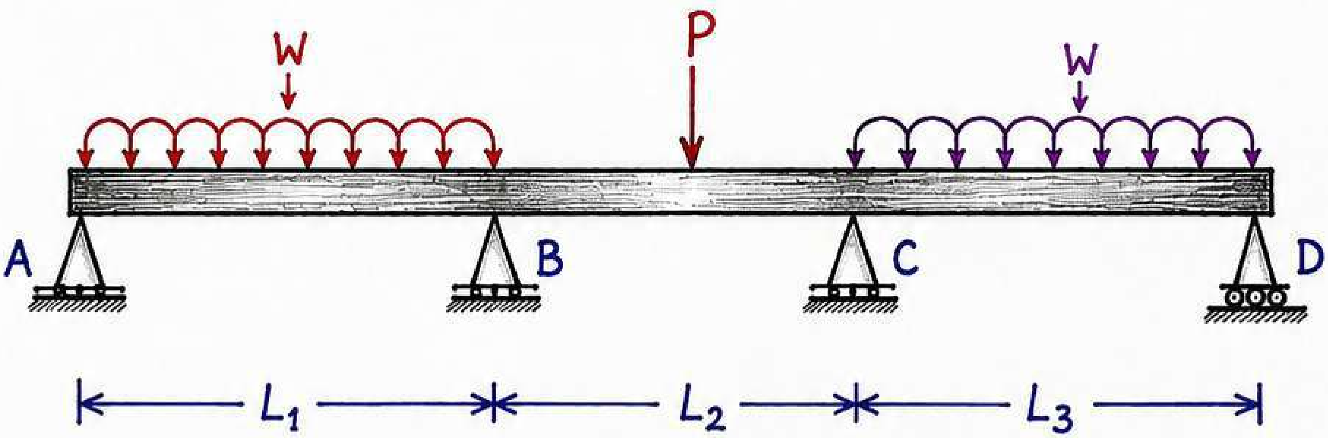
THEORY OF STRUCTURES

UNIT - 03

CONTINUOUS BEAM



TOPICS COVERED

* Definition of Continuous Beam
* Effect of Continuity
* Nature of Moments Induced due to Continuity
* Definition of Elastic Curve
* Clapeyron's Theorem of Three Moments
* Applications / Problems

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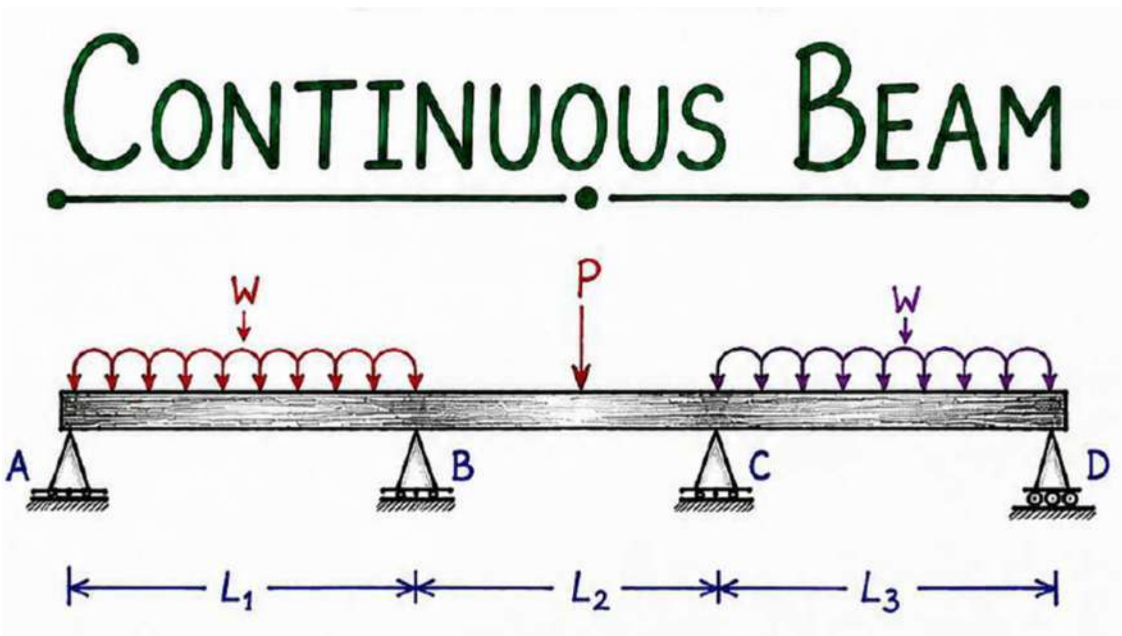
Course Title- Theory of Structure **Course Code -**315313

Programme Name -Civil Engineering **Semester-**Fifth

Unit	Title	COs	Learning hours	R Level	U Level	A Level	Total Marks
III	Continuous Beam	CO3	14	2	8	6	16



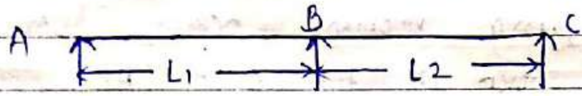
Unit - [redacted]

Unit-03

Continuous Beam

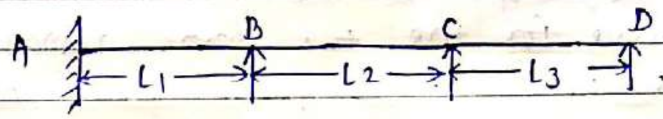
* **Defination of continuous Beam :-**

when a beam is supported by more than two supports then it is called as continuous beam.

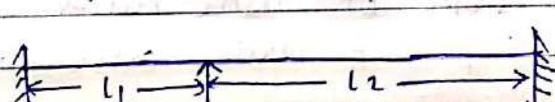


(TWO span)

fig. a) continuous Beam with simply supported end

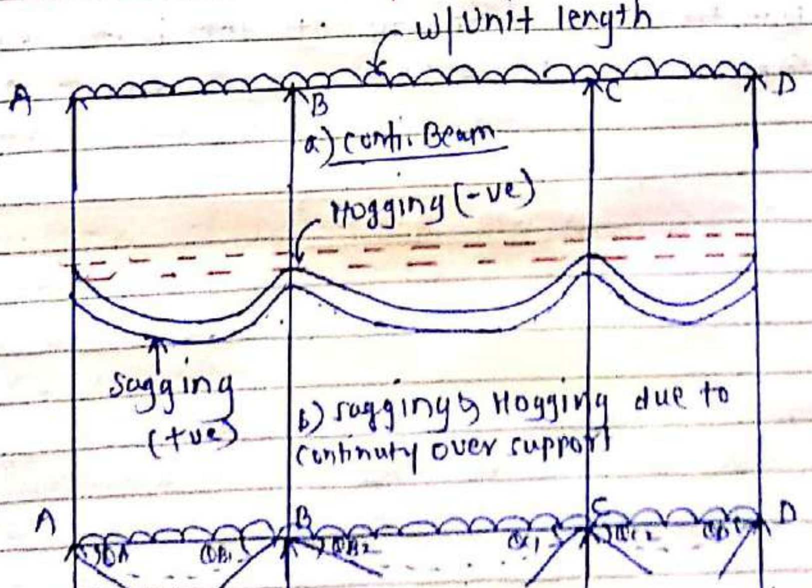


b) continuous Beam with one end fixed & other simply supported.



c) continuous Beam with Both ends fixed.

* **Effect of continuity :->**



c) slopes at intermediate supports remains same At B, $\theta_{B1} = \theta_{B2}$ & At C $\theta_{C1} = \theta_{C2}$

fig. Effect of continuity

fig. 4.2.1(a) shows a beam continuous over the number of supports.
Due to continuity of beam over the supports, the continuous beam are always subjected to sagging moment over the mid-section of the span and hogging moment occurs over the supports as shown in fig. 4.2.1 (b)
fig. 4.2.1 (b) shows the deflection curve for the beam when it is subjected to an external load system such that at the intermediate supports the slope of the deflection curve or elastic curve for the two spans remains same.
The fig. 4.2.1 (c) shows the elastic curve as well as slope at the intermediate support.

for span AB

Let θ_{B1} = slope of elastic curve at support B

for span BC.

Let θ_{B2} = slope of elastic curve at support B.

