The Shirpur Education Society's

R. C. Patel College of Engineering and Polytechnic, Shirpur

Department of Applied Sciences and Humanities

NAME OF COURSE: - Basic Mathematics

CODE OF COURSE: - 311302

SEMESTER: - FY CO/CW/EE/CE/ME- 1K

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Unit-2. Trignometry

ch-1) Trignometric Ratio of compound, allied, multiple and sub-multiple angles.

Compound Angle :-

If A and B are two angles then sum A+B or difference A-B are called compound angles.

Allied Angle :-

If the sum or difference of any two angles is either zero or an integral multiple of $\frac{\pi}{2}$ then the angles are called allied angles.

e.g. $\frac{\pi}{2} \pm \theta$, $\pi \pm \theta$, $\frac{3\pi}{2} \pm \theta$, $2\pi \pm \theta$ --- are allied angles.

* Some standard compound angle formulae

1) $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

2) $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

3) $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

4) $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

5) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

6) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$

Examples on compound angles.

Ex. Find value of without using calculator $\sin 15^\circ$

sol^n:- $\sin 15^\circ = \sin(45-30)$

use $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

$= \sin 45 \cdot \cos 30 - \cos 45 \cdot \sin 30$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Ex. without using calculator, find value of $\cos 75^\circ$

Sol^n:. $\cos 75^\circ = \cos(45+30)$

Use $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$$\cos 75^\circ = \cos 45 \cdot \cos 30 - \sin 45 \cdot \sin 30$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$\cos 75^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Ex. Evaluate $\dfrac{\tan 85^\circ - \tan 40^\circ}{1 + \tan 85 \cdot \tan 40}$

Sol^n:. Given $\dfrac{\tan 85^\circ - \tan 40^\circ}{1 + \tan 85^\circ \cdot \tan 40^\circ}$

Use $\dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B} = \tan(A+B)$

$$= \tan(85^\circ - 40^\circ)$$

$$= \tan 45^\circ$$

$$= 1$$

Ex. If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$ find $\tan(A+B)$

Sol^n:. Given $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$

we know $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$$= \frac{1/2 + 1/3}{1 - 1/2 \cdot 1/3}$$

$$\tan(A+B) = \frac{\frac{3+2}{6}}{\frac{6-1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

Ex. If $\tan(x+y) = \frac{3}{4}$, & $\tan(x-y) = \frac{8}{15}$ then show that $\tan 2x = \frac{77}{36}$.

Sol^n:. we can write

$$2x = (x+y) + (x-y)$$

Apply tan Ratio on both side

$$LHS = \tan 2x = \tan[(x+y) + (x-y)]$$

$$= \frac{\tan(x+y) + \tan(x-y)}{1 - \tan(x+y) \cdot \tan(x-y)}$$

$$= \frac{\frac{3}{4} + \frac{8}{15}}{1 - \frac{3}{4} \times \frac{8}{15}}$$

$$= \frac{\frac{45+32}{60}}{\frac{60-24}{60}}$$

$$= \frac{\frac{77}{60}}{\frac{36}{60}}$$

$$= \frac{77}{36} = RHS.$$

Ex. show that $\tan 3A - \tan 2A - \tan A = \tan A \cdot \tan 2A \cdot \tan 3A$

sol^n:- ~~LHS~~ we have

$$3A = A + 2A$$

Taking tangent Ratio on both side,

$$\tan 3A = \tan(A + 2A)$$

$$= \frac{\tan A + \tan 2A}{1 - \tan A \cdot \tan 2A}$$

$$\tan 3A(1 - \tan A \cdot \tan 2A) = \tan A + \tan 2A$$

$$\tan 3A - \tan A \cdot \tan 2A \cdot \tan 3A = \tan A + \tan 2A$$

$$\tan 3A - \tan 2A - \tan A = \tan A \cdot \tan 2A \cdot \tan 3A$$

hence proof.

Ex. If $\angle A$ & $\angle B$ are both obtuse angles and $\sin A = \frac{12}{13}$ and $\cos B = \frac{-4}{5}$ find $\sin(A+B)$.

sol^n:. Given $\sin A = \frac{12}{13}$, $\cos B = \frac{-4}{5}$

$$\cos^2 A = 1 - \sin^2 A \qquad \sin^2 B = 1 - \cos^2 B$$

$$= 1 - \frac{144}{169} \qquad = 1 - \frac{16}{25}$$

$$\cos^2 A = \frac{25}{169} \qquad \sin^2 B = \frac{9}{25}$$

$$\boxed{\cos A = \frac{-5}{13}} \qquad \boxed{\sin B = \frac{3}{5}}$$

we know

$$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$\sin(A+B) = \frac{12}{13}\cdot\left(\frac{-4}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{3}{5}\right)$$

$$= \frac{-48}{65} - \frac{15}{65}$$

$$\boxed{\sin(A+B) = \frac{-63}{65}}$$

* Allied angles Formulae.

