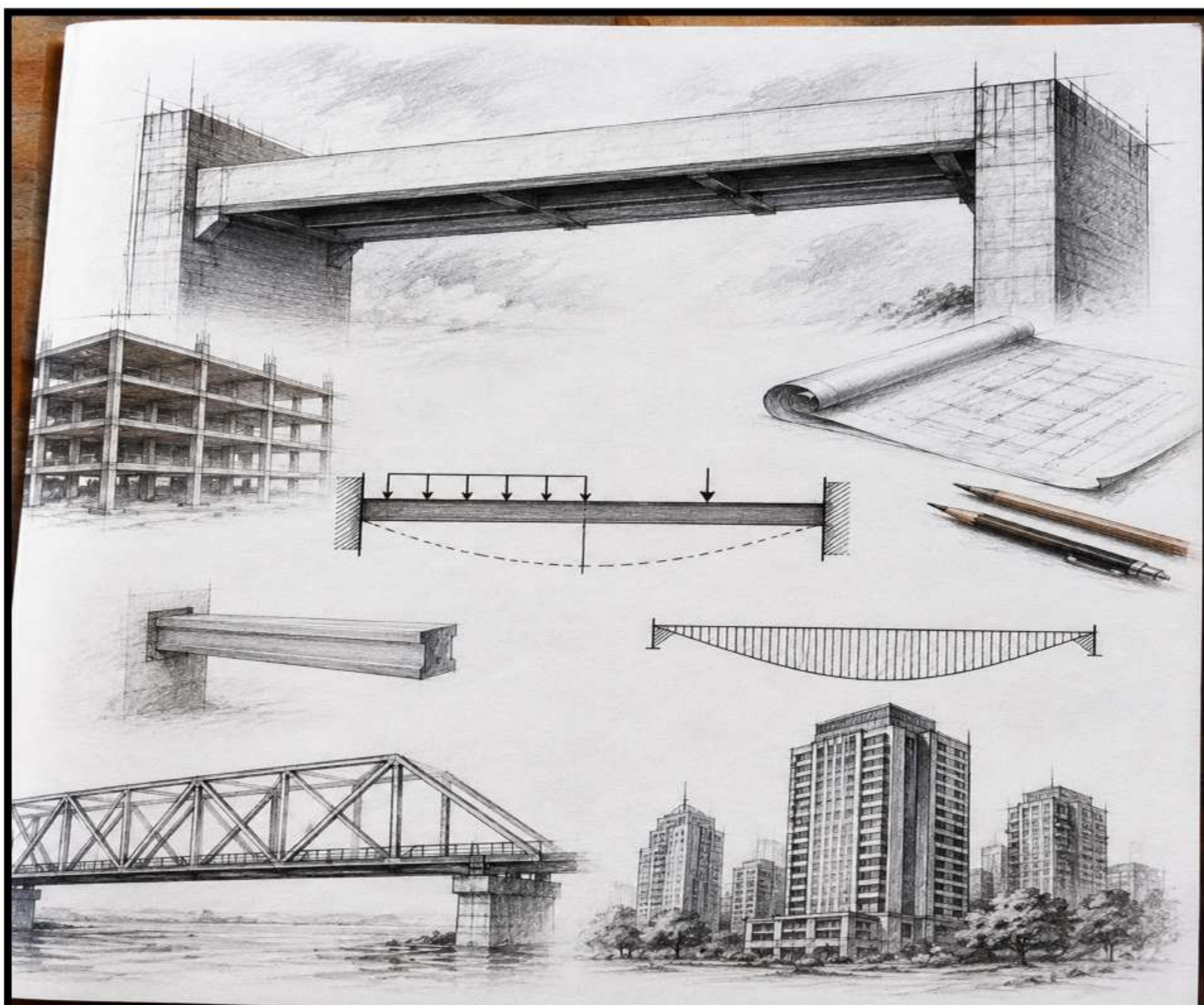
R.C.Patel College Of Engineering & Polytechnic, Shirpur

Department of Civil Engineering

Course Title- Theory of Structure **Course Code** -315313

Programme Name -Civil Engineering **Semester**-Fifth

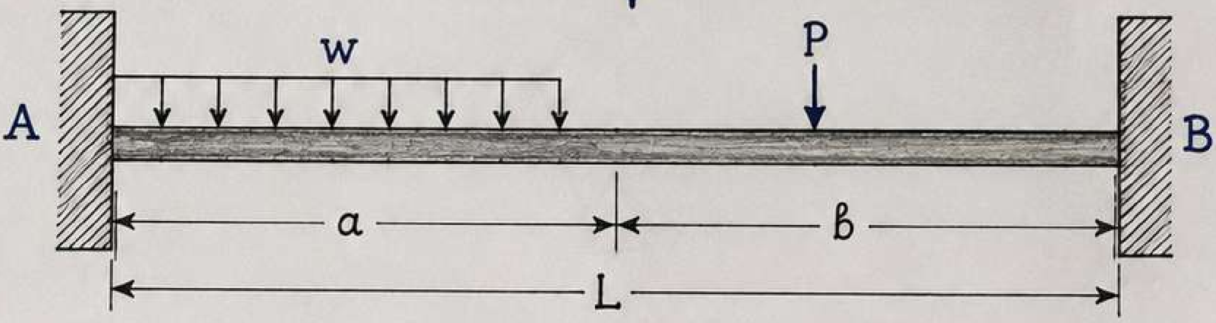
Unit	Title	COs	Learning hours	R Level	U Level	A Level	Total Marks
I	Fixed Beam	CO2	08	2	4	4	10



THEORY OF STRUCTURE

UNIT - II

Fixed Beam



Includes :

- Fixed Beam
- Fixed End Moments
- Slope and Deflection
- Problems based on Fixed Beam

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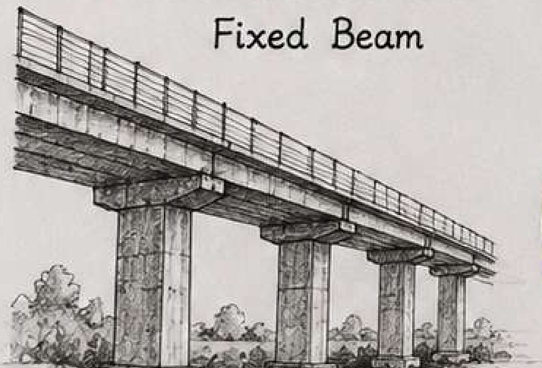
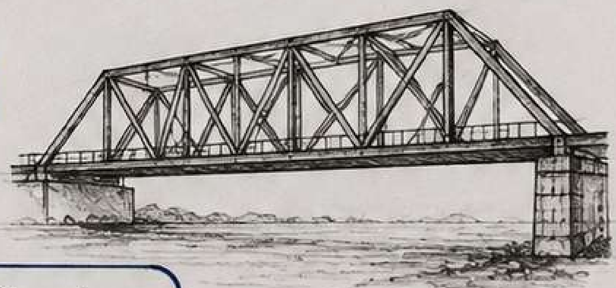




Subject : Theory of Structure

Course Code : 315313

Unit - 02 Fixed Beam

* **Def^n :- Fixed Beam :-** A beam whose end is firmly built in the support like wall, Pillar or any other structure then such beam is called as fixed Beam.

• Fixed end moments are developed at the end supports due to fixity.

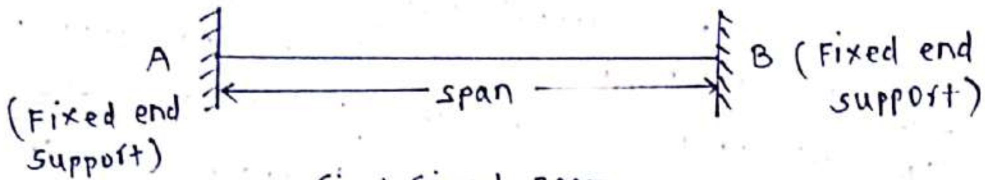
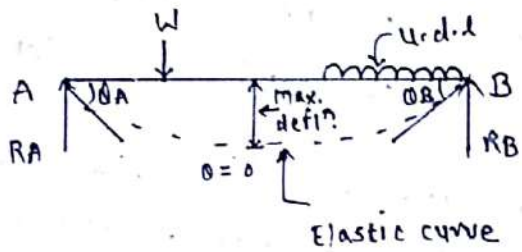


Fig. 1) Fixed Beam.

* **Defination of fixing :-** when a support is restrained against ~~to~~ rotation and vertical movement, then it is called as fixing.

* **concept of fixity, Effect of fixity :-**



θ_A = max. slope at simple support A

θ_B = max. slope at simple support B

fig. 2) simply supported Beam

• The deflected shape or elastic curve of the beam due to the loading is also shown in fig. 2)

• Beam Bends due to the loadings and slope at the support A i.e. θ_A and slope at the support B i.e θ_B and maximum deflection will develop in case of simply supported beam as shown in fig. 2)

• Note that if the ends of the beam is firmly fixed or built in the supports then the behaviour of the elastic curve, slopes at the support, deflection and fixed end moment will be as shown in fig.3>

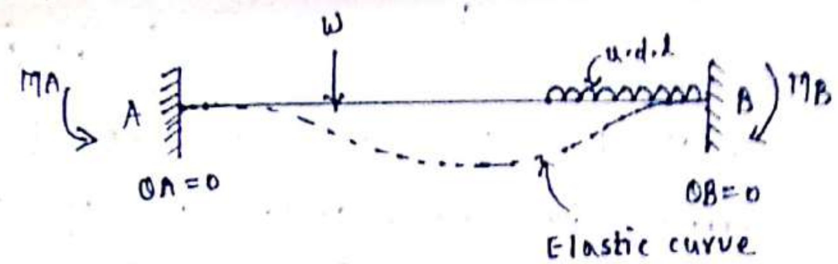


fig.3> Fixed Beam

- Due to absolute fixity at the end supports, the slopes at the support becomes zero i.e. $\theta_A = 0$ and $\theta_B = 0$. Fixity at the end support also induces end moments MA and MB as shown in fig.3> These end moments MA and MB developed due to the fixity are called as Fixed end moments, which are hogging (-ve) in nature prevents the deflection.

