3 E_1 I_1 \cos \phi_1$
2. Electrical power to rotor $P_2 = \omega_s \cdot T = 2\pi \frac{N_s}{60} \times T$
3. Gross mechanical output power $P_m = \omega T = \frac{2\pi N}{60} \times T$
4. Rotor copper loss $P_c = s P_2$
5. Rotor efficiency $\eta_r = (1-s)$
6. Relation beth P_m and P_2 ;

$$P_m = (1-s) P_2$$

* Efficiency of an Induction Motor:

The motor efficiency is defined as,

$$\% \eta_M = \frac{P_{out}}{P_i} \times 100$$

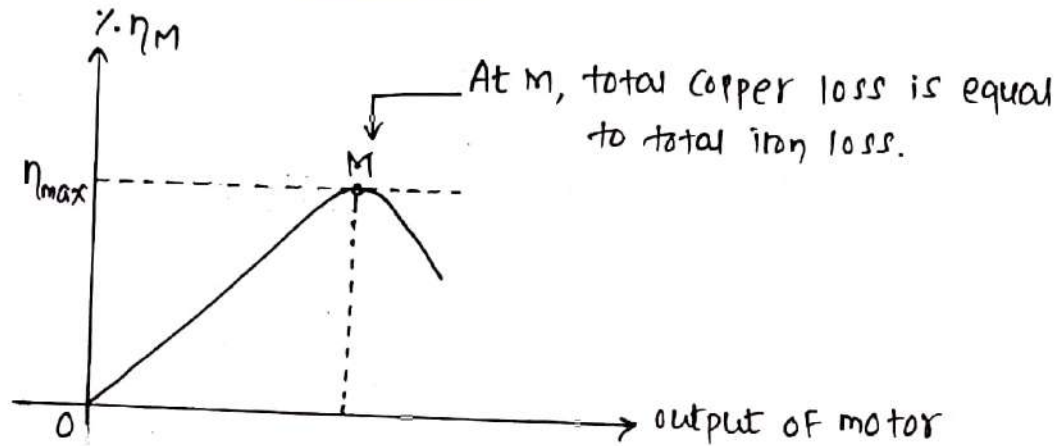


Fig: Variation in efficiency of induction motor.

The motor efficiency does not remain constant, it depends on the load applied on the motor. For example the efficiency is low when the motor is running on no load, because the current drawn by the motor is small.

The efficiency increases with increase in load on load upto a point M as shown in Fig.

The maximum efficiency is obtained when the total copper loss is equal to the total iron loss.

* Numericals:

- ① A 500V, 3 ϕ , 50 Hz induction motor develops an output of 15 kW at 950 rpm. If the input power factor is 0.86 lagging, mechanical losses are 730 W and the stator losses 1500 W. Find:

1. The slip
2. The rotor copper loss
3. The motor input
4. The line current.

(W-14) (4M)

Soln:

Given: $V_L = 500V$

$P_{\text{rotor out}} = 15 \text{ kW}$,

$N = 950 \text{ rpm}$

$\text{P.F} = \cos \phi = 0.861 \text{ lag}$

Mechanical loss = 730 W ,

stator losses = 1500 W .

Find: 1. slip 2. Rotor Copper loss
3. Motor input 4. The line current.

$$\Rightarrow N_s = \frac{120F}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm.}$$
$$s = \frac{N_s - N}{N_s} = \frac{1000 - 950}{1000} = 0.05 = 5\%.$$

$$\begin{aligned} \text{Rotor output} &= P_{\text{out}} + \text{Mechanical loss} \\ &= 15 \text{ kW} + 730 \text{ W} \\ &= 15.73 \text{ kW} \end{aligned}$$

Rotor Copper loss:

$$\frac{\text{Rotor Copper loss}}{\text{Rotor Output}} = \frac{s}{1-s}$$

$$\begin{aligned} \therefore \text{Rotor Copper loss} &= \frac{0.05}{1-0.05} \times 15.73 \text{ kW} \\ &= 827.89 \text{ W} \end{aligned}$$

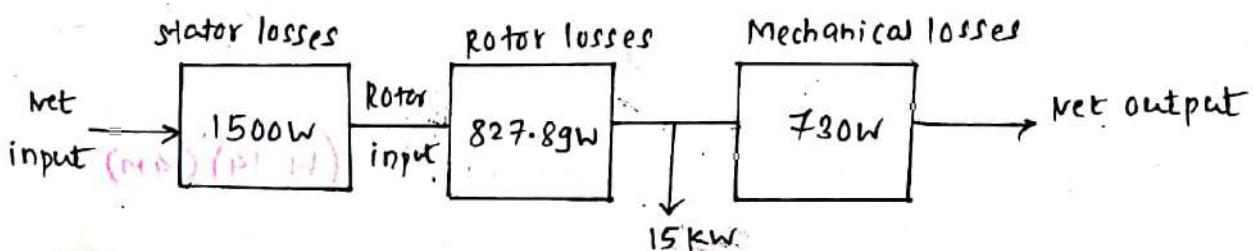


Fig: power flow diagram.

from power flow diagram,

$$\begin{aligned} \text{Motor input } P_{in} &= 15000 + 730 + 1500 + 827.89 \\ &= 18057.89 \text{ W} \\ &= 18.058 \text{ kW} \end{aligned}$$

$$P_{in} = \sqrt{3} I_L V_L \cos \phi$$

$$I_L = \frac{18.058 \times 10^3}{\sqrt{3} \times 500 \times 0.86}$$

$$I_L = 24.25 \text{ A}$$

- ② A 20 HP three phase, 50 Hz, 4 pole induction motor has a full load slip of 4%. The friction and windage losses are 500 watts, calculate the rotor I^2R loss and rotor speed. (W-14) (4M).

Soln:

Given:

$$\begin{aligned} \text{HP} &= 20, \\ F &= 50 \text{ Hz} \\ P &= 4 \\ S &= 4\% = 0.04 \end{aligned}$$

$$\text{Friction and windage loss} = 500 \text{ W}$$

Find:

- Rotor I^2R loss (Copper loss) = ?
- Rotor speed (N) = ?

$\Rightarrow$ Rotor speed (N):

$$N_s = \frac{120F}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\begin{aligned} N &= (1-S) N_s \\ &= (1-0.04) \times 1500 \end{aligned}$$

$$N = 1440 \text{ rpm}$$

Rotor I^2R loss (Rotor Copper loss):

$$\text{Net output power } P_{\text{out}} = 20 \text{ HP}$$

$$= 20 \times 746$$

$$= 14920 \text{ W}$$

$$\therefore \text{Rotor copper loss} = P_{\text{out}} - \text{Friction and windage loss}$$

$$= 14920 - 500$$

$$\boxed{\text{Rotor Copper loss} = 14420 \text{ W}}$$

* Induction Motor as a Transformer:

We know that a transformer has two windings primary and secondary, both windings are magnetically coupled to each other. When primary is connected to the ac supply, emf gets induced into the secondary as per the principle of induction.

The induction motor also operates on the principle of induction because when the stator winding is connected to the three phase ac voltage, an ac voltage is induced into the rotor winding. So the induction motor is called as the rotating transformer.

We know that,

$$E_1 = \text{Stator emf per phase and}$$

$$E_2 = \text{Rotor emf per phase at standstill } (N=0, s=1)$$

Hence similar to the static transformer we can define the transformation ratio (k) as follows:

$$\boxed{k = \frac{E_2}{E_1} = \frac{N_2}{N_1}}$$

Where N_2 = No. of rotor turns/phase

N_1 = No. of stator turns/phase.