1) Ratio of $\left(\frac{\pi}{2}-\theta\right)$

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$

$\cot\left(\frac{\pi}{2}-\theta\right) = \tan\theta$

$\sec\left(\frac{\pi}{2}-\theta\right) = \operatorname{cosec}\theta$

$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right) = \sec\theta$

2) Ratio of $\left(\frac{\pi}{2}+\theta\right)$

$\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$

$\cot\left(\frac{\pi}{2}+\theta\right) = -\tan\theta$

$\sec\left(\frac{\pi}{2}+\theta\right) = -\operatorname{cosec}\theta$

$\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right) = \sec\theta$

3) Ratio of $(\pi-\theta)$

$\sin(\pi-\theta) = \sin\theta$

$\cos(\pi-\theta) = -\cos\theta$

$\tan(\pi-\theta) = -\tan\theta$

$\cot(\pi-\theta) = -\cot\theta$

$\sec(\pi-\theta) = -\sec\theta$

$\operatorname{cosec}(\pi-\theta) = \operatorname{cosec}\theta$

4) Ratio of $(\pi+\theta)$

$\sin(\pi+\theta) = -\sin\theta$

$\cos(\pi+\theta) = -\cos\theta$

$\tan(\pi+\theta) = \tan\theta$

$\cot(\pi+\theta) = \cot\theta$

$\sec(\pi+\theta) = -\sec\theta$

$\operatorname{cosec}(\pi+\theta) = -\operatorname{cosec}\theta$

5) Ratio of $\left(\frac{3\pi}{2}-\theta\right)$

$\sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta$

$\cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta$

$\tan\left(\frac{3\pi}{2}-\theta\right) = \cot\theta$

$\cot\left(\frac{3\pi}{2}-\theta\right) = \tan\theta$

$\sec\left(\frac{3\pi}{2}-\theta\right) = -\operatorname{cosec}\theta$

$\operatorname{cosec}\left(\frac{3\pi}{2}-\theta\right) = -\sec\theta$

6) Ratio of $\left(\frac{3\pi}{2}+\theta\right)$

$\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$

$\cos\left(\frac{3\pi}{2}+\theta\right) = +\sin\theta$

$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$

$\cot\left(\frac{3\pi}{2}+\theta\right) = -\tan\theta$

$\sec\left(\frac{3\pi}{2}+\theta\right) = +\operatorname{cosec}\theta$

$\operatorname{cosec}\left(\frac{3\pi}{2}+\theta\right) = -\sec\theta$

7) Ratio of $(2\pi-\theta)$

$\sin(2\pi-\theta) = -\sin\theta$

$\cos(2\pi-\theta) = \cos\theta$

$\tan(2\pi-\theta) = -\tan\theta$

$\cot(2\pi-\theta) = -\cot\theta$

$\sec(2\pi-\theta) = \sec\theta$

$\operatorname{cosec}(2\pi-\theta) = -\operatorname{cosec}\theta$

8) Ratio of $(2\pi+\theta)$

$\sin(2\pi+\theta) = \sin\theta$

$\cos(2\pi+\theta) = \cos\theta$

$\tan(2\pi+\theta) = \tan\theta$

$\cot(2\pi+\theta) = \cot\theta$

$\sec(2\pi+\theta) = \sec\theta$

$\operatorname{cosec}(2\pi+\theta) = \operatorname{cosec}\theta$

Trignometric Ratio of $(-\theta)$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta$

$\cot(-\theta) = -\cot\theta$

$\sec(-\theta) = \sec\theta$

$\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

Examples on Allied angle formulae.