* Advantages and Disadvantages of fixed Beam over the simply Supported Beam :-

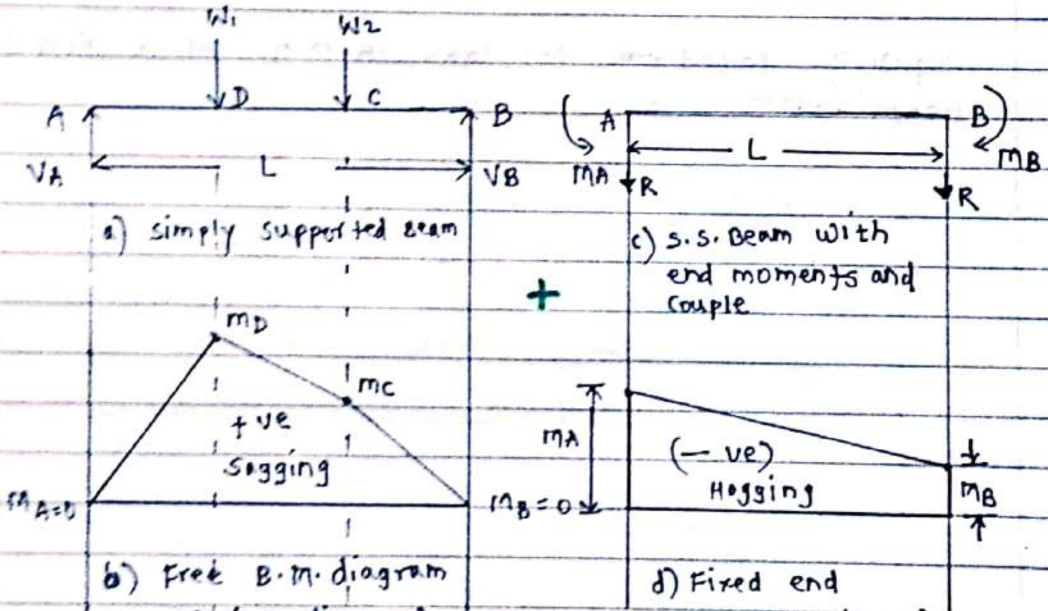
* Advantages :- 1> The slopes at the end support in case of fixed beam are zero but in case of simply supported beam slopes are maximum. 2> In case of fixed beam, the values of Bending moment and deflections are less as compared to a simply supported beam for span and loading.
3> Fixed beam is more strong, stable and stiff than a s.s.B.

* Disadvantages :- 1> If any one of the support sinks to a small extent, it induces additional moment at each end.
2> since the both the end of beam are fixed, temperature stresses are developed due to variation in temperature.
3> The end of Fixed beam can be disturbed due to frequent fluctuations in loading.

* Principle of Superposition :-

It states that " If the Number of forcess or moments are acting simultaneously on a body then their combined effect on the body is equal to the algebric sum of the effects of the individual forces or moments considered seperately."

* Application of principle of superposition for the Analysis of a fixed Beam :-



a) simply supported Beam

b) Free B.M. diagram OR (M- diagram)

c) S.S. Beam with end moments and couple

d) Fixed end moment diagram) OR (M'-diagram)

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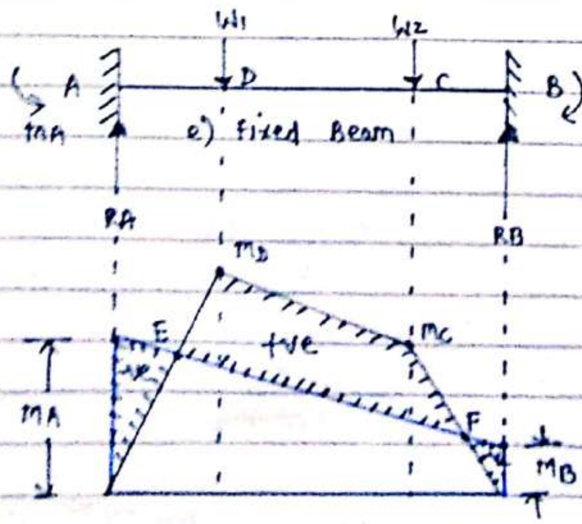


e) fixed Beam

f) Resultant B.M. diagram combined effect of free moments and fixed end moments)

* step wise procedure to Draw S.F.D. of a Fixed Beam :-

step 1) Find the support reactions of a given beam by considering fixed end moments

step 2) Find the shear force on either side of a section of a given beam.

step 3) Draw S.F.D.

* Procedure to find fixed end moments :-

· as per M' diagram we can refering it as shown in fig. (d)

· a' = Area of trapezoidal bending moment diagram of fixed end moment.

$$= -\frac{1}{2}(M_A + M_B) \times L$$

a = Area of M-diagram (sagging)

Now $a = a'$ ... Relation of a and a'

$$= \frac{-1}{2}(M_A + M_B) \times L$$

$$a = \frac{-1}{2}(M_A + M_B) \times L$$

$$\therefore \boxed{M_A + M_B = \frac{-2a}{L}} \quad \text{--- .. (i)}$$

$\bar{X}$ = distance of centroid of M-diagram from the left end.

$\bar{x}'$ = Distance of centroid of M' diagram from the left end.

$$\bar{x}' = \frac{L}{3} \frac{M_A + 2M_B}{M_A + M_B}$$

- Now using the relation

$a\bar{x} = a'\bar{x}'$ we get

$$a\bar{x} = \frac{-1}{2}(M_A + M_B)L \times \frac{L}{3}\left(\frac{M_A + 2M_B}{M_A + M_B}\right)$$

$$= \frac{-L^2}{6}(M_A + 2M_B)$$

$$a\bar{x} = \frac{-L^2}{6}(M_A + 2M_B)$$

$$\therefore \quad M_A + 2M_B = \frac{-6a\bar{x}}{L^2} \quad \text{--- ii)}$$

The fixed end moments M_A and M_B can be determined by solving the simultaneous eqn. (i) and (ii).

* **Fixed End moments by using first Principle**

$$M_A + M_B = -\frac{2a}{L} \quad \text{--- i)}$$

$$M_A + 2M_B = \frac{-6a\bar{x}}{L^2} \quad \text{---- ii)}$$

The above eqn. (i) & (ii) should be used to calculate fixed end moments from the first principle..

Que * Derivation of formula to find fixed end moments ① and S.F.D. and B.M.D for a fixed beam carrying a central point load :-

* Fixed End moments from First principle for Beam subjected to central point load.

fig. (a)

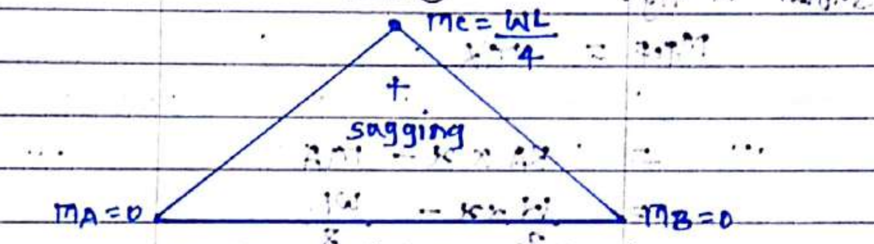


fig. (b) M - diagram

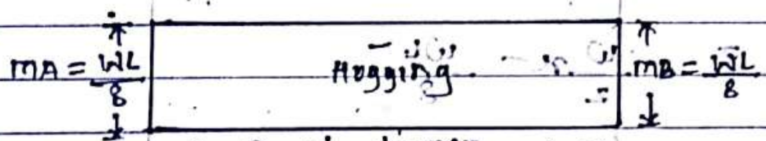


fig. (c) M' - diagram

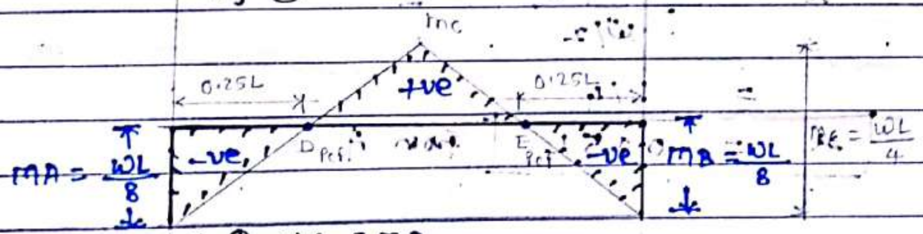


fig. (d) Net B.M.D.

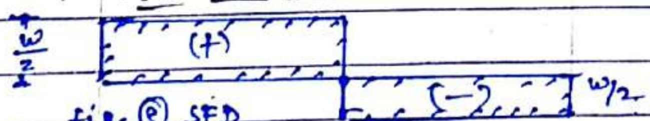


fig. (e) SFD

step 1) To find support Reaction V_A and V_B and free moments by considering the beam to be simply supported.

∴ Support Reactions

$V_A = V_B = \frac{W}{2}$ (due to symmetry)

Free moments

$M_A = 0$, $M_B = 0$, $M_C = \frac{WL}{4}$ (sagging)

(2)

Hence draw M-diagram as shown in fig. (b)

Step 2) To find Fixed end moments i.e. M_A & M_B

Let, M_A = Fixed end moment at A,

M_B = Fixed end moment at B

The M-diagram is a triangle as shown in fig (c)

$\therefore$ a = Area of M-diagram $= \left(\frac{1}{2} \times L \times \frac{WL}{4}\right)$

From the first principle, we have

$$M_A + M_B = \frac{-2a}{L}$$

$$= \frac{-2}{L} \times \left(\frac{1}{2} \times L \times \frac{WL}{4}\right)$$

$$\therefore M_A + M_B = \frac{-WL}{4}$$

But $M_A = M_B$ due to symmetry,

$$\therefore M_A + M_A = \frac{-WL}{4}$$

$$\therefore 2M_A = \frac{-WL}{4}$$

$$\therefore M_A = \frac{-WL}{8}$$

$$\therefore M_B = \frac{-WL}{8} \quad \ldots \text{(Fixed end moments)}$$

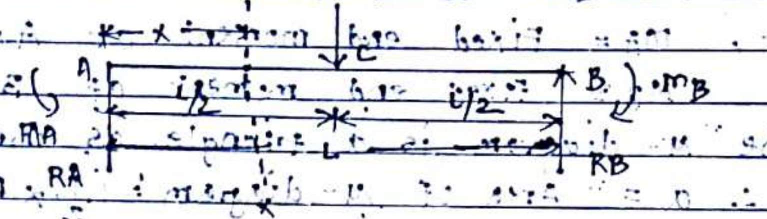
$(\because M_A = M_B)$

... (-ve sign indicate Hogging)

$$M_A = M_B = \frac{WL}{8} \text{ (Hogging)}$$

Hence draw M' diagram as shown in fig. (c) and superimpose it over M-diagram and draw net B.M.D. of a fixed beam as shown in fig. (d)

Step 3) To find support Reactions RA and RB by considering Fixed end moments.



$\therefore \quad RA + RB = W \quad \ldots (1)$

Taking moment at A we get,

$$\Sigma MA = -MA + W \times \frac{L}{2} - RB \times L + MB$$

$$0 = -\cancel{\frac{WL}{8}} + \frac{WL}{2} - RBL + \cancel{\frac{WL}{8}}$$

$$\therefore \quad RBL = \frac{WL}{2}$$

$$\therefore \quad RB = \frac{W}{2}$$

Put the value of RB in eqn. (1) we get

$$RA + \frac{W}{2} = W$$

$$RA = W - \frac{W}{2} \quad \therefore RA = \frac{W}{2}$$

Note: $VA = RA = \frac{W}{2}$ and

$VB = RB = \frac{W}{2}$ because $MA = MB$

also if loading is symmetrical, no reactions will be developed at the support in case of Fixed beam.