Let θ_{C1} = slope of elastic curve at support C

for span CD

Let θ_{C2} = slope of elastic curve at support C.

Due to the continuity of the beam over the supports, $\theta_{B1} = \theta_{B2}$ and $\theta_{C1} = \theta_{C2}$

* Nature of moments induced due to continuity :

There are two types of moments namely sagging and Hogging induced into the continuous beam due to continuity over the supports for the usual downward loadings.

sagging moment produces convexity in downward direction at the mid-section of the span whereas hogging moments produces convexity in upward direction at the intermediate supports as shown in fig. 4.2.1 (b).

Nature of moments induced due to continuity
1. sagging (at mid-section of span)
2. Hogging (at the intermediate supports

* Defination of Elastic curve or deflected shape

when the beam is ~~loaded~~ subjected to the usual downward loadings then longitudinal axis of the beam bends or deflects in the form of a curve called elastic curve or deflected shape of the beam.

* clapeyron's Theorem of Three moments :-

1) statement of clapeyron's theorem of three moment with uniform MI. :-

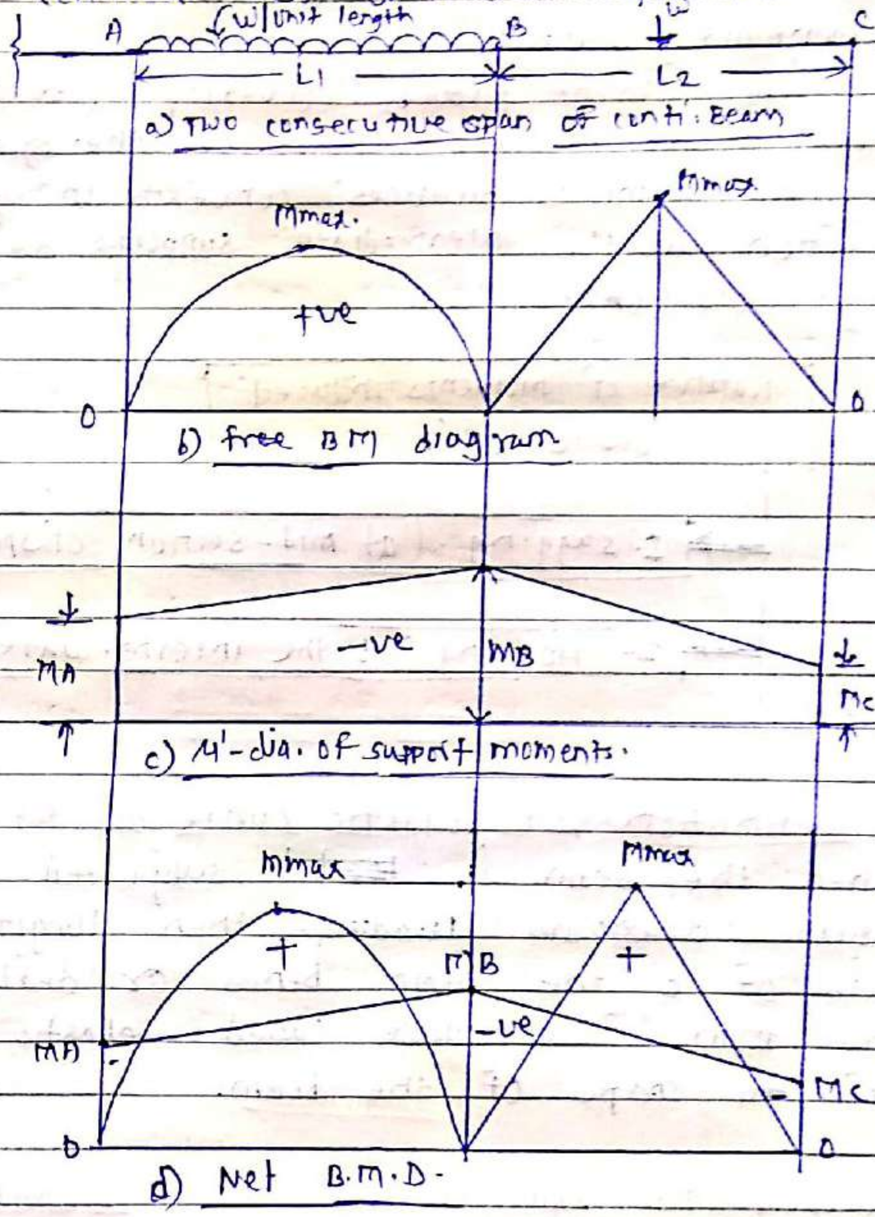
Refer fig 4.5.1 (a).

for any two consecutive spans of a continuous beam subjected to an external loading and having uniform moment of inertia the support moments M_A, M_B and M_C at the support A, B & C resp. are given by the following relation as.

$$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = -\left(\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right)$$

* Where MA = support moment at A
MB = support moment at B
MC = support moment at C

L_1 = length of span AB ; L_2 = length of span BC
a_1 = Area of M diagram sagging free B.M. – AB span
a_2 = Area of M – diagram sagging f.B.M. – BC span
$\bar{x}_1$ = centroidal dist. of M-dia. span AB
$\bar{x}_2$ = centroidal dist. of M-dia. span BC



* 2) Three moment theorem for different MI :-

$$M_A \frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C \frac{L_2}{I_2} = -\left(\frac{6a_1\bar{x}_1}{L_1 I_1} + \frac{6a_2\bar{x}_2}{L_2 I_2}\right)$$

where I_1 = M.I. for span AB
I_2 = M.I for span BC.

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* Position of centroid '$\bar{x}$' and Area 'a' of 'M' diagram (sagging) for some standard cases.

* Case - I simply supported Beam subjected to central point load

$$\bar{x} = \frac{L}{2} \text{ from A and B}$$

$$\boxed{a = \left(\frac{1}{2} \times L \times \frac{WL}{4}\right)}$$

* Case II :- simply supported Beam subjected to an eccentric load :-

$$\bar{x} = \left(\frac{L+a}{3}\right) \text{ from A}$$

$$\bar{x} = \left(\frac{L+b}{3}\right) \text{ from B}$$

$$\boxed{a = \left(\frac{1}{2} \times L \times \frac{Wab}{L}\right)}$$

* Case III :- simply supported Beam subjected to a u.d.l. over entire span :-

$$\bar{x} = \frac{L}{2} \text{ from A and B}$$

* Analysis of Continuous Beam :

step :- 1) Each span of a continuous beam is assumed to be a simply supported beam. find the free B. M. and draw the M - diagram for a given loading.

step : 2) calculate a $\bar{x}$ of M diagram

step : 3) find the support moments by applying the clapeyron's theorem of three moments. Draw M - diagram superimpose the M' - diagram over the M - diagram so as to get Net BM

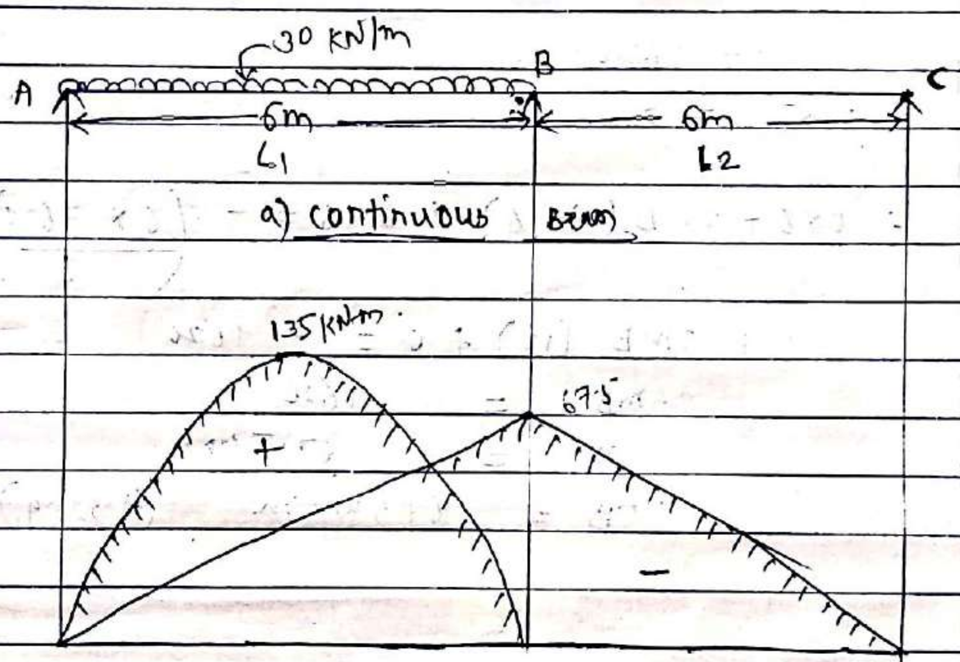
step 4) find the support reactions by considering the support moments.

step 5) DO the S.F. calculations according to sign convention considered then draw S.F. diagram

* Note that overhanging portion of continuous beam is considered as a cantilever.