Ex. Find without Using calculator $\sin(210^\circ)$

sol^n:- $\sin(210^\circ) = \sin(180+30)$

use $\sin(\pi+\theta) = -\sin\theta$

$= -\sin 30^\circ$

$$\boxed{\sin(210^\circ) = -\frac{1}{2}}$$

Ex. find value $\cos 330^\circ$

sol^n:- $\cos(330^\circ) = \cos(360-30)$

Use $\cos(2\pi-\theta) = \cos\theta$

$= \cos 30^\circ$

$$\boxed{\cos(330^\circ) = \frac{\sqrt{3}}{2}}$$

Ex. find value of $\sec(3660^\circ)$

sol^n:- $\sec(3660^\circ) = \sec(20\times180+60)$

use $\sec(20\pi+\theta) = \sec\theta$

$= \sec 60^\circ$

$$\boxed{\sec(3660^\circ) = 2}$$

Ex. find value of $\sin(-765^\circ)$

$\sin(-765) = -\sin 765^\circ$

$= -\sin(4\times180+45)$

Use $\sin(4\pi+\theta) = \sin\theta$

$= -\sin 45^\circ$

$= -\frac{1}{\sqrt{2}}$

$$\boxed{\sin(-765^\circ) = -\frac{1}{\sqrt{2}}}$$

Ex. Evaluate $\dfrac{\tan 66 + \tan 69}{1-\tan 66 \cdot \tan 69}$

Sol^n:. Given $\dfrac{\tan 66 + \tan 69}{1-\tan 66 \cdot \tan 69}$

$= \tan(66+69)$

$= \tan(135^\circ)$

$= \tan(90+45)$

Use $\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$

$= -\cot 45$

$= -1$

$$\therefore \boxed{\frac{\tan 66 + \tan 69}{1-\tan 66 \cdot \tan 69} = -1}$$

* multiple angle :- let A be the angle then $2A, 3A, 4A, \ldots$ are called multiple angle.

* sub-multiple angle :- let A be the Given angle then $\frac{A}{2}, \frac{A}{3}, \ldots$ are called sub-multiple angle.

* Some multiple angle formulae.

1) $\sin 2A = 2 \sin A \cdot \cos A$

$$= \frac{2 \tan A}{1 + \tan^2 A}$$

2) $\cos 2A = \cos^2 A - \sin^2 A$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

$$= \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

3) $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

4) $\sin 3A = 3 \sin A - 4 \sin^3 A$

5) $\cos 3A = 4 \cos^3 A - 3 \cos A$

6) $\tan 3A = \dfrac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$

* some sub-multiple angle formulae.

1) $\sin A = 2 \sin A/2 \cdot \cos A/2 = \dfrac{2 \tan A/2}{1 + \tan^2 A/2}$

2) $\cos A = \cos^2 A/2 - \sin^2 A/2$

$= 2\cos^2 A/2 - 1$

$= 1 - 2\sin^2 A/2$

$$= \frac{1 - \tan^2 A/2}{1 + \tan^2 A/2}$$

3) $$\tan A = \frac{2\tan A/2}{1 - \tan^2 A/2}$$

Examples on multiple & sub-multiple angles.

Ex. If $\sin A = \frac{1}{2}$ find $\sin 3A$

Sol^n:- Given $\sin A = \frac{1}{2}$

we know $\sin 3A = 3\sin A - 4\sin^3 A$

$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$

$= \frac{3}{2} - \frac{1}{2}$

$= \frac{2}{2}$

$\therefore \boxed{\sin 3A = 1}$

Ex. If $\cos A = \frac{1}{2}$ find $\cos 3A$

Sol^n:- Given $\cos A = \frac{1}{2}$

we know $\cos 3A = 4\cos^3 A - 3\cos A$

$$\cos 3A = 4\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)$$

$$= \frac{1}{2} - \frac{3}{2}$$

$$= \frac{-2}{2}$$

$$\therefore \boxed{\cos 3A = -1}$$

Ex. If $A = 30^\circ$ verify results $\sin 3A = 3\sin A - 4\sin^3 A$

sol^n:- let $L.H.S = \sin 3A$

$$= \sin 3(30^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$R.H.S = 3\sin A - 4\sin^3 A$$

$$= 3\sin 30 - 4(\sin 30)^3$$

$$= 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

$$= \frac{3}{2} - \frac{1}{2}$$

$$= \frac{2}{2}$$

$$= 1$$

$\therefore$ $L.H.S = R.H.S$

$\sin 3A = 3\sin A - 4\sin^3 A$ is verified.

Ex. If $A = 30^\circ$, verify results.

i) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$ ii) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$

sol^n:. Given $A = 30^\circ$

i) $LHS = \sin 2A$

$= \sin 2(30)$

$= \sin 60^\circ$

$= \frac{\sqrt{3}}{2}$

i) $RHS = \frac{2\tan A}{1+\tan^2 A}$

$= \frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$

$= \frac{2\left(\frac{\cancel{1}}{\sqrt{3}}\right)}{\cancel{1+\left(\frac{\sqrt{3}}{3}\right)^2}} = \frac{{}^{1}\cancel{2}}{\sqrt{3}} \times \frac{3}{\cancel{4}_{2}} = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}\cdot\cancel{\sqrt{3}}}{2\cancel{\sqrt{3}}}$

$= \frac{\frac{\cancel{2}}{\cancel{\sqrt{3}}}}{\frac{\cancel{4}}{\cancel{\sqrt{3}}}} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

$= \frac{\cancel{2}}{\cancel{4}}$ $\therefore$ $LHS = RHS$

$= \frac{\cancel{1}}{\cancel{2}}$ $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$ is verified.

ii) $LHS = \cos 2A$

$= \cos 2(30)$

$= \cos 60$

$= \frac{1}{2}$

$RHS = \cancel{2}\ \frac{1-\tan^2 A}{1+\tan^2 A}$

$$= \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30}$$

$$= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{3-1}{3}}{\frac{3+1}{3}} = \frac{2}{4} = \frac{1}{2}$$

LHS = RHS

$\cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$ is verified

Ex. If $\tan \frac{\alpha}{2} = \frac{1}{\sqrt{3}}$ find value of $\sin \alpha$.