Step 4) S.F. calculation

$SFA_L = 0$, $SFA_R = \frac{W}{2}$

$SFC_L = \frac{W}{2}$

$SFC_R = \frac{W}{2} - W = \frac{-W}{2}$, $SFB_L = \frac{-W}{2}$

$SFB_R = 0$

from above S.F. calculation, draw S.F.D. as shown in fig (e)

step (5) To locate the point of contraflexure. i.e Pcf.

Let x be the position of Pcf from A since the Pcf lies in portion AC, take section x-x at Pcf at dist. x in portion AC as shown in fig.

$$M_{pcf} = M_x$$

$$= R_A \times x - M_A$$

$$= \frac{W}{2} \times x - \frac{WL}{8}$$

But at point contraflexure $M_{pcf} = 0$

$$\therefore 0 = \frac{W}{2} x - \frac{WL}{8}$$

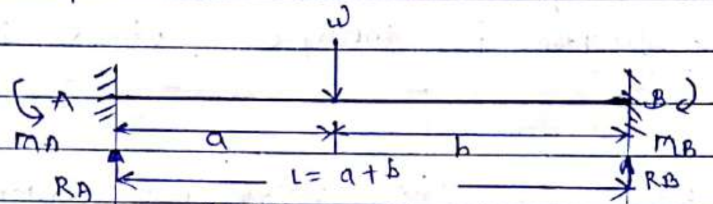
$$x = \frac{WL/8}{W/2}$$

$$= L/4$$

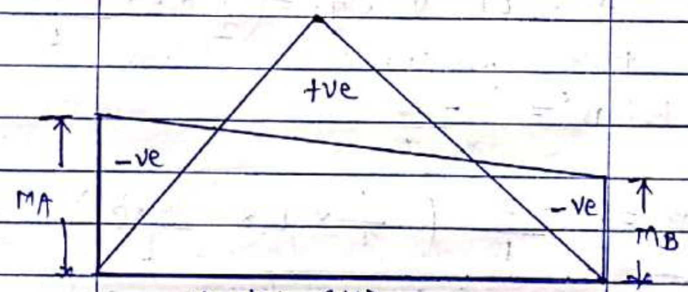
$$\boxed{x = 0.25 L} \text{ from A}$$

* **Fixed End Moments from First principle for Fixed Beam subjected to point load other than mid span :-**

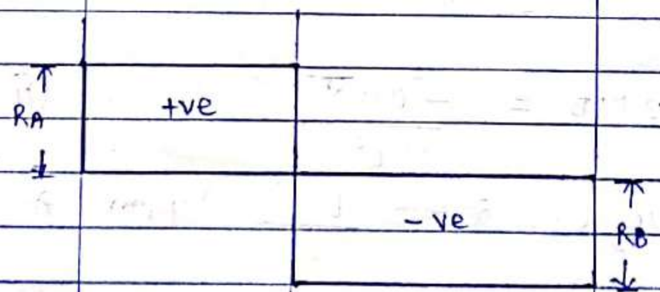
* Derivation of formula to find fixed end moments and S.F.D. and B.M.D. for fixed Beam carrying an Eccentric point load i.e. point load other than mid-span



a) Fixed Beam



b) Net BMD



c) S.F.D.

Consider a fixed Beam AB of span L carrying an eccentric point load as shown in fig.

step 1) To find support Reactions VA and VB and free moments be considering the beam to be simply supported.

$$V_A = \frac{Wb}{L} \quad \text{and} \quad V_B = \frac{Wa}{L}$$

free moments:

$$M_A = 0, \quad M_B = 0, \quad M_C = \frac{Wab}{L} \text{ (sagging)}$$

Draw M-diagram as per above calculation.
the M-diagram is triangle whose height is $\frac{Wab}{L}$

* step 2) To find fixed end moments M_A & M_B

$$a = \text{Area of M dig.} = \left(\frac{1}{2} \times L \times \frac{Wab}{L}\right)$$

from first principle we have

$$M_A + M_B = \frac{-2a}{L}$$

$$\therefore M_A + M_B = \frac{-2}{L}\left(\frac{1}{2} \times L \times \frac{Wab}{L}\right)$$

$$\therefore \boxed{M_A + M_B = \frac{-Wab}{L}} \quad \text{— (i)}$$

$$M_A + 2M_B = \frac{-6a\bar{x}}{L^2} \quad \text{(ii)}$$

For triangle, $\bar{x} = \frac{L+a}{3}$ from A

put the values of a and $\bar{x}$ in equation (ii), we get:

$$\therefore M_A + 2M_B = \frac{-6}{L^2}\left(\frac{1}{2} \times \cancel{L}\, \frac{Wab}{\cancel{L}}\right) \times \left(\frac{L+a}{\cancel{3}}\right)$$

$$\therefore M_A + 2M_B = \frac{-(L+a) \times Wab}{L^2} \quad \text{— (iii)}$$

solve the simultaneous eqn. (i) and (iii) as follows

$$M_A + M_B = -wab/L$$

$$M_A + 2M_B = -\frac{(L+a)\,wab}{L^2}$$

$$0 - M_B = -\frac{wab}{L} + \frac{(L+a)\,wab}{L^2}$$

$$= \frac{-wab\times L + wab\times L + wa^2b}{L^2}$$

$$\boxed{M_B = \frac{wba^2}{L^2}}$$ ... fixed end moment at B

... (-ve sign indicate Hogging moment)

substituting the value of M_B in eqn. (i) we get

$$M_A - \frac{wba^2}{L^2} = -\frac{wab}{L}$$

$$M_A = \frac{-wab}{L} + \frac{wba^2}{L^2}$$

$$= -\frac{wab\times L + wba^2}{L^2}$$

$$= -\frac{wab\,(L-a)}{L^2}$$

$$M_A = \frac{-wab\,(\cancel{a}+b-\cancel{a})}{L^2} \qquad (\because L = a+b)$$

$$= \frac{-wab}{L^2}\times b$$

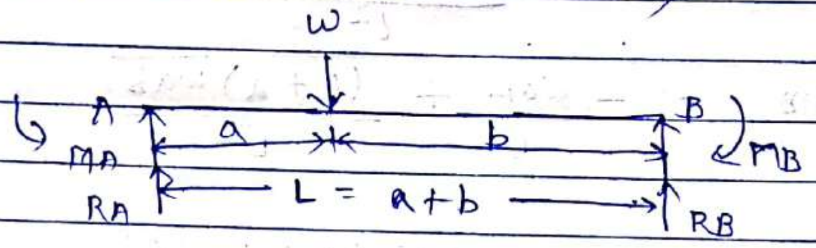
$$= \frac{-wab^2}{L^2}$$

$$\therefore \boxed{M_A = \frac{-wab^2}{L^2}}$$ ... Fixed end moment at A

(-ve sign indicate hogging moment)

step 3) To find Reactions RA and RB considering the fixed end moments MA and MB

$RA + RB = W$ — iv)



Taking moment at A we get

[+) (-] $\Sigma MA = -MA + Wa - RB \times L + MB$

$$0 = \frac{-Wab^2}{L^2} + Wa - RB \times L - \frac{Wba^2}{L^2} \; ; (\Sigma MA = 0)$$

$$RB \times L = \frac{-Wab^2}{L^2} + Wa - \frac{Wba^2}{L^2}$$

$$\therefore \boxed{RB = \frac{1}{L}\left(-\frac{Wab^2}{L^2} + Wa - \frac{Wba^2}{L^2}\right)} \quad \text{(v)}$$

substituting the value of RB from eqn (v) in eqn. (iv) we get

$$RA = W - RB$$

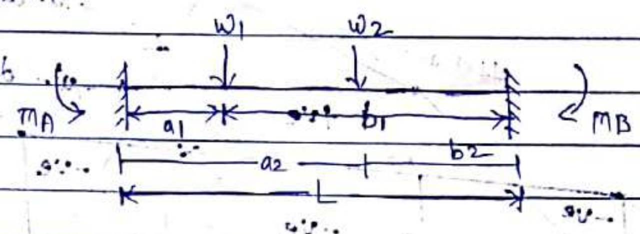
$$\therefore RA = W - \frac{1}{L}\left(\frac{-Wab^2}{L^2} + Wa - \frac{Wba^2}{L^2}\right)$$

$$\therefore \boxed{RA = W - \frac{1}{L}\left(-\frac{Wab^2}{L^2} + Wa - \frac{Wba^2}{L^2}\right)} \quad \text{— (vi)}$$

step 4) Draw S.F.D. from the RA & RB as show in fig

* Fixed Beam carrying Two point loads or concentrated loads :-

Important formulae :-



MA = fixed end moment at A

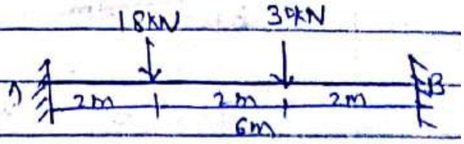
$$= -\frac{W_1 a_1 b_1^2}{L^2} - \frac{W_2 a_2 b_2^2}{L^2}$$

M_B = Fixed end moment at 'B'

$$= -\frac{W_1 b_1 a_1^2}{L^2} - \frac{W_2 b_2 a_2^2}{L^2}$$

Ex:-1) A fixed beam 6m in span is subjected to two point loads of 18 KN and 30 KN at 2m and 4m from left hand support, calculate Net B.M. under point load and draw B.M. diagram showing net bending moments. Draw S.F diagram.

→ 1) Assume that the beam is simply supported and calculate the support reaction.