* solved examples Based on clapeyron's Theorem of three moment for a continuous Beam with simply supported Ends.

Ex.-1) A continuous Beam ABC, simply supported at A, B and C such that $l(AB) = l(BC) = 6m$. span AB subjected to 30 kN/m UDL, find moment at B. flexural rigidity EI is same throughout.



a) continuous Beam

b) BMD in kN·m.

* step 1) Assuming each span to be simply supported, find free moments.

free moments :-

Span AB : $M_A = 0$, $M_B = 0$,

$$M_{max.} = \frac{WL^2}{8} = \frac{30 \times 6^2}{8} = 135 \text{ kN·m. (+ve)}$$

Span BC : $M_B = 0$, $M_C = 0$, $M_{max} = 0$ (no load on BC)

Step 2> To find support moment at B i.e M_B

Applying clapeyron's theorem of three moments

Note that $M_A = 0$ and $M_C = 0$ due to simply support at A & B resp. EI is throughout same.

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = -\left(\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right)$$

$$a_1\bar{x}_1 = \left(\frac{2}{3} \times 6 \times 135\right) \times \frac{6}{2} \qquad \because \bar{x}_1 = \frac{6}{2} \text{ from A}$$

$$= 1620$$

$$a_1\bar{x}_2 = 0 \qquad \because \text{no loads on BC}$$

$$\therefore 0 \times 6 + 2M_B(6+6) + 0 \times 6 = -\left(\left\{\frac{6 \times 1620}{6}\right\} + 0\right)$$

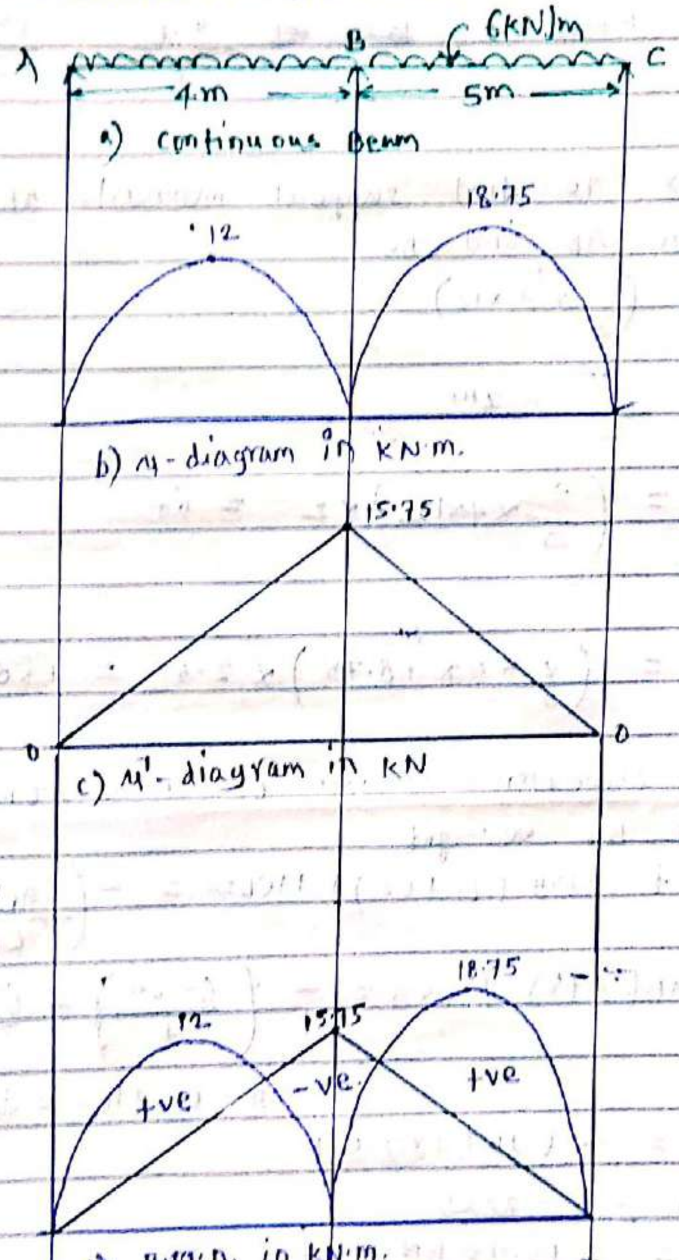
$$0 + 2M_B(12) + 0 = -1620$$

$$24 M_B = -1620$$

$$M_B = -67.5 \text{ kNm}$$

$$\therefore M_B = 67.5 \text{ kN m.} \quad \text{(Hogging)}$$

Ex: 7) Using theorem of three moments calculate support moments and draw BMD giving net B.M. only for a continuous beam as shown in fig.



a) continuous beam
b) M-diagram in KN.m.
c) M'- diagram in KN
d) B.M.D. in KN.m.

Given :- $w = 6KN/m$ over span AB and BC
L_1 = span AB = 4m , L_2 = span BC = 5m.

Step 1) Assuming each span of a beam to be simply supported, find the free B.M.

for span AB : $M_A = 0$; $M_B = 0$

$$m_{max} = \frac{wL_1^2}{8} = \frac{6\times4^2}{8} = 12\ kN\cdot m.$$

for span BC : $m_B = 0$; $m_C = 0$

$$m_{max} = \frac{wL_2^2}{8} = \frac{6\times5^2}{8} = 18.75\ kN\cdot m$$

* step 2) To find support moments at A, B and C for span AB and BC

$$a_1 = \left(\frac{2}{3}\times4\times12\right)$$

$$\bar{x}_1 = \frac{4}{2} = 2m$$

$$\therefore a_1\bar{x}_1 = \left(\frac{2}{3}\times4\times12\right)\times2 = 64$$

$$a_2\bar{x}_2 = \left(\frac{2}{3}\times5\times18.75\right)\times2.5 = 156.25$$

Applying clapeyron's theorem of three moments for span AB and BC we get.

$$M_AL_1 + 2M_B(L_1+L_2) + M_CL_2 = -\left(\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right)$$

$$0\times4 + 2M_B(4+5) + 0\times5 = -\left(\frac{6\times64}{4} + \frac{6\times156.25}{5}\right)$$

($\because$ $M_A = 0$, $M_C = 0$ due to S.S.)

$$18M_B = -(96+187.5)$$

$$\therefore 18M_B = -283.5$$

$$\therefore \boxed{M_B = -15.75\ kN\cdot m.}$$ Ans.