Sol^n:. Given $\tan \frac{\alpha}{2} = \frac{1}{\sqrt{3}}$

we know $\sin \alpha = \frac{2 \tan \alpha/2}{1 + \tan^2 \alpha/2}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$$

If $\tan\frac{A}{2} = \frac{1}{\sqrt{3}}$ find value of $\cos A$

sol^n:- Given $\tan\frac{A}{2} = \frac{1}{\sqrt{3}}$.

we know $\cos A = \dfrac{1-\tan^2 A/2}{1+\tan^2 A/2}$

$$= \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{1-\frac{1}{3}}{1+\frac{1}{3}}$$

$$= \frac{\frac{2}{3}}{\frac{4}{3}}$$

$$= \frac{2}{3} \times \frac{3}{4}$$

$$= \frac{1}{2}$$

Ex. prove that $\dfrac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2}$

sol^n:- $LHS = \dfrac{\sin\theta}{1+\cos\theta}$

$$= \frac{2\sin\theta/2 \cdot \cos\theta/2}{2\cos^2\theta/2} = \frac{\sin\theta/2}{\cos\theta/2} = \tan\frac{\theta}{2}$$

$= RHS.$

Ex. prove that $\dfrac{\cos A}{1-\sin A} = \sec A + \tan A$.

Sol^n:. LHS $= \dfrac{\cos A}{1-\sin A}$

$$= \frac{\cos A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}$$

$$= \frac{\cos A(1+\sin A)}{1-\sin^2 A}$$

$$= \frac{\cos A(1+\sin A)}{\cos^2 A}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

= R.H.S.

Ex. prove that $\dfrac{1+\cos A}{\sin A} = \cot \dfrac{A}{2}$

Sol^n:. LHS $= \dfrac{1+\cos A}{\sin A}$

$$= \frac{2\cos^2 A/2}{2\cos A/2 \cdot \sin A/2}$$

$$= \frac{\cos A/2}{\sin A/2}$$

$= \cot A/2 =$ RHS.

Ex. prove that $\dfrac{\cos A}{1+\cos A} = \dfrac{1}{2}\left[1-\tan^2\dfrac{A}{2}\right]$

sol^n:. $LHS = \dfrac{\cos A}{1+\cos A}$

$$= \frac{\cos^2\frac{A}{2} - \sin^2\frac{A}{2}}{2\cos^2\frac{A}{2}}$$

$$= \frac{1}{2}\left[\frac{\cos^2\frac{A}{2} - \sin^2\frac{A}{2}}{\cos^2\frac{A}{2}}\right]$$

$$= \frac{1}{2}\left[\frac{\cos^2\frac{A}{2}}{\cos^2\frac{A}{2}} - \frac{\sin^2\frac{A}{2}}{\cos^2\frac{A}{2}}\right]$$

$$= \frac{1}{2}\left[1 - \tan^2\frac{A}{2}\right]$$

$= R.H.S.$

ch-2> Factorization & Defactorization.

* Defactorization formulae.

1> $2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$

2> $2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$

3> $2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$

4> $2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$

* Factorization formulae.

1> $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

2> $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

3) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$

4) $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

Examples on Defactorization formulae.

Ex. prove that $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = \frac{3}{16}$

sol^n:. $LHS = \sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ$

$= \sin 20^\circ \cdot \sin 40^\circ \cdot \frac{\sqrt{3}}{2} \cdot \sin 80^\circ$

multiply & Devide by 2

$= \frac{\sqrt{3}}{2} \times \frac{1}{2} (2\sin 20^\circ \cdot \sin 40^\circ) \cdot \sin 80^\circ$

$= \frac{\sqrt{3}}{4} \{ \cos(20-40) - \cos(\overline{20+40}) \} \sin 80^\circ$

$= \frac{\sqrt{3}}{4} \{ \cos(-20) - \cos 60^\circ \} \sin 80^\circ$

$$= \frac{\sqrt{3}}{4}\left\{ \cos 20 - \frac{1}{2} \right\} \sin 80^\circ$$

$$= \frac{\sqrt{3}}{4}\left\{ \cos 20 \cdot \sin 80 - \frac{1}{2}\sin 80 \right\}$$

multiply & devide by 2

$$= \frac{\sqrt{3}}{8}\left\{ 2\cos 20 \cdot \sin 80 - \sin 80 \right\}$$

$$= \frac{\sqrt{3}}{8}\left\{ \sin(20+80) - \sin(20-80) - \sin 80^\circ \right\}$$