$\Sigma fy = 0$; $R_A + R_B = 18 + 30 = 48 kN$ — i)

$\Sigma M_A = 0$; $(18 \times 2) + 30 \times 4 - R_B \times 6 = 0$

$$\therefore R_B = \frac{36 + 120}{6}$$

$\therefore$ $\boxed{R_B = 26 kN}$ $\therefore$ $R_A = 48 - 26$

$\boxed{R_A = 22 kN}$

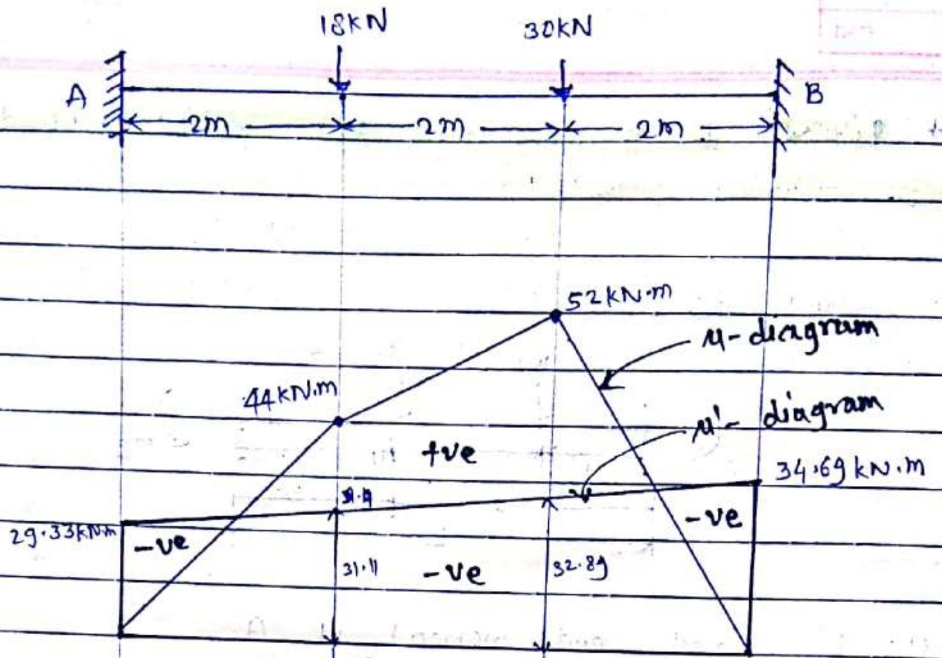


fig. Final BMD

$M_A = M_B = 0$

$\therefore M_C = 22 \times 2 = 44$ kN.m

$M_D = 26 \times 2 = 52$ kN.m

step 2) calculate the fixed end moments

Here, $w_1 = 18$ kN , $w_2 = 30$ kN.

$a_1 = 2m$, $a_2 = 4m$

$b_1 = 4m$, $b_2 = 2m$

$$\therefore M_A = -\frac{w_1 a_1 b_1^2}{L^2} - \frac{w_2 a_2 b_2^2}{L^2}$$

$$= \frac{-18 \times 2 \times 4^2}{6^2} - \frac{30 \times 4 \times 2^2}{6^2}$$

$$\boxed{M_A = -29.33 \text{ kN m}}$$

$$M_B = \frac{-w_1 b_1 a_1^2}{L^2} - \frac{w_2 b_2 a_2^2}{L^2}$$

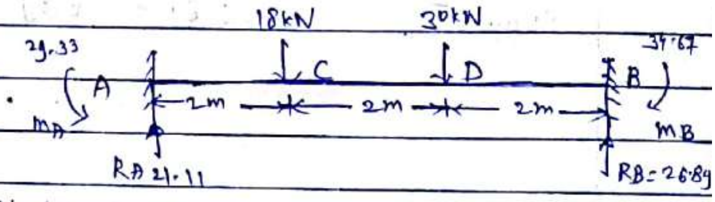
$$= \frac{-18 \times 4 \times 2^2}{6^2} - \frac{30 \times 2 \times 4^2}{6^2}$$

$$\boxed{M_B = -34.67 \text{ kN.m}}$$

3) Calculate Reaction of a fixed Beam

$\sum F_y = 0$; $R_A + R_B = 48 \text{ kN}$

$\sum M_A = 0$



$-29.33 + (18 \times 2) + (30 \times 4) + 34.67 - R_B \times 6 = 0$

$$\therefore R_B = \frac{-29.33 + (36) + 120 + 34.67}{6}$$

$$\boxed{R_B = 26.89 \text{ kN}}$$

$\therefore R_A = 48 - 26.89$

$$\boxed{R_A = 21.11 \text{ kN}}$$

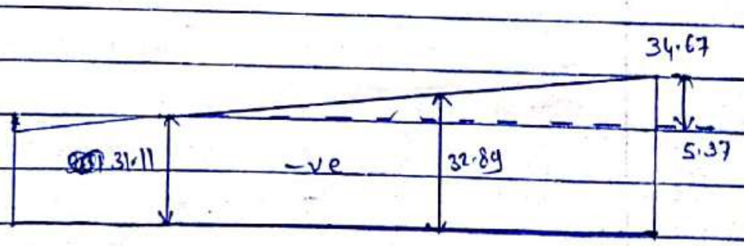
4) Net B.M. at C

$M_C = -29.33 + R_A \times 2 = -29.33 + 21.11 \times 2$

$M_C = +12.89 \text{ kN.m}$

5) Net B.M. at D

$M_D = -34.99 + R_B \times 2 = -34.99 + 26.89 \times 2 = 19.11 \text{ kN.m}$



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SF calculation

$SF_{AL} = 0$

$SF_{AR} = R_A = 21.11\ kN$

$SF_{CL} = 21.11\ kN$

$SF_{CR} = 21.11 - 18 = 3.11\ kN$

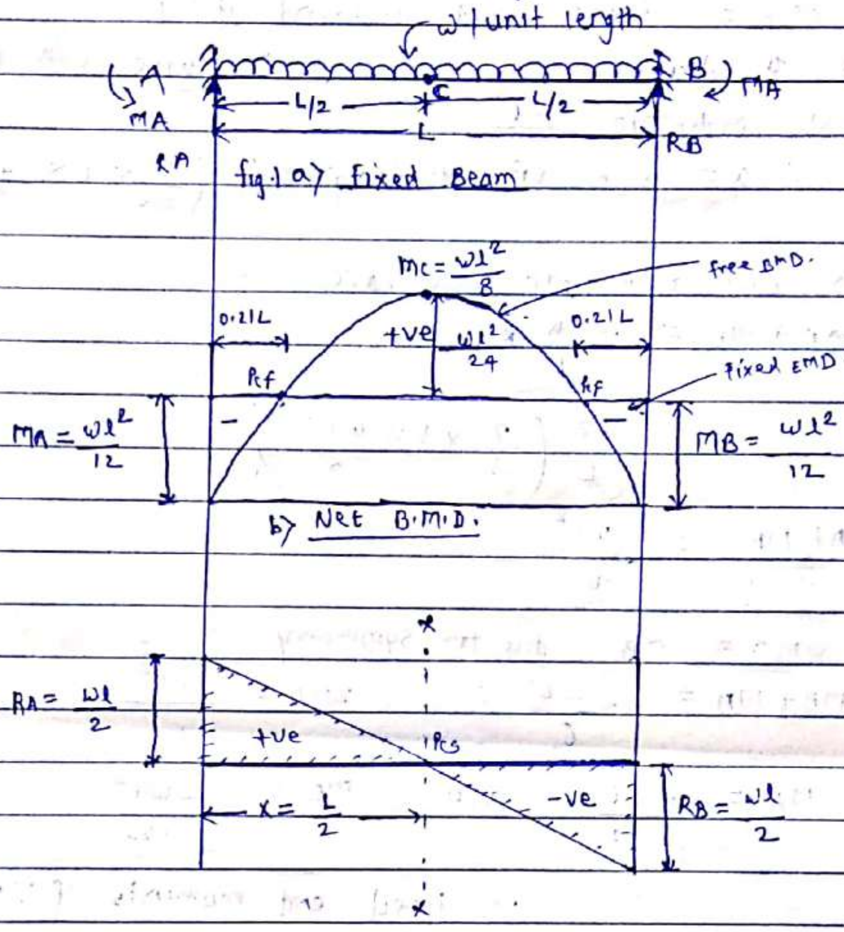
$SF_{DL} = 3.11\ kN$

$SF_{DR} = 3.11 - 30 = -26.89\ kN$

$SF_{BL} = -26.89\ kN$

$SF_{BR} = -26.89 + 26.89 = 0$

* Fixed End moments from first principle for fixed Beam subjected to udl over entire span:



c) S.F.D.

Consider a fixed Beam AB of span 'L' carrying the uniformly distributed load (udl) over entire span as shown in fig. ⓐ

Step 1) To find support reaction V_A and V_B and free moments by considering the beam to be simply supported.

* Support Reactions $V_A = V_B = \frac{wL}{2}$ (due to symmetry)

* free moments = $M_A = M_B = 0$

$M_C = M_{max} = \frac{wL^2}{8}$ (sagging)

Hence draw M diagram which will be parabola with central ordinate $\frac{wL^2}{8}$ as shown in Fig. ⓑ

step 2 To find fixed end moments MA and MB

Let M_A = Fixed end moment at A.

M_B = Fixed end moment at B

The M diagram is a parabolic curve with the central ordinate $\frac{WL^2}{8}$

$\therefore a$ = area of M-diagram $= \left(\frac{2}{3} \times L \times \frac{WL^2}{8}\right)$

From First principle, we have

$$M_A + M_B = \frac{-2a}{L}$$

$$= \frac{-2}{L}\left(\frac{2}{3} \times L \times \frac{WL^2}{8}\right)$$

$$\therefore M_A + M_B = \frac{-WL^2}{6}$$

But $M_A = M_B$ due to symmetry

$\therefore M_A + M_A = \frac{-WL^2}{6}$, $2M_A = \frac{-WL^2}{6}$

$\therefore M_A = \frac{-WL^2}{12}$ and $M_B = \frac{-WL^2}{12}$

... Fixed end moments ($\because M_A = M_B$)

... (-ve sign indicate hogging moment)

$\therefore M_A = M_B = \frac{WL^2}{12}$ (Hogging)

Due to symmetry of loading, $M_A = M_B$, hence M'-diagram is a rectangle. superimpose both the diagrams i.e. M-diagram and M' diagram so as to get net BMD as shown in Fig. b step ③ : To find reactions RA & RB by considering fixed end moments

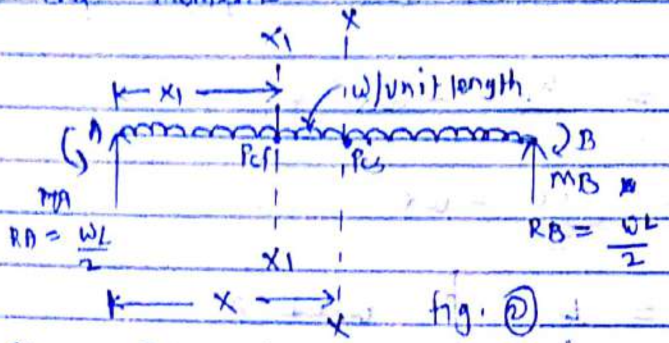


fig. ②

Refer fig. for further calculations.

$$R_A + R_B = \omega L \quad \text{—i)}$$

(+) Taking moment at 'A'

$$\Sigma M_A = -M_A + w \times L \times \frac{L}{2} - R_B \times L + M_B$$

$$0 = -\frac{\cancel{wL^2}}{\cancel{12}} + \frac{wL^2}{2} - R_B \times L + \frac{\cancel{wL^2}}{\cancel{12}} \qquad (\Sigma M_A = 0)$$

$$0 = \frac{wL^2}{2} - R_B \times L \quad \therefore R_B = \frac{wL}{2}$$

put RB in eqn. ①, we get

$$R_A + \frac{wL}{2} = wL \qquad \therefore \boxed{R_A = \frac{wL}{2}}$$

* Note → $V_A = R_A = \frac{wL}{2}$, $V_B = R_B = \frac{wL}{2}$

because $M_A = M_B$.