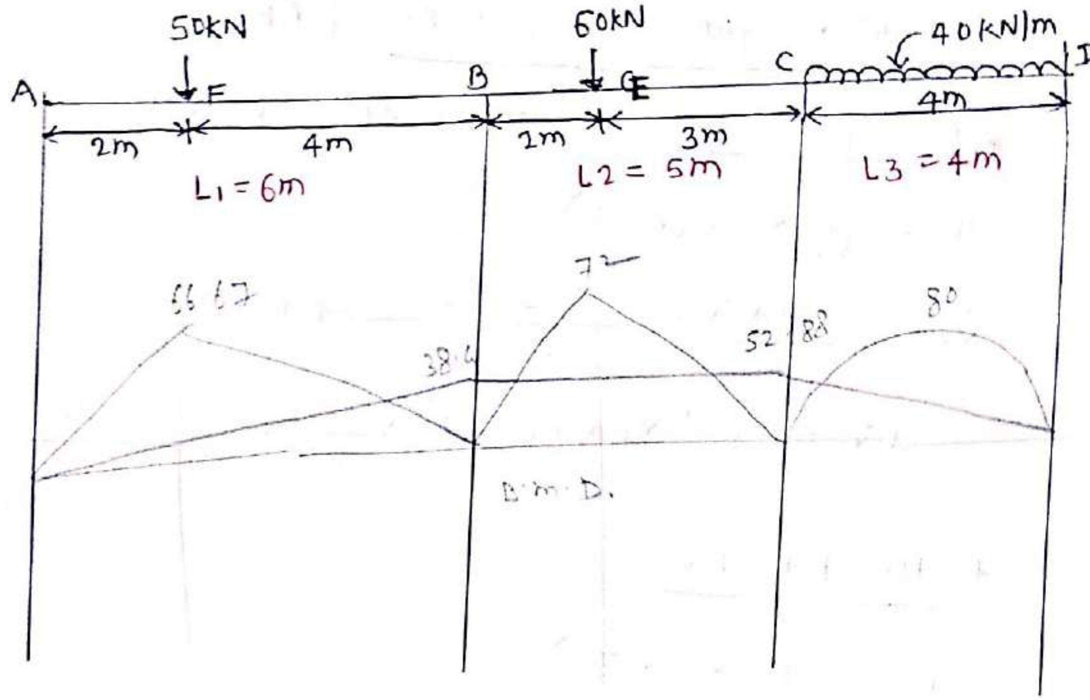
$M_A = 0$, $M_C = 0$ Ans.

Draw the M' - diagram from calculated support moments as shown in fig. (c)

step 3) superimpose M' - diagram over the M - diagram

Ex:-1) A continuous Beam ABCD 15m long rest on supports A, B, C and D all at same level. AB = 6m, BC = 5m and CD = 4m. It carries two concentrated load 50KN and 60 KN at 2m and 8m from A and UDL of 40 kN/m over CD. find moments and reactions at the supports. Draw BMD using three moments theorem only.

⟹



Given L_1 = span AB = 6m, L_2 = span BC = 5m

L_3 = span CD = 4m W_1 = 50KN, W_2 = 60KN

ω = 40KN/m.

*step 1) Assume each span of beam to be simply supported and find free B.M.

Free B.M. * for span AB :- a = 2m, b = 4m, W = 50KN

$M_A = 0, \quad M_B = 0, \quad M_F = \frac{Wab}{L} = \frac{50 \times 2 \times 4}{6} = 66.67 \text{ kN.m}$

* for span BC

$M_B = 0, \quad M_E = \frac{Wab}{L} = \frac{60 \times 2 \times 3}{5} = 72 \text{ kN.m}$

* For span CD $L = 4m$, $w = 40\ kN/m$, $M_C = 0$, $M_D = 0$

$$M_{max} = \frac{wL^2}{8} = \frac{40 \times 4^2}{8} = 80\ kN.m.$$

* step 2) To find support moments at A, B, C and D

consider span AB and BC (ABC)

* for span AB : $a = 2m$, $b = 4m$, $L = 6m$

$$a_1 = \left(\frac{1}{2} \times 6 \times 66.67\right)$$

$$\bar{x}_1 = \left(\frac{L+a}{3}\right) = \left(\frac{6+2}{3}\right) = \frac{8}{3} \leftarrow \text{from A}$$

$$\therefore a_1\bar{x}_1 = \left(\frac{1}{2} \times 6 \times 66.67\right) \times \frac{8}{3} = 533.36$$

* for span BC

$$a_2 = \left(\frac{1}{2} \times 5 \times 72\right)$$

$$\bar{x}_2 = \left(\frac{L+b}{3}\right) = \left(\frac{5+3}{3}\right) = \frac{8}{3} \leftarrow \text{from 'C'}$$

$$\therefore a_2\bar{x}_2 = \left(\frac{1}{2} \times 5 \times 72\right) \times \frac{8}{3} = 480$$

* Apply Clapeyron's theorem of three moments for span AB and BC :

$L_1 = 6m$, $L_2 = 5m$

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = -\left(\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right)$$

$$0 \times 6 + 2M_B(6+5) + \underset{M_C}{0} \times 5 = -\left(\frac{6 \times 533.36}{6} + \frac{6 \times 480}{5}\right)$$

$$22M_B + 5M_C = -(533 + 576)$$

$$22M_B + 5M_C = -1109.36 \text{ --- i)}$$

1110.74

* consider span BC and CD (BCD)

for span BC $\quad a = 2m,\ b = 3 \quad L = 5m$

$$a_2 = \left(\frac{1}{2} \times 5 \times 72\right)$$

$$\bar{x}_2 = \left(\frac{L+a}{3}\right) = \frac{5+2}{3} = \frac{7}{3} \leftarrow \text{(from end B)}$$

$$\therefore a_2\bar{x}_2 = \left(\frac{1}{2} \times 5 \times 72\right) \times \frac{7}{3} = 420$$

for span CD $\quad L_3 = 4m$

$$a_3 = \left(\frac{2}{3} \times 4 \times 80\right)$$

$$\bar{x}_3 = \frac{4}{2} = 2 \leftarrow \text{from end D}$$

$$\therefore a_3\bar{x}_3 = \left(\frac{2}{3} \times 4 \times 80\right) \times 2 = 426.67$$

Apply clapeyron's theorem for span BC and CD

$L_2 = 5m \quad , L_3 = 4m$

$$M_B L_2 + 2M_C (L_2 + L_3) + M_D L_3 = -\left(\frac{6a_2\bar{x}_2}{L_2} + \frac{6a_3\bar{x}_3}{L_3}\right)$$

$$M_B \times 5 + 2M_C(5+4) + 0 \times 4 = -\left(\frac{6 \times 420}{5} + \frac{6 \times 426.67}{4}\right)$$

$$5M_B + 18M_C = -(504 + 640)$$

$$5M_B + 18M_C = -1144 \quad \text{— ii)}$$

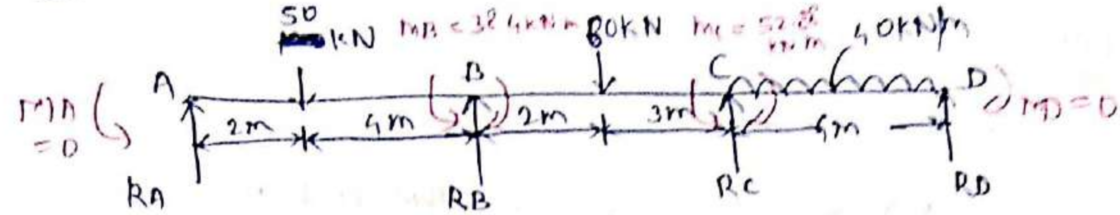
solving eqn. (i) and (ii) simultaneously we get

$M_B = -38.4$ kN·m Ans (-ve sign indicate

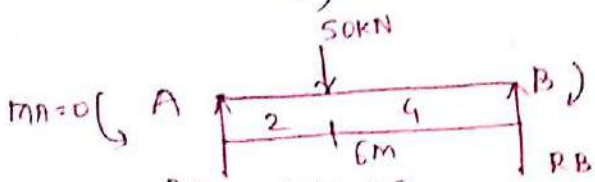
$M_C = -52.88$ kN·m Ans. Hogging B.M.)