$$= \frac{\sqrt{3}}{8}\left\{ \sin 100^\circ - \sin(-60) - \sin 80 \right\}$$

$$= \frac{\sqrt{3}}{8}\left\{ \sin(180-80) + \sin 60 - \sin 80 \right\}$$

use $\sin(\pi - \theta) = \sin\theta$

$$= \frac{\sqrt{3}}{8}\left\{ \cancel{\sin 80} + \frac{\sqrt{3}}{2} - \cancel{\sin 80} \right\}$$

$$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2}$$

$$= \frac{3}{16}$$

= R.H.S.

Ex. prove that $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$.

Sol^n:- $LHS = \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ$

$$= \cos 20 \cdot \cos 40 \cdot \frac{1}{2} \cdot \cos 80^\circ$$

$$= \frac{1}{2} \times \frac{1}{2} \times 2\cos 20 \cdot \cos 40 \cdot \cos(90-10)$$

$$= \frac{1}{4} \{ \cos(20+40) + \cos(20-40) \cdot \sin 10^\circ$$

$$= \frac{1}{4} \{ \cos 60^\circ + \cos(-20) \} \cdot \sin 10^\circ$$

$$= \frac{1}{4} \{ \frac{1}{2} + \cos 20^\circ \} \sin 10^\circ$$

$$= \frac{1}{4} \{ \frac{1+2\cos 20}{2} \} \sin 10^\circ$$

$$= \frac{1}{8} \{ 1 + 2\cos 20 \} \sin 10^\circ$$

$$= \frac{1}{8} \{ \sin 10 + 2\cos 20 \cdot \sin 10^\circ \}$$

$$= \frac{1}{8} \{ \sin 10^\circ + \sin(20+10) - \sin(20-10) \}$$

$$= \frac{1}{8} \{ \cancel{\sin 10} + \sin 30 - \cancel{\sin 10} \}$$

$$= \frac{1}{8} \{ \sin 30 \}$$

$$= \frac{1}{8} \{ \frac{1}{2} \}$$

$$= \frac{1}{16}$$

$= R \cdot H \cdot S$.

Ex. prove that $\cos 15^\circ \cdot \cos 30^\circ \cdot \cos 60^\circ \cdot \cos 75^\circ = \frac{\sqrt{3}}{16}$

sol^n:. $LHS = \cos 15^\circ \cdot \cos 30^\circ \cdot \cos 60^\circ \cdot \cos 75^\circ$

$= \cos 15^\circ \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} \cdot \cos 75^\circ$

$= \frac{\sqrt{3}}{4} \cos 15^\circ \cdot \cos 75^\circ$

multiply & devide by 2

$= \frac{\sqrt{3}}{4} \times \frac{1}{2} \times 2\cos 15^\circ \cdot \cos 75^\circ$

$= \frac{\sqrt{3}}{8} \cdot 2\cos 15 \cdot \cos 75^\circ$

$= \frac{\sqrt{3}}{8} \left\{ \cos(15+75) + \cos(15-75) \right\}$

$= \frac{\sqrt{3}}{8} \left\{ \cos 90 + \cos(-60) \right\}$

$= \frac{\sqrt{3}}{8} \left\{ \cos 90 + \cos 60 \right\}$

$= \frac{\sqrt{3}}{8} \left\{ 0 + \frac{1}{2} \right\}$ $\therefore \cos 90 = 0$

$\therefore \cos 60 = \frac{1}{2}$

$= \frac{\sqrt{3}}{8} \left\{ \frac{1}{2} \right\}$

$= \frac{\sqrt{3}}{16}$

$= R \cdot H \cdot S$.

Examples on Factorization formulae.

Ex. prove that $\dfrac{\sin 19^\circ + \cos 11^\circ}{\cos 19 - \sin 11^\circ} = \sqrt{3}$

Sol^n:. $LHS = \dfrac{\sin 19^\circ + \cos 11^\circ}{\cos 19^\circ - \sin 11^\circ}$

$$= \frac{\sin(90-71) + \cos 11^\circ}{\cos(90-71) \cancel{+} - \sin 11^\circ}$$

$$= \frac{\cos 71^\circ + \cos 11^\circ}{\cancel{co}\ \sin 71^\circ - \sin 11^\circ}$$

$$= \frac{2\cos\left(\frac{71+11}{2}\right)\cdot \cos\left(\frac{71-11}{2}\right)}{2\cos\left(\frac{71+11}{2}\right)\cdot \sin\left(\frac{71-11}{2}\right)}$$

$$= \frac{2\cos\left(\frac{82}{2}\right)\cdot \cos\left(\frac{60}{2}\right)}{2\cos\left(\frac{82}{2}\right)\cdot \sin\left(\frac{60}{2}\right)}$$

$$= \frac{2\cos 41^\circ \cdot \cos 30^\circ}{2\cos 41^\circ \cdot \sin 30^\circ}$$

$$= \frac{\cos 30^\circ}{\sin 30^\circ}$$

$$= \cot 30^\circ$$

$$= \sqrt{3}$$

$= R.H.S.$

Ex. prove that $\dfrac{\sin 8\theta + \sin 2\theta}{\cos 8\theta + \cos 2\theta} = \tan 5\theta$.