Step 4) S.F. calculation

$SF_{L} = 0$

$SF_{R} = \frac{wL}{2}$

$SF_{BL} = \frac{wL}{2} - w = -\frac{wL}{2}$

$SF_{BR} = 0$

from above calculation draw S.F.D.

step 5) To locate the point of zero shear Pcs :

let x be position or location of Pcs from A.
consider a section x-x passing through Pcs as shown in fig. ②

$\therefore F_x = F_{pcs}$

= shear force at point of contra shear

$\therefore F_x = \frac{wL}{2} - wx$

$0 = \frac{wL}{2} - wx$

$\therefore \boxed{x = \frac{L}{2}}$ w.r.t. A.

* step ⑥ To locate Point of contraflexure (Pcf)

let x be the position of pcf from A. consider a section $x_1 - x_1$ passing through Pcf as shown in fig. ②

let $M_{pcf} = M_{x_1}$

= Bending at the point of contraflexure.

$\therefore M_{pcf} = M_{x_1} = -M_A + \frac{wL}{2} \times x_1 - \left(w \times x_1 \times \frac{x_1}{2}\right)$

$= \frac{-wL^2}{12} + \frac{wL}{2} x_1 - \frac{w x_1^2}{2}$

$0 = \frac{-wL^2}{12} + \frac{wL}{2} x_1 - \frac{w x_1^2}{2}$ $(\because M_{Pcf} = M_{x_1} = 0)$

* Note → that at the point of contraflexure, B.M. is zero i.e. $M_{pcf} = 0$

Dividing the above eqn both sides by $\frac{w}{2}$ we get

$Lx - x^2 - \frac{L^2}{6} = 0$ $\therefore x^2 - Lx + \frac{L^2}{6}$

This is a quadratic equation.

Comparing this eqn with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -L$ and $c = L^2/6$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

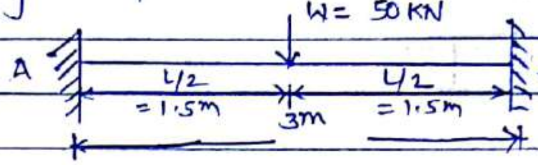
$= \frac{-(-L) \pm \sqrt{(-L)^2 - 4 \times 1 \times L^2/6}}{2 \times 1}$

$= \frac{L \pm \sqrt{L^2 - \frac{2L^2}{3}}}{2}$

$\therefore x = \frac{L \pm 0.577L}{2} = \frac{L}{2}(1 \pm 0.58)$

$\therefore x = 0.79L$ OR $0.21L$ Take $x = 0.21L$

Ex.1) A point load of 50 kN acts at centre on a fixed beam of span 3m. Using first principle, determine fixed end moment. Draw net BMD indicating important values.



Given :- $l = 3\,m$, $W = 50\,kN$

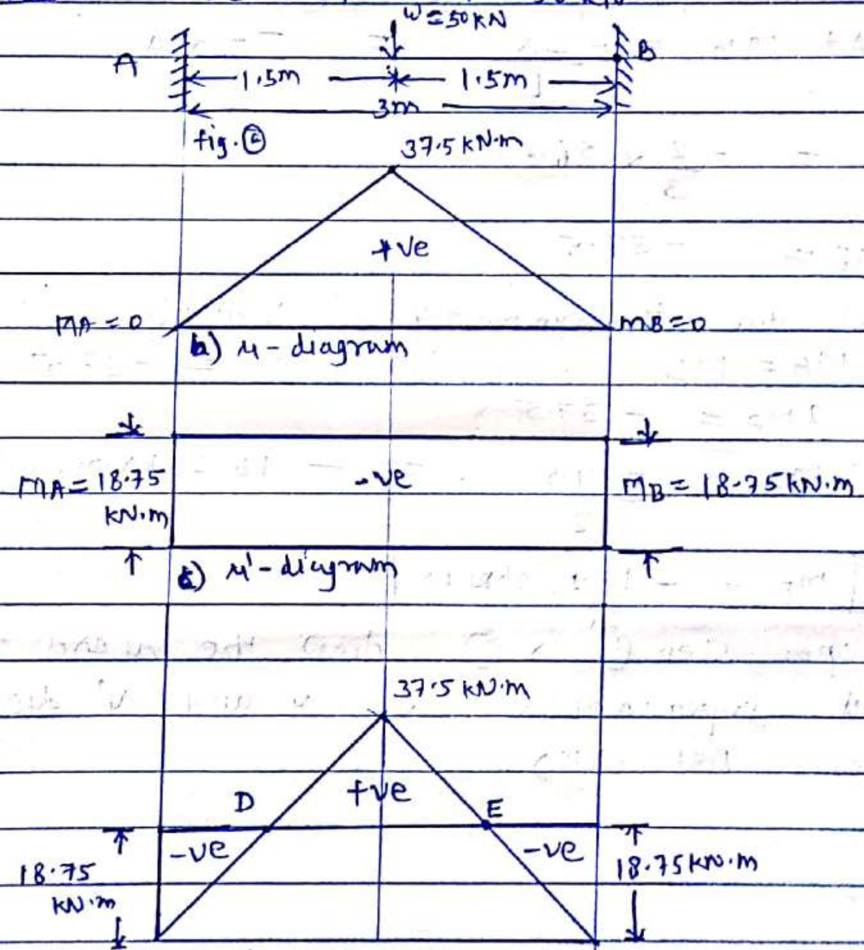


fig d) Net B.M.D.

step 1) To find free B.M. by considering the beam to be simply supported.

$\therefore$ free B.M.D. = $M_A = M_B = 0$

$$M_C = M_{max} = \frac{WL}{4} = \frac{50 \times 3}{4} = 37.5\,kN{\cdot}m \ (+ve)$$

step 2) To find fixed end moments M_A and M_B by first principle.

$a =$ Area of M diagram

$$a = \frac{1}{2} \times 3 \times 37.5$$

$$\boxed{a = 56.25}$$

from first principle we have,

$$M_A + M_B = \frac{-2a}{L} = \frac{-2}{3} \times a$$

$$= \frac{-2}{3} \times 56.25$$

$$M_A + M_B = -37.5$$

But due to symmetry of loading,

$M_A = M_B$ ∴ $M_A + M_A = -37.5$

$$\therefore 2M_A = -37.5$$

$$\therefore M_A = \frac{-37.5}{2} = -18.75 \text{ kN.m.} \quad \text{Ans.}$$

$$\therefore \boxed{M_B = -18.75 \text{ kN.m}}$$

as per step ① & ② draw the M and M' diagram and superimpose the M and M' diagram gives net BMD.

Ex:1) A fixed beam of span 4m carries a point load of 20 kN at 1m from left hand support. find the fixed end moments by first principle and draw S.F.D. & B.M.D.

→

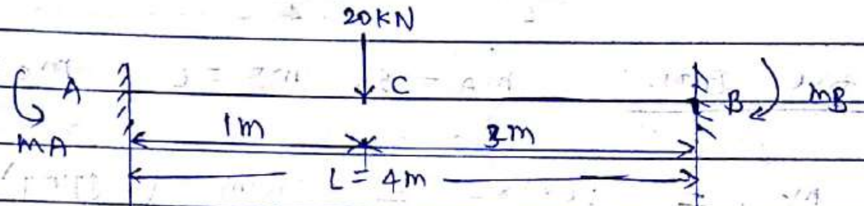


fig. (a) fixed beam

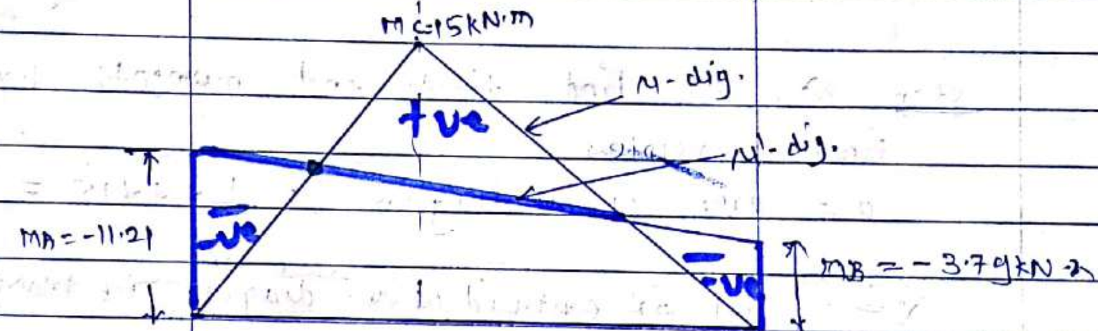


fig. (b) Net B.M.D.

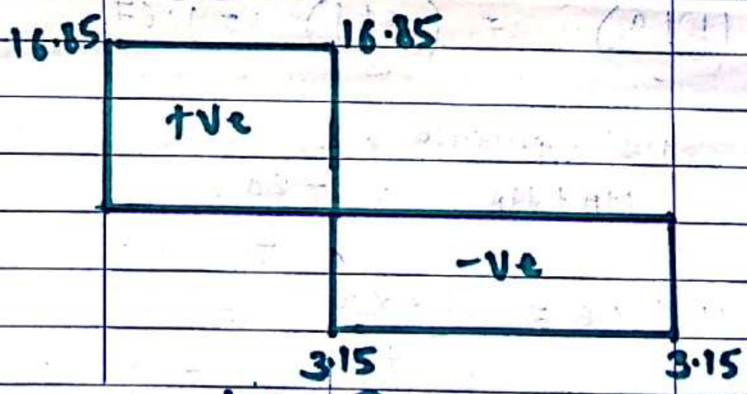
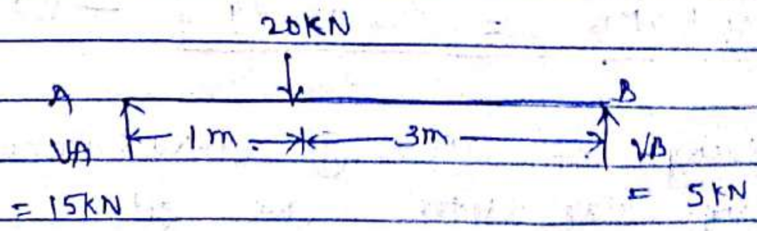


fig. (c) S.F.D.

Given :- $L = 4m$, $W = 20kN$, $a = 1m$, $b = 3m$.

Step 1) To find supports Reactions and free B.M. by assuming the beam to be simply supported.