$M_A = 0, \quad M_D = 0$

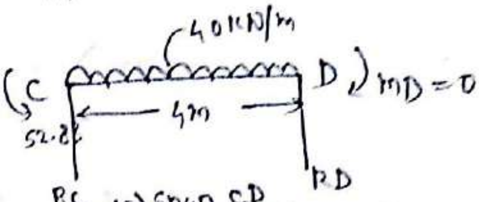
step 3) To find support Reactions at A, B, C and D



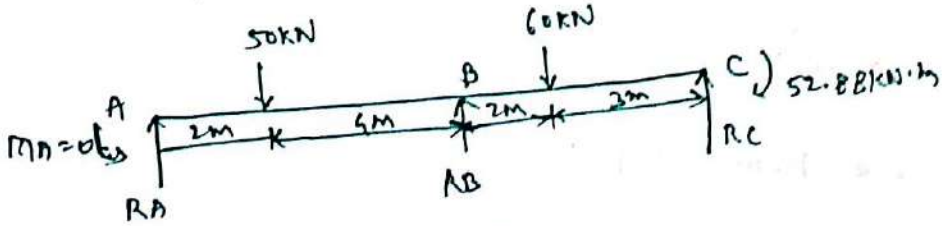
iii) conti. Beam with support moments



iv) Span AB



v) Span CD



vi) span ABC

consider span AB fig. iv)

$$\Sigma M_B = R_A \times 6 - 50 \times 4 + 38.4$$

$$0 = 6R_A - 161.6$$

$$\therefore \boxed{R_A = 26.93 \text{ kN}}$$

consider span CD fig v)

$$\Sigma M_C = -52.88 + (40 \times 4 \times 2) - R_D \times 4$$

$$\therefore \boxed{R_D = 66.78 \text{ kN}}$$

consider span ABC fig. vi)

$$\Sigma M_C = 52.88 - 60 \times 3 + R_B \times 5 - 50 \times 9 + R_A \times 11$$

$$0 = 52.88 - 180 + 5R_B - 450 + 26.93 \times 11$$

$$0 = 5R_B - 280.89 \qquad \boxed{\therefore R_B = 56.17 \text{ kN}}$$

$$\Sigma F_y = R_A + R_B + R_C + R_D - 50 - 60 - (40 \times 4)$$

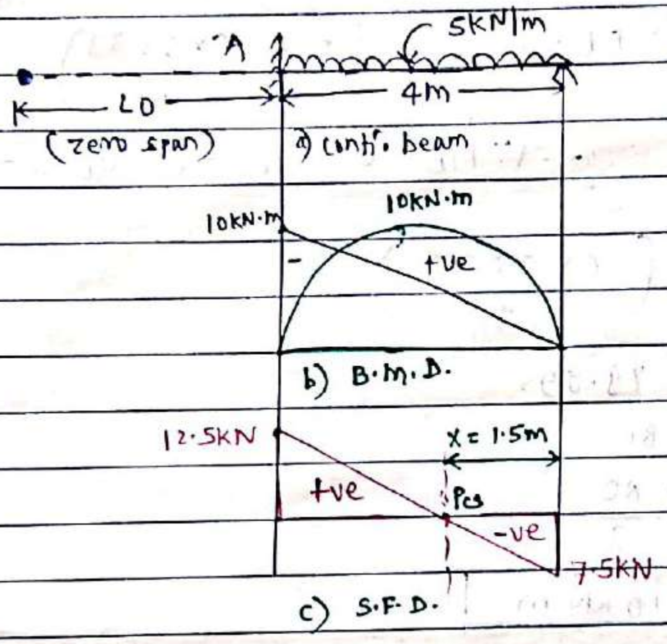
$$0 = 26.93 + 56.178 + R_C + 66.78 - 270$$

$$0 = 120.112 + R_C$$

$$\boxed{R_C = 120.112 \text{ kN}} \text{ Ans.}$$

* Example based on clapeyron's Theorem of Three moments for a continuous Beam with one End fixed and others simply supported. (7)

Ex:- Draw S.F.D. and B.M.D. by using clapeyron's three moment theorem for a beam AB of span 4m carrying u.d.l. 5kN/m in which support A is fixed and B is simply supported.



* step 1) Assume the beam to simply supported and find free B.M.

free B.M. : $M_A = 0$, $M_B = 0$

$$M_{maxi} = \frac{wL^2}{8} = \frac{5 \times 4^2}{8} = 10 \text{ kN.m.}$$

* step 2) To find the support moment M_A and M_B

since the end A of a given beam is fixed, assume an imaginary span AA_0 to the left of A. so as to apply clapeyron's theorem for span AA_0 and AB.

for span AA_0 and AB

$L_0 = 0$, $L = 4m$

$$a = \frac{2}{3} \times 4 \times 10 \quad ; \quad \bar{x} = \frac{4}{2} = 2$$

$$\therefore a\bar{x} = \left(\frac{2}{3} \times 4 \times 10\right) \times 2 = 53.33 \text{ m}^3.$$

Applying clapeyron's theorem of three moment for span A A0 and AB we get

$$M_0 L_0 + 2M_A(L_0 + L) + M_B \times L = -\left(\frac{6a_0\bar{x}_0}{L_0} + \frac{6a\bar{x}}{L}\right)$$

$$\therefore 0 + 2M_A(0+4) + 0 \times 4 = -\left(0 + \frac{6 \times 53.33}{4}\right)$$

$$(\because M_B = 0, M_0 L_0 = 0, a_0 x_0 = 0)$$

$$\therefore 8M_A = -\left(\frac{6 \times 53.33}{4}\right)$$

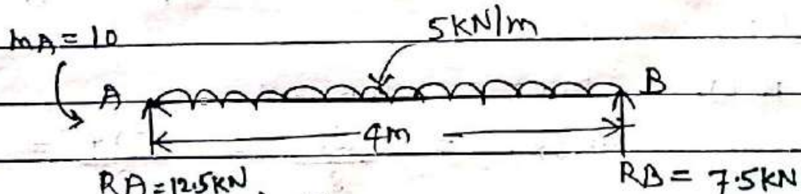
$$= -79.995$$

$$8M_A = -80$$

$$M_A = \frac{-80}{8}$$

$$\boxed{M_A = -10\ kN \cdot m}$$

* step 3) To find support reaction by considering to support moments :-



$+\uparrow \ -\downarrow$

$\Sigma f_y = 0$

$R_A + R_B - 5 \times 4 = 0$

$$\boxed{R_A + R_B = 20\ kN} \quad \text{---} 1)$$

$\circlearrowright +\ \circlearrowleft -,\quad \Sigma M_A = -10 + 5 \times 4 \times 4/2 - R_B \times 4$

$= -10 + 40 - 4R_B$

$$\therefore R_B = \frac{30}{4}$$

$$\boxed{R_B = 7.5\ kN}$$

substituting the value of of RB in eqn (i) we get

$R_A + 7.5 = 20$

$R_A = 20 - 7.5$

$$\boxed{R_A = 12.5\ kN}$$

* step - 4 S.F. calculation + ↑ - ↓

$SFAL = 0$

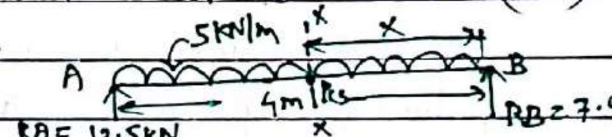
$SFAR = 12.5 kN$

$SFBL = 12.5 - 5 \times 4 = -7.5$

$SFBR = 7.5 - 7.5 = 0$

* step - 5 To locate the point of zero shear (PCS) :-

Let x be the position of point of contrashear (PCS) from B as shown in fig.



$F_{PCS} = F_x =$ S.F. at point of contrashear or at $x-x$

$\therefore F_{PCS} = -7.5 + 5 \times x$

$0 = -7.5 + 5x$

$\therefore 5x = 7.5$

$\boxed{x = 1.5m}$ from B

* ## Concept of zero span :-

* when the ends of the continuous beam are fixed, then an imaginary zero span is taken or considered to the left or right of the support as the case may be and the clapeyron's theorem is applied to an imaginary span and its adjacent span.

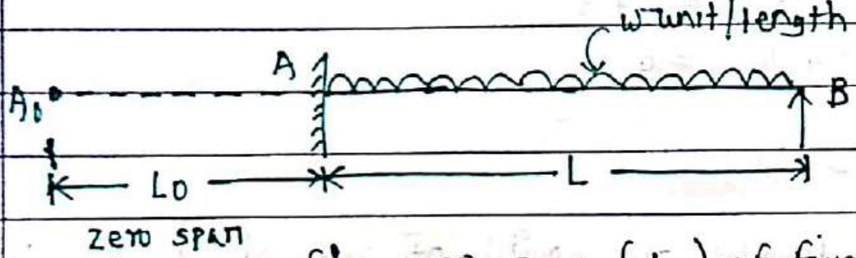


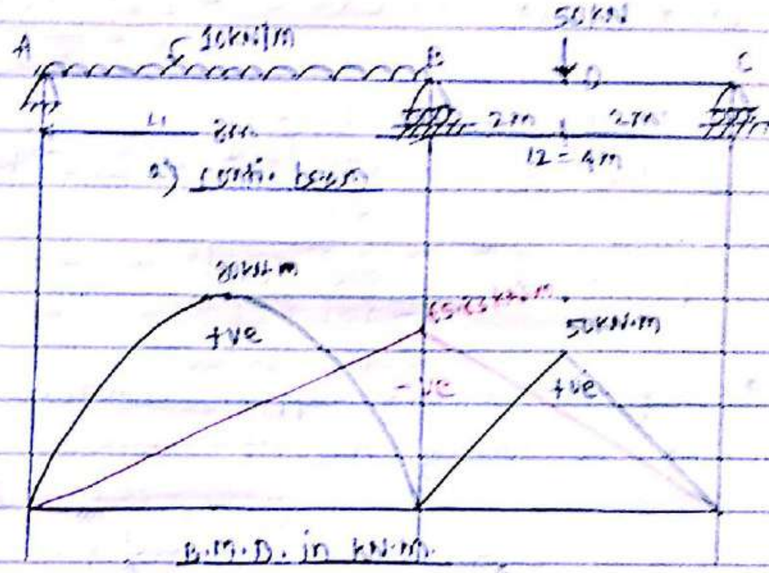
fig. zero span (L_0) of fixed end of conti. beam.