Solⁿ:. $LHS = \dfrac{\sin 8\theta + \sin 2\theta}{\cos 8\theta + \cos 2\theta}$

$$= \frac{2\sin\left(\frac{8\theta+2\theta}{2}\right)\cdot\cos\left(\frac{8\theta-2\theta}{2}\right)}{2\cos\left(\frac{8\theta+2\theta}{2}\right)\cdot\cos\left(\frac{8\theta-2\theta}{2}\right)}$$

$$= \frac{2\sin 5\theta\cdot\cos 3\theta}{2\cos 5\theta\cdot\cos 3\theta}$$

$$= \frac{\sin 5\theta}{\cos 5\theta}$$

$= \tan 5\theta$

$= R.H.S.$

Ex. prove that $\dfrac{\sin 3A + \sin 2A}{\cos 3A + \cos 2A} = \tan\left(\dfrac{5A}{2}\right)$

Solⁿ:. $L.H.S = \dfrac{\sin 3A + \sin 2A}{\cos 3A + \cos 2A}$

$$= \frac{2\sin\left(\frac{3A+2A}{2}\right)\cdot\cos\left(\frac{3A-2A}{2}\right)}{2\cos\left(\frac{3A+2A}{2}\right)\cdot\cos\left(\frac{3A-2A}{2}\right)}$$

$$= \frac{2\sin\left(\frac{5A}{2}\right)\cdot\cos\left(\frac{A}{2}\right)}{2\cos\left(\frac{5A}{2}\right)\cdot\cos\left(\frac{A}{2}\right)}$$

$$= \frac{\sin\left(\frac{5A}{2}\right)}{\cos\left(\frac{5A}{2}\right)}$$

$$= \tan\left(\frac{5A}{2}\right)$$

$= R.H.S.$

Ex. prove that $\frac{\sin 7x + \sin x}{\cos 5x - \cos 3x} = \sin 2x - \cos 2x \cdot \cot x$

Sol^n: $L.H.S = \frac{\sin 7x + \sin x}{\cos 5x - \cos 3x}$

$$= \frac{2\sin\left(\frac{7x+x}{2}\right)\cdot\cos\left(\frac{7x-x}{2}\right)}{2\sin\left(\frac{3x+5x}{2}\right)\sin\left(\frac{3x-5x}{2}\right)}$$

$$= \frac{2\sin 4x \cdot \cos(-3x)}{2\sin 4x \cdot \sin(-x)}$$

$$= \frac{\cos 3x}{-\sin x}$$

$$= \frac{\cos(2x+x)}{-\sin x} = \frac{\cos 2x \cdot \cos x - \sin 2x \cdot \sin x}{-\sin x}$$

$$= -\frac{\cos 2x \cdot \cos x}{\sin x} + \frac{\sin 2x \cdot \sin x}{\sin x}$$

$$= -\cos 2x \cdot \cot x + \sin 2x$$

$$= \sin 2x - \cos 2x \cdot \cot x = R.H.S.$$

Ex. prove that $\dfrac{\cos 3A - \cos 7A}{\sin 9A + \sin A} = \cos 2A \cdot \tan 4A - \sin 2A$

$$LHS = \frac{\cos 3A - \cos 7A}{\sin 9A + \sin A}$$

$$= \frac{-2\sin\left(\frac{3A+7A}{2}\right) \cdot \sin\left(\frac{3A-7A}{2}\right)}{2\sin\left(\frac{9A+A}{2}\right) \cdot \cos\left(\frac{9A-A}{2}\right)}$$

$$= \frac{-2\sin 5A \sin(-2A)}{2\sin 5A \cdot \cos 4A}$$

$$= \frac{\sin 2A}{\cos 4A} \qquad \therefore 2A = 4A - 2A$$

$$= \frac{\sin(4A-2A)}{\cos 4A}$$

$$= \frac{\sin 4A \cdot \cos 2A - \cos 4A \cdot \sin 2A}{\cos 4A}$$

$$= \frac{\sin 4A \cdot \cos 2A}{\cos 4A} - \frac{\cos 4A \cdot \sin 2A}{\cos 4A}$$

$$= \cos 2A \cdot \tan 4A - \sin 2A$$

$= R.H.S.$

Ex. prove that $\dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$.