Support Reactions

$$V_A = \frac{Wb}{L} = \frac{20 \times 3}{4} = 15\,KN$$

$$V_B = \frac{Wa}{L} = \frac{20 \times 1}{4} = 5\,KN$$

Free B.M. $\quad M_A = 0, \quad M_B = 0, \quad M_C = \frac{Wab}{L}$

$$M_C = \frac{20 \times 1 \times 3}{4} = 15\,KN{\cdot}m \quad \text{(sagging)}$$

step 2) To find fixed end moments by First principle.

a = area of M diagram $= \frac{1}{2} \times 4 \times 15 = 30$

$\bar{X}$ = dist. of centroid of M-diagram i.e triangle from A

$$\bar{x} = \left(\frac{L+a}{3}\right) = \left(\frac{4+1}{3}\right) = 1.67$$

by first principle,

$$M_A + M_B = \frac{-2a}{L}$$

$$M_A + M_B = \frac{-2 \times 30}{4}$$

$$\boxed{M_A + M_B = -15} \quad \text{— i)}$$

$$M_A + 2M_B = \frac{-6a\bar{x}}{L^2}$$

$$M_A + 2M_B = \frac{-6 \times 30 \times 1.67}{4^2}$$

$$\boxed{M_A + 2M_B = -18.79} \quad \text{— ii)}$$

solving eqn. (i) & (ii) simultaneously or by using calculator we get

$$M_A + M_B = -15$$

$$\underline{M_A + 2M_B = -18.79}$$

$$-M_B = 3.79$$

$\boxed{M_B = -3.79\ kN\cdot m}$ (-ve sign indicate Hogging B.m.)

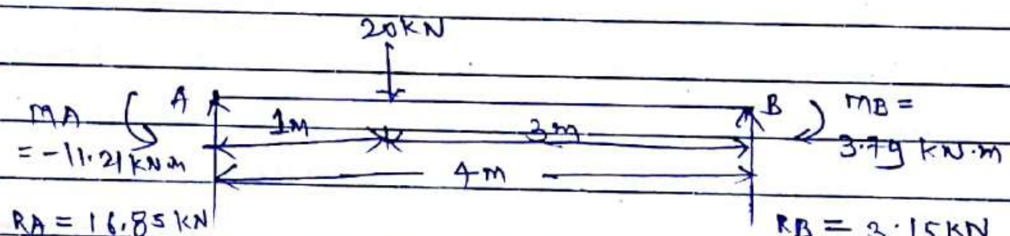
put in eqn. (1) we get MA

$M_A - 3.79 = -15$

$M_A = -15 + 3.79$

$\boxed{M_A = -11.21\ kN\cdot m}$

Step (3) to find support Reactions RA and RB by considering the fixed end moments.



$R_A + R_B = 20\ kN$ ——— (III)

↻ (-ve) taking moment @ A

↺ (+ve) $\Sigma M_A = -11.21 + 20\times 1 - \cancel{3.15}^{R_B} \times 4 + 3.79$

$0 = 12.58 - 4R_B$

$4R_B = 12.58$

$\boxed{R_B = 3.15\ kN}$

$\therefore R_A = 20 - R_B = 20 - 3.15$

$\boxed{R_A = 16.85\ kN}$

Step (4) S.F.D. calculation

$SF_{AL} = 0$

$SF_{AR} = 16.85\ kN$

$SF_{CL} = 16.85$

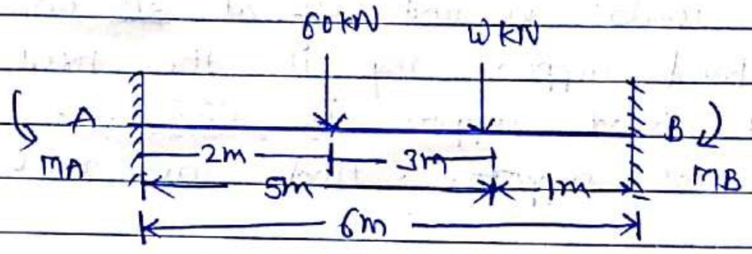
$SF_{CR} = -3.15\ kN$ [16.85 - 20]

$SF_{BL} = -3.15\ kN$

$SF_{BR} = 0$

Ex: A fixed Beam of span 6m carries two point loads 60 kN and 'W' kN at 2m and 5m from left support resp. If fixed end moments at both ~~ends~~ supports are equal, calculate 'W'

→



Given :- $L = 6m$, $w_1 = 60\ kN$, $w_2 = W$

$a_1 = 2m$, $b_1 = 4m$

$a_2 = 5m$, $b_2 = 1m$, $M_A = M_B$

where, M_A = fixed end moment at A

M_B = fixed end moment at B

To find 'W'

$$M_A = -\frac{w_1 a_1 b_1^2}{L^2} - \frac{w_2 a_2 b_2^2}{L^2}$$

$$= -\frac{60 \times 2 \times 4^2}{6^2} - \frac{W \times 5 \times 1^2}{6^2}$$

$$M_A = -53.33 - 0.1388\,W \quad \text{--- i)}$$

$$M_B = -\frac{w_1 b_1 a_1^2}{L^2} - \frac{w_2 b_2 a_2^2}{L^2}$$

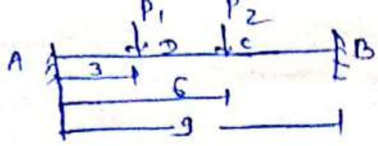
$$= -26.666 - 0.6944\,W \quad \text{--- ii)}$$

But $M_A = M_B$ which is given, hence equating eqn. (i) and (ii) we get

$$-53.33 - 0.1388\,W = -26.666 - 0.6944\,W$$

$$-53.33 + 26.66 = -0.6944\,W + 0.1388\,W$$

$$-26.667 = W(-0.6944 + 0.1388)$$



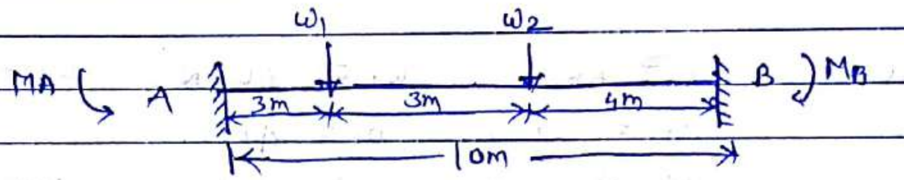
$\frac{P_1}{P_2} = 0.4$

$$-26.667 = W \times (-0.5556)$$

$$W = \frac{-26.667}{-0.5556}$$

$$\boxed{W = 47.99\,kN \cong 48\,kN}$$

S-18 Ex:- A fixed beam span 10m carries two point loads, w_1 and w_2 at 3m and 6m from left hand support resp. if the fixed end moment at left hand support is 1.25 times that of right hand support. find the ratio of w_1 and w_2.



Given :- $M_A = 1.25\, M_B$

$a_1 = 3,\ b_1 = 7$
$a_2 = 6,\ b_2 = 4$

$$\therefore M_A = -\frac{w_1 a_1 b_1^2}{L^2} - \frac{w_2 a_2 b_2^2}{L^2}$$

$$= -\frac{w_1 \times 3 \times 7^2}{10^2} - \frac{w_2 \times 6 \times 4^2}{10^2}$$

$$M_A = -1.47 w_1 - 0.96 w_2 \quad \text{--- i)}$$

$$M_B = -\frac{w_1 b_1 a_1^2}{L^2} - \frac{w_2 b_2 a_2^2}{L^2}$$

$$= -\frac{w_1 \times 7 \times 3^2}{10^2} - \frac{w_2 \times 4 \times 6^2}{10^2}$$

$$M_B = -0.63 w_1 - 1.44 w_2 \quad \text{--- ii)}$$

But $M_A = 1.25\, M_B$

equating eqn. (i) & (ii)

$$-1.47 w_1 - 0.96 w_2 = [-0.63 w_1 - 1.44 w_2] \times 1.25$$

$$-1.47 w_1 - 0.96 w_2 = -0.7875 w_1 - 1.8 w_2$$

$$-1.47 w_1 + 0.7875 w_1 = -1.82 w_2 + 0.96 w_2$$

$$\omega_1(-1.47 + 0.7875) = \omega_2(-1.8 + 0.96)$$

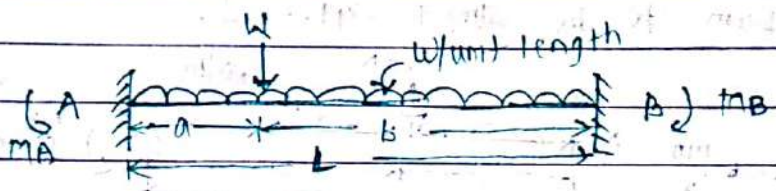
$$-0.6825\omega_1 = -0.84\omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{-0.84}{0.682}$$

$$\boxed{\frac{\omega_1}{\omega_2} = 1.23}$$

* Examples on a fixed Beam carrying u.d.l of w/unit length over Entire span an Eccentric point load 'W'

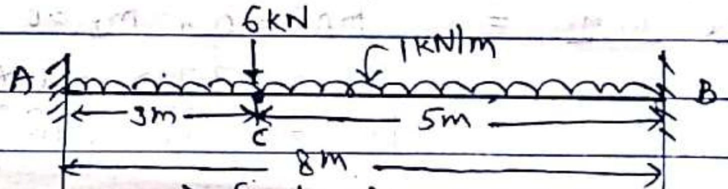
Important formulae :



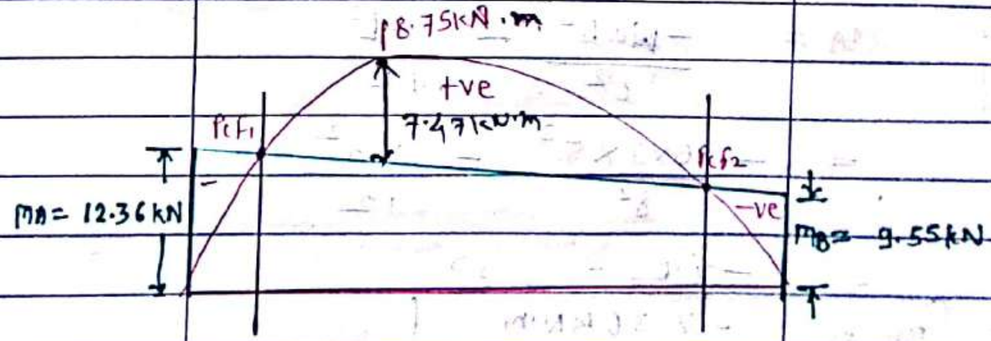
$$M_A = -\frac{Wab^2}{L^2} - \frac{wL^2}{12}$$

$$M_B = -\frac{Wba^2}{L^2} - \frac{wL^2}{12}$$

Ex:- A fixed beam 8m span carries a u.d.l. of 1KN/m and a point load of 6KN at 3m from left support. calculate fixed end moments and draw BMD. also calculate support reactions.



a) fixed Beam



step 01 → To find support Reactions VA and VB by assuming the beam to be simply supported.