In fig. the end A is fixed, hence assume an imaginary span AA_0 (called as zero span) to the left of A so as to apply clapeyron's theorem for span AA_0 and AB.

* Example based on Theorem of three moments or Clapeyron's Theorem for a continuous beam with pinned or hinged and rolled supported Ends. Pins can resist both V & H forces but not moment

unit-III

Ex:1) A continuous beam of uniform section is pinned at A and supported on rollers at B and C, AB = 8m, BC = 4m. it is subjected to u.d.l. of 10 kN/m on span AB and a central point load of 50kN on BC. using Clapeyron's theorem, calculate the support moment at B. Draw SFD & BMD.



* step :- 1) Assuming the each span of a beam to be simply supported, find the free B.M.

free B.M. ⇒ for span AB

$m_A = 0$, $m_B = 0$, $m_{max} = \frac{wl_1^2}{8} = \frac{10 \times 8^2}{8} = 80 kN \cdot m$

for span BC

$m_B = 0$, $m_C = 0$, $m_D = \frac{Wl_2}{4} = \frac{50 \times 4}{4} = 50 kN \cdot m$

* Step :- 2) To find support moments at A, B and C for span AB and BC

$a_1 = \left(\frac{2}{3} \times 8 \times 80\right)$; $\bar{x}_1 = \frac{l_1}{2} = \frac{8}{2} = 4m$

$\therefore a_1\bar{x}_1 = \left(\frac{2}{3} \times 8 \times 80\right) \times 4 \Rightarrow 1706.67 m^3$

$a_2 = \left(\frac{1}{2} \times 4 \times 50\right)$; $\bar{x}_2 = \frac{l+2}{3} = \frac{6}{3} = 2$

$\therefore a_2\bar{x}_2 = \left(\frac{1}{2} \times 4 \times 50\right) \times 2 = 200 m^3.$

Applying clapeyron's theorem of three moment for span AB and BC we get

$$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 = -\left(\frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2}\right)$$

$$0\times 8 + 2M_B(8+4) + 0\times 4 = -\left(\frac{6\times 1706.67}{8} + \frac{6\times 200}{4}\right)$$

$$24M_B = -1580$$

$$\therefore M_B = \frac{-1580}{24}, \quad \boxed{M_B = -65.83 \text{ KN·m.}}$$

$$\boxed{M_A = 0 \; ; \; M_C = 0}$$

* step 3) to find support reactions RA, RB and RC by considering support moments.

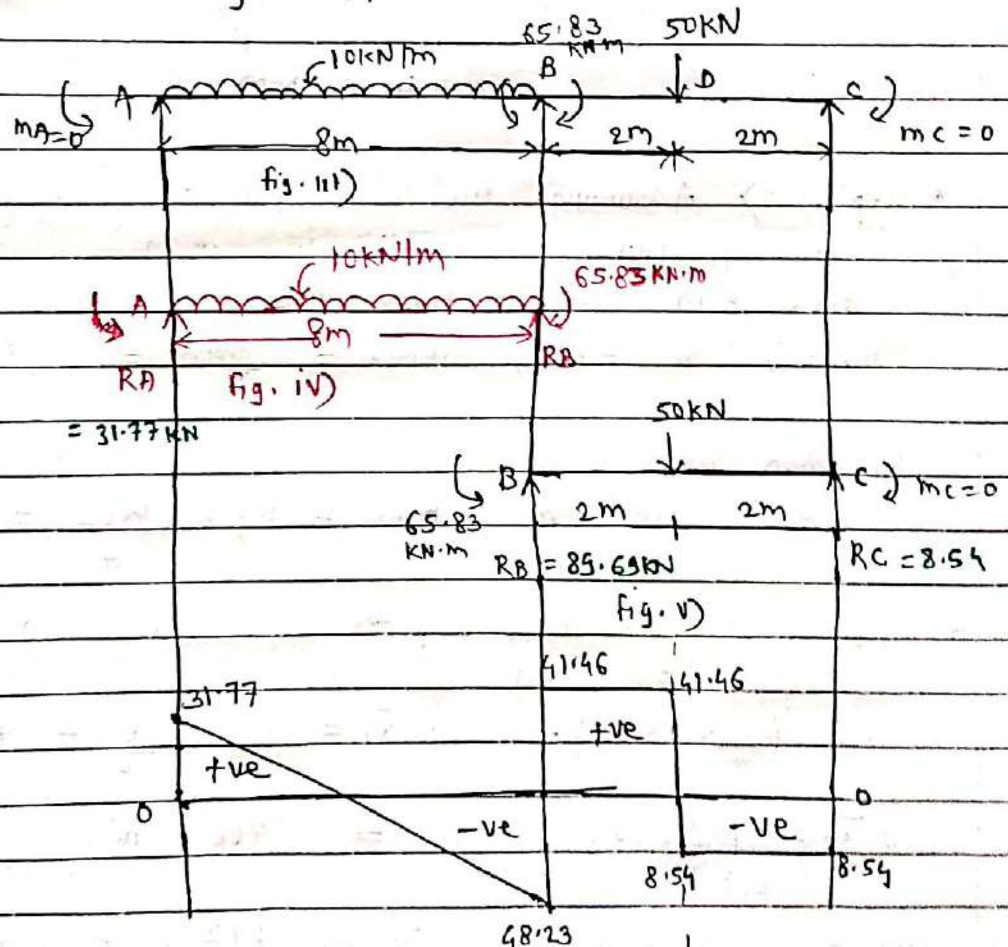


fig. S.F.D in KN

* Consider span AB fig. (iv)

$\circlearrowright^{+}\ \circlearrowleft^{-}$, $\Sigma M_B = R_A \times 8 - 10 \times 8 \times 8/2 + 65.83$

$= 8R_A - 320 + 65.83$

$= R_A = \dfrac{+254.17}{8}$

$$\boxed{R_A = 31.77\ kN}$$

* consider span BC fig (V)

$\circlearrowright^{+}\ \circlearrowleft^{-}$, $\Sigma M_B = -65.83 + 50 \times 2 - R_C \times 4$

$= -65.83 + 100 - 4R_C$

$= 34.17 - 4R_C$

$\therefore R_C = \dfrac{34.17}{4}$

$$\therefore \boxed{R_C = 8.54\ kN}$$

$\uparrow^{+}\ \downarrow^{-}$, $\Sigma f_y = R_A - (10 \times 8) + R_B - 50 + R_C$