sol^n :- $L.H.S = \dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A}$

$$= \frac{\sin A + \sin 5A + \sin 3A}{\cos A + \cos 5A + \cos 3A}$$

$$= \frac{2\sin\left(\frac{A+5A}{2}\right).\cos\left(\frac{A-5A}{2}\right) + \sin 3A}{2\cos\left(\frac{A+5A}{2}\right)\cdot\cos\left(\frac{A-5A}{2}\right) + \cos 3A}$$

$$= \frac{2\sin 3A\cdot\cos(-2A) + \sin 3A}{2\cos 3A\cdot\cos(-2A) + \cos 3A}$$

$$= \frac{\sin 3A\,[2\cos(-2A)+1]}{\cos 3A\,[2\cos(-2A)+1]}$$

$$= \frac{\sin 3A}{\cos 3A}$$

$= \tan 3A$

$= R.H.S.$

Ex. prove that $\dfrac{\sin A + \sin 2A + \sin 3A + \sin 4A}{\cos A + \cos 2A + \cos 3A + \cos 4A} = \tan\dfrac{5A}{2}$

Sol^n :- $L.H.S = \dfrac{\sin A + \sin 4A + \sin 2A + \sin 3A}{\cos A + \cos 4A + \cos 2A + \cos 3A}$

$$= \frac{2\sin\left(\frac{A+4A}{2}\right)\cos\left(\frac{A-4A}{2}\right) + 2\sin\left(\frac{2A+3A}{2}\right)\cos\left(\frac{2A-3A}{2}\right)}{2\cos\left(\frac{A+4A}{2}\right)\cdot\cos\left(\frac{A-4A}{2}\right) + 2\cos\left(\frac{2A+3A}{2}\right)\cos\left(\frac{2A-3A}{2}\right)}$$

$$= \frac{2\sin\left(\frac{5A}{2}\right)\cos\left(\frac{-3A}{2}\right) + 2\sin\left(\frac{5A}{2}\right)\cos\left(\frac{-A}{2}\right)}{2\cos\left(\frac{5A}{2}\right)\cos\left(\frac{-3A}{2}\right) + 2\cos\left(\frac{5A}{2}\right)\cos\left(\frac{-A}{2}\right)}$$

$$= \frac{2\sin\left(\frac{5A}{2}\right)\left[\cos\left(-\frac{3A}{2}\right) + \cos\left(-\frac{A}{2}\right)\right]}{2\cos\left(\frac{5A}{2}\right)\left[\cos\left(-\frac{3A}{2}\right) + \cos\left(-\frac{A}{2}\right)\right]}$$

$$= \frac{\sin\left(\frac{5A}{2}\right)}{\cos\left(\frac{5A}{2}\right)}$$

$$= \tan\left(\frac{5A}{2}\right)$$

$= R.H.S.$

ch-3> Inverse Trignometric Ratios.

Definition :-

If $\sin\theta = x$ then θ is called sine inverse of x and it is denoted by

$$\theta = \sin^{-1}x$$

Thus, if $\sin\theta = x$ then $\theta = \sin^{-1}x$

Similarly If $\cos\theta = x$ then $\theta = \cos^{-1}x$

$\tan\theta = x$ then $\theta = \tan^{-1}x$

$\cot\theta = x$ then $\theta = \cot^{-1}x$

$\sec\theta = x$ then $\theta = \sec^{-1}x$

$\operatorname{cosec}\theta = x$ then $\theta = \operatorname{cosec}^{-1}x$

The functions $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$, $\cot^{-1}x$, $\sec^{-1}x$ and $\operatorname{cosec}^{-1}x$ are called inverse trignometric functions.

Note that $\sin^{-1}x \neq \frac{1}{\sin x}$

Properties of inverse functions:-

1> $\sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}$

$\operatorname{cosec}^{-1}x = \sin^{-1}\left(\frac{1}{x}\right)$

2) $\cos^{-1}x = \sec^{-1}\left(\frac{1}{x}\right)$

$\sec^{-1}x = \cos^{-1}\left(\frac{1}{x}\right)$

3) $\tan^{-1}x = \cot^{-1}\left(\frac{1}{x}\right)$

$\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$

4) $\sin^{-1}(-x) = -\sin^{-1}x$

5) $\cos^{-1}(-x) = \pi - \cos^{-1}x$

6) $\tan^{-1}(-x) = -\tan^{-1}x$

7) $\cot^{-1}(-x) = \pi - \cot^{-1}x$

8) $\sec^{-1}(-x) = \pi - \sec^{-1}x$

9) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

10) $\sin^{-1}(\sin\theta) = \theta$

11) $\cos^{-1}(\cos\theta) = \theta$

12) $\tan^{-1}(\tan\theta) = \theta$

13) $\cot^{-1}(\cot\theta) = \theta$

14) $\sec^{-1}(\sec\theta) = \theta$

15) $\operatorname{cosec}^{-1}(\operatorname{cosec}\theta) = \theta$

16) $\sin(\sin^{-1}x) = x$

17) $\cos(\cos^{-1}x) = x$

18) $\tan(\tan^{-1}x) = x$

19) $\cot(\cot^{-1}x) = x$

20) $\sec(\sec^{-1}x) = x$

21) $\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x$

22) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

23) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

24) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

25) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \quad x, y > 0 \ \& \ xy < 1$