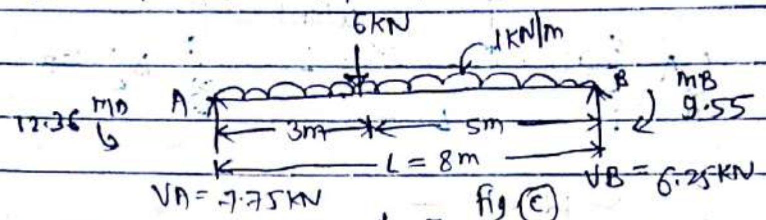


fig (C)

Support Reactions :- $\uparrow + \downarrow -$, $\Sigma F_y = V_A - 6 - (1\times8) + V_B$

$0 = V_A - 14 + V_B$

$\therefore V_A + V_B = 14 \quad \text{—i)}$

$\Sigma M_A = 6\times3 + 1\times8\times8/2 - V_B\times8$

$0 = 18 + 32 - 8V_B$

$\therefore \boxed{V_B = 6.25\,KN}$

substituting the value of V_B in eqn. (i)

$V_A + 6.25 = 14 \quad \therefore \boxed{V_A = 7.75\,KN}$

free B.M. = $M_A = 0,\ M_B = 0$

$M_C = 7.75\times3 - (1\times3)\times1.5$

$\boxed{M_C = 18.75\,KN\cdot m}$

step 2> To find fixed end moments MA and MB

$M_A = -\dfrac{Wab^2}{L^2} - \dfrac{wL^2}{12}$

$= -\dfrac{6\times3\times5^2}{8^2} - \dfrac{1\times8^2}{12}$

$= -7.03 - 5.33$

$\boxed{M_A = -12.36\,KN\cdot m}$

$M_B = -\dfrac{Wba^2}{L^2} - \dfrac{wL^2}{12}$

$= -\dfrac{6\times5\times3^2}{8^2} - \dfrac{1\times8^2}{12}$

$= -4.218 - 5.33$

$= -9.548\ KNm$

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step (3) To calculate support Reactions R_A and R_B by considering the fixed end moments.

Refer fig. (c)

$$R_A + R_B = 6 + (1\times8) = 14\,kN \quad \text{--- ii)}$$

$\Sigma M_A = -12.36 + 6\times3 + (1\times8)\times 8/2 - R_B\times8 + 9.55$

$0 = 47.19 - 8R_B \quad \therefore 8R_B = 47.19$

$\therefore \boxed{R_B = 5.89\,kN}$

substitute the value of R_B in eqn. (ii) we get

$R_A + 5.89 = 14$

$R_A = 14 - 5.89 = 8.11\,kN$

* step (4) To locate the point of contraflexure P_{cf_1} & P_{cf_2}

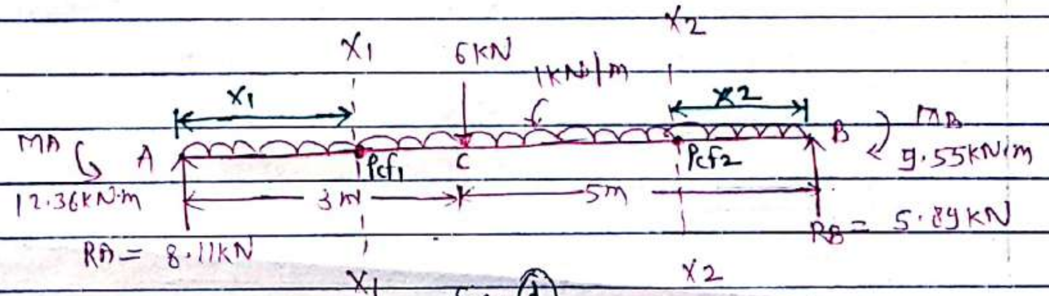


fig. (d)

consider a section $x_1 - x_1$ at P_{cf_1} in portion AC at a distance x_1 from A and a section $x_2 - x_2$ at P_{cf_2} in portion BC at a distance x_2 from B as shown in fig. (d)

$M_{Pcf_1} = -12.36 + 8.11\times x_1 - 1\times x_1\times \frac{x_1}{2}$

$0 = -12.36 + 8.11x_1 - 0.5x_1^2 \quad (\because M_{Pcf_1} = 0)$

$\therefore 0.5x_1^2 - 8.11x_1 + 12.36 = 0$

solving above eqn. on calculator we get

$x_1 = 14.5\,m$ or $x_1 = 1.7\,m$.

but $x_1 = 14.5\,m >$ span AC, hence not valid.

$\therefore$ Take $x_1 = 1.7\,m$ from A.

$M_{Pcf_2} = -9.55 + 5.89 \times x_2 - \left(1 \times x_2 \times \frac{x_2}{2}\right)$

$0 = -9.55 + 5.89 x_2 - 0.5 x_2^2 \quad (\because M_{Pcf_2} = 0)$

$\therefore \quad 0.5 x_2^2 - 5.89 x_2 + 9.55 = 0$

solving above eqn. on calculator we get

$x_2 = 1.94m$ or $x_2 = -9.83m$.

$x_2 = -9.83m$ not valid, hence take $x_2 = 1.94m$ from B.

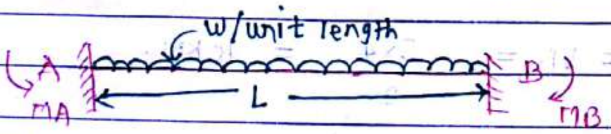
step 5 : To find Net B.M. at C

Net B.M. at C $= -12.36 + 8.11 \times 3 - (1 \times 3) \times 3/2$

$M_C = 7.47$ kN.m (sagging)

* Application of standard formulae in finding End moments, End Reactions and Drawing S.F. diagram showing point of contra shear and B.M. and point of contra flexure for a fixed Beam :-

* Examples Based on Fixed Beam subjected to udl over entire span :-



Imp. formulae :-

i) Fixed end moments :-

M_A = Fixed end moments at $A = \dfrac{-wL^2}{12}$

M_B = Fixed end moments at $B = -\dfrac{wL^2}{12}$

... (-ve sign indicates hogging nature)

ii) free moments :-

$M_A = M_B = 0$; $M_{max} = M_{center} = \dfrac{wL^2}{8}$

Ex: 1) Draw SFD and BMD for a fixed Beam of span 4m carrying u.d.l. of 5 kN/m over entire span.

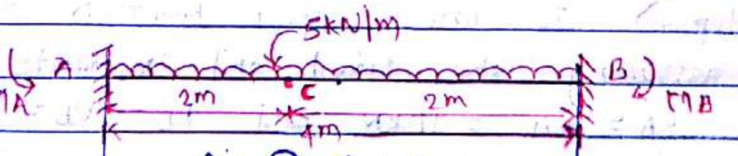


fig. (a) fixed Beam

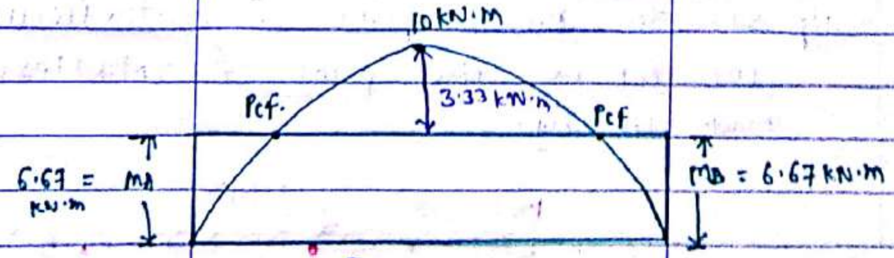
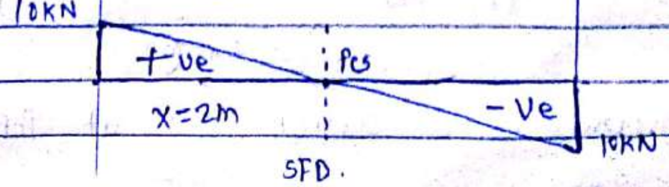


fig. (b) B.M.D.



SFD.

step 1) To find support Reactions V_A and V_B by considering the beam to be simply supported.

Support Reactions $V_A = V_B = \frac{wL}{2}$ due to symmetry of loading

$\therefore V_A = V_B = \frac{5 \times 4}{2} = 10 \text{ kN}.$

free B.M. = $M_A = M_B = 0$

$M_C = \frac{w \times l^2}{8} = \frac{5 \times 4^2}{8} = 10 \text{ kN·m}$ (sagging)

step 2) To find fixed end moments

$M_A = \frac{-wL^2}{12} = -\frac{5 \times 4^2}{12} = -6.67 \text{ kN·m}$

$M_B = \frac{-wL^2}{12} = -\frac{5 \times 4^2}{12} = -6.67 \text{ kN·m}.$

–ve sign indicate Hogging B.M.

step 3) To find Net B.M. at C

Net B.M. at C = Max. sagging B.M. − Fixed end moment

$= 10 - 6.67$

$= 3.33 \text{ kN·m}$ (sagging)

step 4) To find support Reaction RA & RB by assuming the fixed end moments.

$R_A = V_A = 10 \text{ kN}$ and $R_B = V_B = 10 \text{ kN}$ due to symmetry of loading and equal values of FEM.

step 5) To locate point of contraflexure Pcf.

Let Pcf be the point of contraflexure as shown in fig.

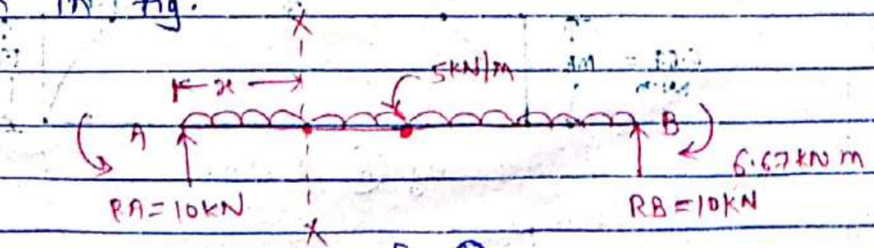


fig. C

consider a section X–X at Pcf as shown in fig. C

$M_{Pcf} = -6.67 + 10 \times x - 5 \times x \times x/2$

$0 = -6.67 + 10x - 2.5x^2$

Note → B.M. at Pcf is zero, hence $M_{Pcf} = 0$

$2.5x^2 - 10x + 6.67 = 0$

solving we get

$x = 3.15m$ or $x = 0.845m$.