$= 31.77 - 80 + R_B - 50 + 8.54$

$= -89.69 + R_B$

$$\therefore \boxed{R_B = 89.69\ kN}$$ ✓

* step 4) S.F calculation $\uparrow^{+}\ \downarrow^{-}$

$SF_{AL} = 0$

$SF_{AR} = 31.77\ kN$

$SF_{BL} = 31.77 - 10 \times 8 = -48.23\ kN$

$SF_{BR} = -48.23 + 89.69 = +41.46\ kN$

$SF_{DL} = +41.46$

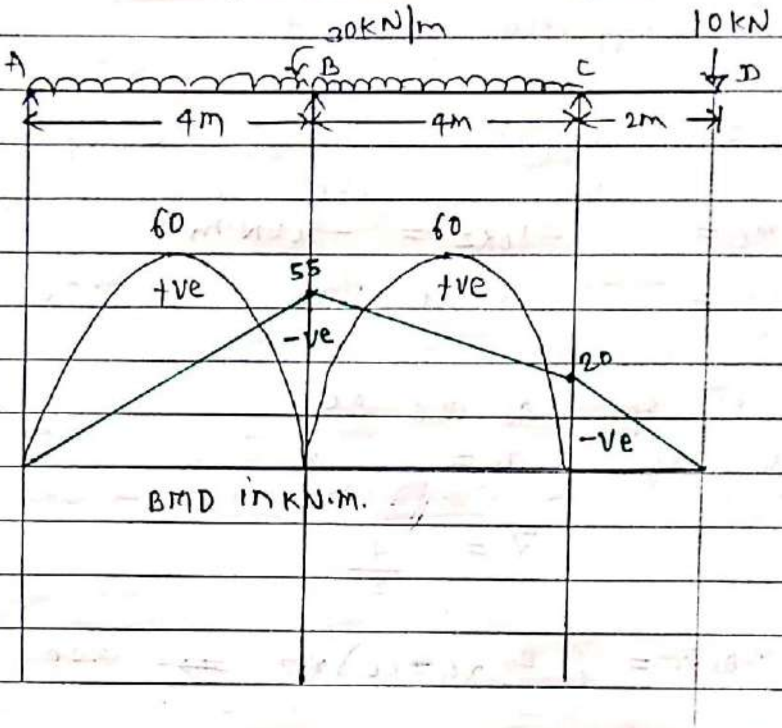
$SF_{DR} = 41.46 - 50 = -8.54\ kN$

$SF_{CL} = -8.54$

$SF_{CR} = -8.54 + 8.54 = 0$

Example Based on clapeyron's Theorem of three moment for a beam with one end simply supported and other end span overhanging

Ex:- calculate support moment using clapeyrons theorem also draw B.M.D.



BMD in KN.m.

Assume over hang span CD is 2m, since it is not given in question.

Given :- $w = 30$ kN/m over span ABC

$W = 10$ kN at free end D of an overhang CD

$L_1 = 4$ m , $L_2 = 4$ m

step 1) Assume span AB and BC to be simply supported and find free B.M.

free B.M. for span AB

$L_1 = 4$ m $\quad w = 30$ kN/m $\quad \therefore$ $M_A = 0$, $M_B = 0$

$$M_{max} = \frac{wL_1^2}{8} = \frac{30 \times 4^2}{8} = 60 \text{ kN.m.}$$

* for BC span

$L_2 = 4$ m $\quad w = 30$ kN/m

$\therefore$ $M_B = 0$, $M_C = 0$

$$M_{max} = \frac{wL_2^2}{8} = \frac{30 \times 4^2}{8} = 60 \text{ kN m}$$

+ step 2) To find support moment at A, B, and C

+ ~~for span AB and BC~~

M_A = support moment at A = 0 being simply supported end A

M_B = support moment at B

M_C = support moment at C by considering the cantilever action of part CD

$\therefore M_C = -10 \times 2 = -20 \, kN \cdot m$

M_D = B.M. at free end D = 0

+ for span AB and BC

span AB $\quad a_1 = \left(\frac{2}{3} \times 4 \times 60\right)$

$\bar{x}_1 = \frac{4}{2}$

$\therefore a_1 \bar{x}_1 = \left(\frac{2}{3} \times 4 \times 60\right) \times 2 \Rightarrow 320$

+ span BC $\quad a_2 = \left(\frac{2}{3} \times 4 \times 60\right)$

$\bar{x}_2 = \frac{4}{2}$

$\therefore a_2 \bar{x}_2 = \left(\frac{2}{3} \times 4 \times 60\right) \times 2 \Rightarrow 320$

Applying the clapeyron's theorem of three moment to span AB and BC we get

$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = -\left(\frac{6 a_1 \bar{x}_1}{L_1} + \frac{6 a_2 \bar{x}_2}{L_2}\right)$$

$$0 \times 4 + 2M_B (4+4) + (-20 \times 4) = -\left(\frac{6 \times 320}{4} + \frac{6 \times 320}{4}\right)$$

$\because (M_A = 0, M_C = -20 \, kN \cdot m)$

$16 M_B - 80 = -(480 + 480)$

$16 M_B = -960 + 80$

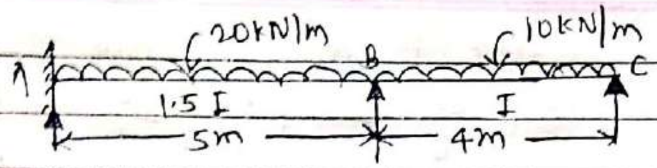
$16 M_B = -880$

$M_B = -880/16$

$\boxed{M_B = -55 \, kN \cdot m}$ $\quad M_A = 0, \; M_C = -20 \, kN \cdot m, \; M_D = 0$

* Example Based on Clapeyron's Theorems of Three moments for a continuous Beam with varying moment of inertia.

Q:- Calculate support moment using clapeyron's theorem. also draw B.M.D



Given :- $w = 20$ kN/m over span

$w = 10$ kN/m over span BC

Let $L_1 = AB = 5m$, $L_2 = BC = 4m$, $I_1 = 1.5I$, $I_2 = I$

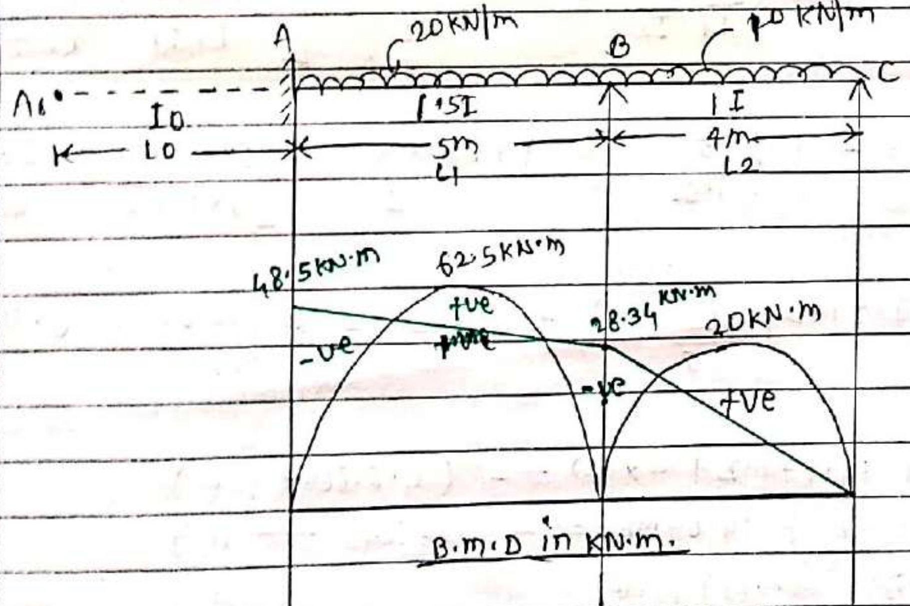
step 1) Assume each span of a continuous beam to be simply supported and find free BMD.

free B.M. for span AB :- $M_A = 0$, $M_B = 0$

$$M_{max.} = \frac{wL_1^2}{8} = \frac{20 \times 5^2}{8} = 62.5 \text{ kN.m.}$$

* for span BC :- $M_B = 0$ $M_C = 0$

$$M_{max.} = \frac{wL_2^2}{8} = \frac{10 \times 4^2}{8} = 20 \text{kN.m.}$$



B.M.D in kN.m.

* step 2) To find support moments at A, B and C since A is fixed assume an imaginary span A_0A to the left of A as shown in fig.