∴ position of Pcf is at distance of 0.85m and 3.15m from A as shown in fig (b)

step (c) S.F. calculation

+ ↑ ↓

$SF_{AL} = 0$

$SF_{AR} = 10$

$SF_{BL} = -10$

$SF_{BR} = -10 + 10 = 0$

$SF_{CL} = 10$

$SF_{CR} = 10 - 5 \times 4 = -10$

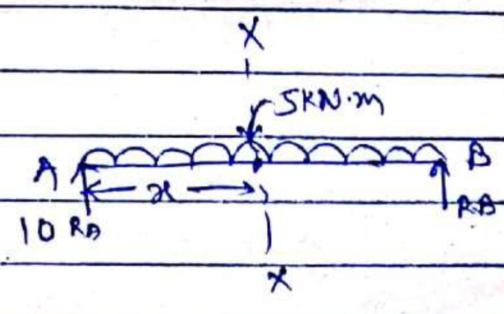
let Pcs be the shear force at point of contrashear Pcs

$F_{pcs} = 10 - 5 \times x$ But $F_{Pcs} = 0$

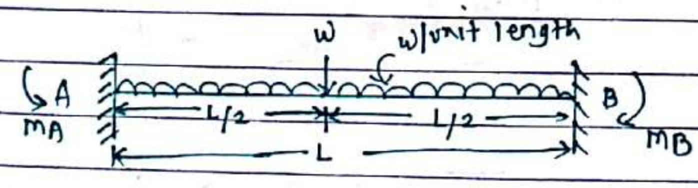
$0 = 10 - 5x$ ∴ $10 = 5x$

∴ $\boxed{x = 2m}$

+ ↑ ↓



* Example on a fixed Beam carrying u.d.l of w/unit length over entire span and a central point load :-



i) Fixed end moments

$$M_A = -\frac{WL}{8} - \frac{wL^2}{12}$$

$$M_B = -\frac{WL}{8} - \frac{wL^2}{12}$$ (-ve sign indicate Hogging nature)

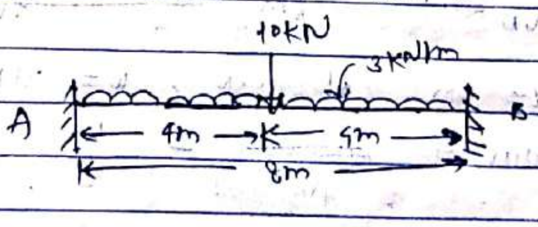
ii) Free end moments

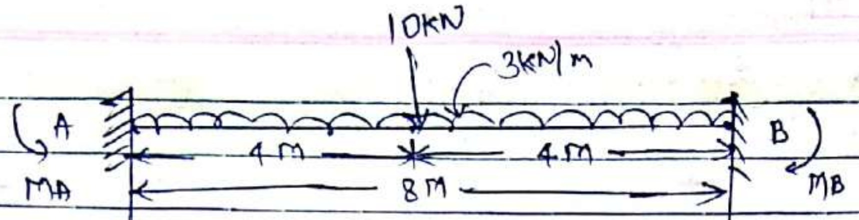
$M_A = M_B = 0$ $\quad M_{max} = M_{center} = \frac{WL}{4} + \frac{wL^2}{8}$

(+ve sagging)

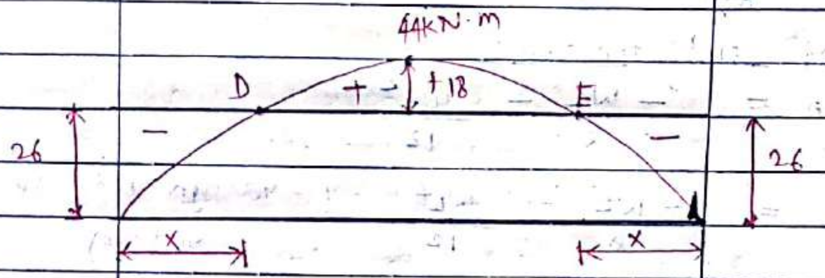
Ex:- A fixed Beam of 8m span is subjected to a u.d.l. of 3 kN/m over entire span with additional point load of 10 kN acting at centre. calculate Net B.M. at centre and with this value draw diagram. Also draw S.F diagram.

→ Given: Fixed Beam, L = 8m, w = 3kN/m
W = 10kN at centre.

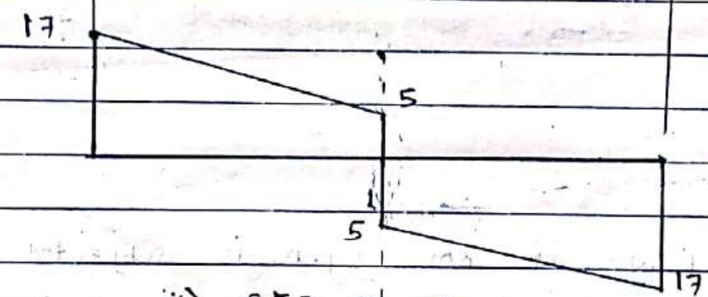




i) Fixed Beam

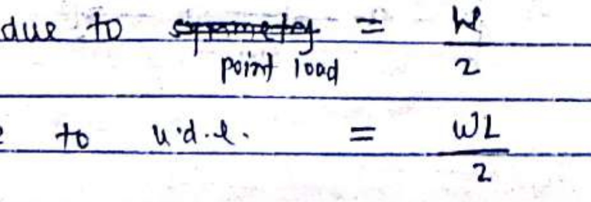


ii) Net B.M.D in KN·m.



iii) SFD in KN

Step 1) Calculate support Reaction assuming Beam as a simply supported.



Here $L = 8m$, $W = 10KN$, $w = 3KN/m$

VA due to ~~symmetry~~ point load $= \frac{W}{2}$

VA due to u.d.l. $= \frac{wL}{2}$

$\therefore VA = \frac{W}{2} + \frac{wL}{2} = \frac{10}{2} + \frac{3 \times 8}{2} = 17KN$

Due to symmetry of loading $VB = 17KN$

Step 2) Calculate free Bending moments

$M_A = M_B = 0$

$M_{max} = M_{centre} = M_c = \frac{wl^2}{8} = \cancel{\frac{3 \times l^2}{8}}$

$M_c = \frac{WL}{4} + \frac{wL^2}{8}$

$= \frac{10 \times 8}{4} + \frac{3 \times 8^2}{8}$

$M_c = 44 \text{ KN.m}$ (sagging)

or $= M_c = 17 \times 4 - (3 \times 4) \times \frac{4}{2} = 44 \text{ kN.m}$

Step 3) Calculate fixed end moments M_A & M_B

due to symmetry of loading

$M_A = M_B = -\frac{wL^2}{12} - \frac{WL}{8} = -\frac{3 \times 8^2}{12} - \frac{10 \times 8}{8}$

$= -26 \text{ KN.m}$ (Hogging)

∴ Net B.M. at centre 'C' = Free B.M. + Fixed end moment

$= 44 - 26$

$= 18 \text{ KN.m}$ (sagging)

* Note → In this case, since M_A & M_B are equal no reactions will be induced due to difference of fixed end moments.

∴ Reactions of fixed beam = Reaction of S.S.B.

∴ $R_A = V_A$; $R_B = V_B$

∴ $R_A = V_A = 17 \text{ kN}$; $R_B = V_B = 17 \text{ KN}$.

* Step 4) Point of contraflexure : A

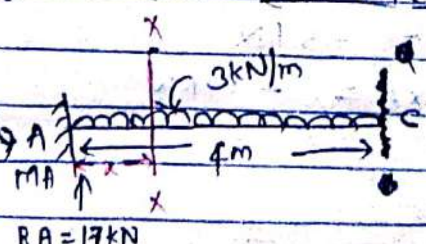


fig. iv)

from B.M. diagram of fixed Beam fig.(ii) it is clear that there exists two points of contraflexure D and E in portion AC and CB resp. Due to symmetry of loading these points are equidistant from the ends.

To locate D :- Refer to the portion AC of a fixed Beam as shown in fig. (iv)

B.M. at any section xx in portion AC at a distance x from A is given by,

$$M_x = -M_A + 17 \times x - \frac{(3 \times x) \times x}{2}$$

$$= -26 + 17 \cdot x - 1.5x^2$$

At the point of contraflexure B.M. = 0

∴ equating M_x to zero we have,

$$-26 + 17x - 1.5x^2 = 0$$

Dividing both sides by (−1.5) we have.

$$x^2 - 11.33x + 17.33 = 0$$

$$\therefore x = \frac{-(-11.33) \pm \sqrt{(-11.33)^2 - 4 \times 1 \times 17.33}}{2 \times 1}$$

$$= \frac{11.33 \pm 7.68}{2}$$

x = 1.825m or 9.505m

x = 9.505m is not valid.

∴ Distance of D from A = 1.825m.

Similarly distance of E from B = 1.825m.

* step 5) Calculation of S.F.

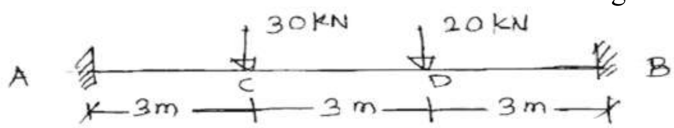
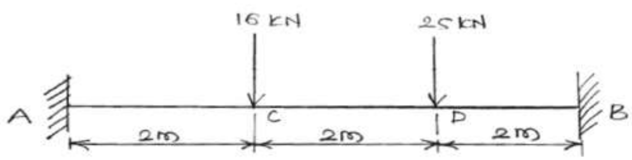
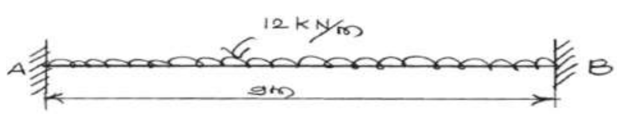
$SF_A = R_A = 17kN$

$SF_{CL} = 17 - 3 \times 4 = 5kN$

$SF_{CR} = 5 - 10 = -5kN$

$SF_{BR} = -5 - 3 \times 4 = -17kN$

Unit-2 Fixed Beam (Question Bank)

Que.No.	**2 marks Questions**
1	Define fixed Beam.
2	State any two advantages and disadvantages of fixed beam over S.S. beam.
3	State the principle of superposition with one example.
	4 marks Questions
4	A fixed beam of span 6 m carries an udl of 20 kN/m over entire span. Find fixed end moment from first principle and draw BMD.
5	State advantages and disadvantages of fixed beam over simply supported beam.
6	Explain the principle of super position with an example.
7	Calculate fixed end moment and draw BMD for a fixed beam as shown a Figure No. 2.  **Fig. No. 2**
8	Fixed Beam 8 m span carries a udl of 1 kN/m over entire span and a point load of 6 KN at 3 m from left support. Calculate fixed end moment and Draw BMD.
9	State any two advantages and two disadvantage of fixed beam over simply supported beam.
10	A fixed beam 6 m span carries a point load of 30kN at2 m from left support. Draw BMD and locate the point of contraflexure.
11	A fixed beam 8 m span carries a udl of 1 kN/m over the entire span and a point load of 6 kN at 3 m from left support. Calculate fixed end moments and draw BMD.
12	Calculate fixed end moment and draw BMD for a fixed beam as shown in Figure No. 1  **Fig. No. 1**
13	Calculate fixed end moments and draw BMD for a beam as shown in Figure No. 3. Use first principle method.  **Fig. No. 3**
14	A fixed beam of span 6 m carries a central point load of 120 KN using first principle. Calculate the fixed end moments.
15	Calculate fixed end moments and draw B.M.D. for a fixed beam subjected to u.d.l. W kN/m over entire span *l* from first principle.
16	Calculate fixed end moments and draw B.M.D. for a fixed beam as shown in Figure