M_0 = support moment at A_0 = 0 due to zero span

M_A = support moment at A

M_B = support moment at B

* for span A_0A and span AB

$a_0\bar{x}_0 = 0$ (∵ zero span)

$a_1\bar{x}_1 = \left(\frac{2}{3}\times 5\times 62.5\right)\times(2.5) = 520.83$

Applying clapeyron's theorem for span A_0A and AB

$$M_0\frac{L_0}{I_0} + 2M_A\left(\frac{L_0}{I_0}+\frac{L_1}{I_1}\right) + M_B\frac{L_1}{I_1} = -\left[\frac{6a_0\bar{x}_0}{L_0I_0} + \frac{6a_1\bar{x}_1}{L_1I_1}\right]$$

$$0 + 2M_A\left(0+\frac{5}{1.5I}\right) + M_B\left(\frac{5}{1.5I}\right) = -\left[0 + \frac{6\times 520.83}{5\times 1.5I}\right]$$

$$+6.67M_A + 3.33M_B = -416.664 \quad \text{— i)}$$

* for span AB and BC

$a_1\bar{x}_1 \Rightarrow \left(\frac{2}{3}\times 5\times 62.5\right)\times 2.5 \Rightarrow 520.83$

$a_2\bar{x}_2 \Rightarrow \left(\frac{2}{3}\times 4\times 20\right)\times 2 \Rightarrow 106.67$

Applying clapeyron's theorem for span AB and BC

$$M_A\frac{L_1}{I_1} + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\times\frac{L_2}{I_2} = -\left[\frac{6a_1\bar{x}_1}{L_1I_1} + \frac{6a_2\bar{x}_2}{L_2I_2}\right]$$

$$M_A\frac{5}{1.5I_1} + 2M_B\left(\frac{5}{1.5I}+\frac{4}{I}\right) + M_C\times\frac{4}{I} = -\left[\frac{6\times 520.83}{5\times 1.5I} + \frac{6\times 106.67}{4I}\right]$$

$$M_A\times\frac{5}{1.5} + 2M_B\left(\frac{5}{1.5}+4\right) + 4M_C = -\left(\frac{6\times 520.83}{5\times 1.5} + \frac{6\times 106.67}{4}\right)$$

$$3.33M_A + 14.67M_B + 4\times(0) = -(416.664 + 160)$$

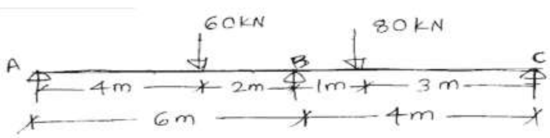
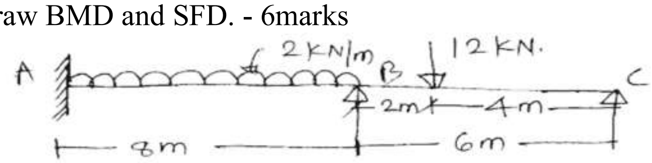
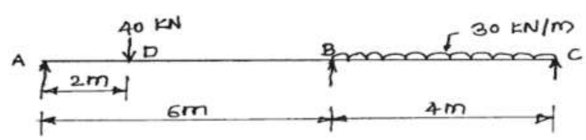
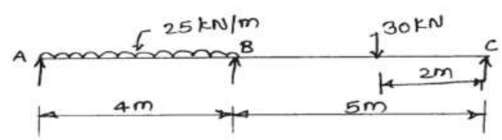
$$3.33M_A + 14.67M_B = -576.664 \quad \text{— ii)}$$

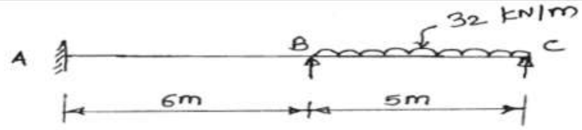
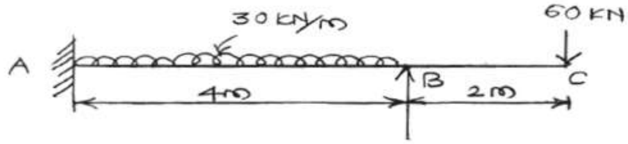
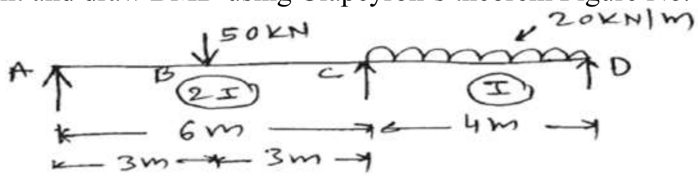
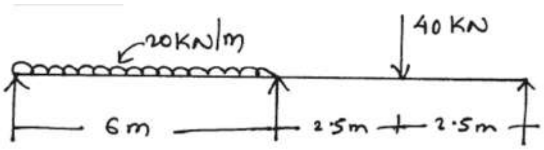
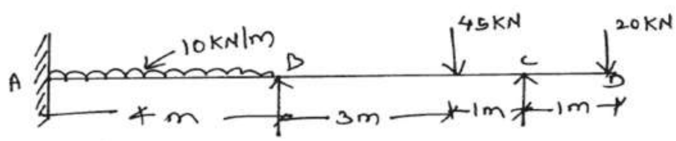
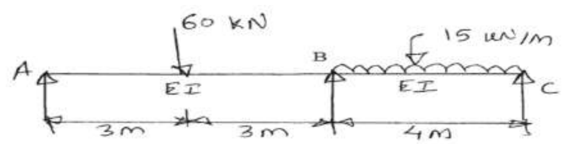
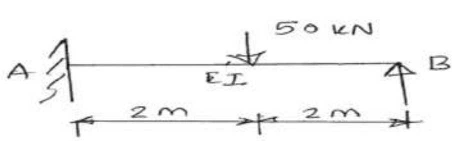
solving eqn. (i) & (ii)

$M_A = -48.319\ \text{KN·m}$, $M_B = -28.34\ \text{KN·m}$

$M_C = 0$ Ans.

Unit-3 Continuous Beam (Question Bank)

Que.No.	**2 marks Questions**
1	Define continuous beam and draw sketch of it.
2	State the effect of continuity in case of continuous beam
3	State the nature of moment induced due to continuity in beams
4	State Clapeyron's theorem
	4 or 6 marks Questions
5	State and explain Clapeyron's theorem of moments,for equal M.I.
6	Write the Clapeyron's three moments theorem for beam with different M.I. giving meaning of each term.draw a neat sketch.
7	State the concept of zero span or imaginary span in case of Clapeyron's theorem
8	A continuous beam ABC is fixed at A and supported at B and C such that AB = 6 m, BC = 5m. Span BC carries a udl of 45 kN/m. Calculate support moments using Clapeyron's theorem and draw B.M. diagram.
9	Draw BMD for continuous beam ABC as shown in Figure No. 3 calculate B.M. at support B using the theorem of three moments.  Fig. No. 3
10	Beam ABCD is simply supported at A and D and continuous over B and C. Calculate distribution factors. Take AB = BC = CD = 6 m and IAB = I, IBC = 2 I,ICD = 1.5 I.
11	For the continuous Beam ABC as shown in Figure No. 5.Calculate the support moments by three moments theorem and draw BMD and SFD. - 6marks  Fig. No. 5
12	A continuous Beam ABC consists of two spans AB and BC of 6 m and 4 m respectively. M.I. of section in AB is twice that of section BC. Span AB carries a central point load of 100 kN and a Udl of 25 kN/m is acting over the entire span BC Considering A and C as simple supports. Draw SFD and BMD giving all its values. Use three moment theorem - 6marks
13	Calculate support moment and draw BM diagram for a continuous beam as shown in Figure No. 1. Use three moments Theorem.  Fig. No. 1
14	Draw SFD for continuous beam as shown in Figure No. 2. Also calculate B.M. at support 'B'. Use three moments theorem.  Fig. No. 2
15	A continuous beam ABC is fixed at A and supported at B and C such that AB = 6 m, BC = 5 m, BC carries a udl of 32 kN/m. Calculate support moments using Clapeyron's theorem and draw B.M. diagram Figure No. 4 -6marks

	 Fig. No. 4
16	Calculate support moments and draw BMD of a beam as shown in Figure No. 4 Use three moment theorem method.  Fig. No. 4
17	Determine support moment and draw BMD using Clapeyron's theorem Figure No. 7. -6amrks  Fig. No. 7
18	Using theorem of three moment, draw SFD for a continuous beam as shown in Fig. No. 1 having negative B.M. at support B as 66.14 KN/m  Fig. No. 1
19	A continuous beam ABCD is loaded and supported as shown in Fig. No. 4. Using three moments theorem. Calculate support moment and draw B.M. Diagram.-6marks  Fig. No. 4
20	Calculate support moment and draw B.M.D. for a continuous beam by using three moment theorem as shown in Fig. No. 2.  Fig. No. 2
21	Calculate support moment and draw BMD and SFD for a beam as shown in Figure No. 6 by using three moment theorem. 6 marks  Fig. No. 6