26) $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ $x, y > 0$ & $xy > 1$

27) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ $x, y > 0$ & $xy < 1$

29) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$

30) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]$

31) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]$

32) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left[xy + \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]$

Simple examples on inverse trignometric function.

Ex. Find the principal value of $\cos\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{2}\right)\right]$

sol^n:- Given $\cos\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{2}\right)\right]$

put $\sin^{-1}\left(\frac{1}{2}\right) = \theta$

$= \cos\left(\frac{\pi}{2} - \theta\right)$

$= \sin\theta$

$= \sin\left[\sin^{-1}\left(\frac{1}{2}\right)\right]$ $\sin(\sin^{-1}x) = x$

$= \frac{1}{2}$

$\therefore$ principal value of $\cos\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1}{2}\right)\right] = \frac{1}{2}$.

Ex. find the principal value of $\sin\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{2}\right)\right]$

sol^n:- Given $\sin\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{2}\right)\right]$

put $\cos^{-1}\left(\frac{1}{2}\right) = \theta$

$= \sin\left[\frac{\pi}{2} - \theta\right]$ $\therefore$ Allied angle.

$= \cos\theta$

$= \cos\left[\cos^{-1}\left(\frac{1}{2}\right)\right]$ $\therefore \cos(\cos^{-1}x) = x$

$= \frac{1}{2}$

$\therefore$ principal value of $\sin\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{2}\right)\right] = \frac{1}{2}$.

Ex. Find the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

sol^n:- We know $\sin^{-1}(-x) = -\sin^{-1}(x)$

$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\therefore\ -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \theta$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\theta$

$\frac{\sqrt{3}}{2} = \sin(-\theta)$ $\therefore \sin 60 = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = -\sin(\theta)$

$\therefore \boxed{\theta = -60}$

Ex. prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$.

sol^n:-

$$L.H.S = \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$$

Use $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ $\quad x, y > 0$, $xy < 1$

$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3+2}{6}}{1-\frac{1}{6}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{6-1}{6}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right)$$

$$= \tan^{-1}\left(\frac{5}{6}\times\frac{6}{5}\right)$$

$$= \tan^{-1}(1)$$

$\because \tan\frac{\pi}{4} = 1$

$$= \frac{\pi}{4}$$

$\therefore \frac{\pi}{4} = \tan^{-1}(1)$

$= R.H.S.$

Ex. Show that $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

Solⁿ:- $LHS = \tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right)$

Use $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$= \tan^{-1}\left[\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{48+77}{264}}{1-\frac{14}{264}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{125}{264}}{\frac{264-14}{264}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{125}{264}}{\frac{250}{264}}\right]$$

$$= \tan^{-1}\left(\frac{125}{250}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

$= R.H.S.$

Ex. Prove that $\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \cot^{-1}\left(\frac{9}{2}\right)$

Sol^n:- LHS $= \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right)$

Use prop. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$= \tan^{-1}\left[\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\cdot\frac{1}{13}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{13+7}{91}}{1-\frac{1}{91}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{20}{91}}{\frac{91-1}{91}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{20}{91}}{\frac{90}{91}}\right]$$

$$= \tan^{-1}\left(\frac{20}{90}\right)$$

$$= \tan^{-1}\left(\frac{2}{9}\right)$$

$\therefore \tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right)$.

$$= \cot^{-1}\left(\frac{9}{2}\right)$$

$=$ R.H.S.

Ex. prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \cot^{-1}(2)$

sol^n:- LHS = $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$

Use $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$= \tan^{-1}\left[\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{9+8}{36}}{1-\frac{2}{36}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{17}{36}}{\frac{36-2}{36}}\right]$$

$$= \tan^{-1}\left(\frac{\frac{17}{36}}{\frac{34}{36}}\right)$$

$$= \tan^{-1}\left(\frac{17}{34}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right) \quad \therefore \tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x$$

$$= \cot^{-1}(2)$$

= R.H.S.

Ex. prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$

Sol^n:- LHS $= \tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$

Use prop. $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$x, y > 0$, and $xy > 1$

$= \tan^{-1}\left(\frac{1+2}{1-1\times 2}\right) + \pi + \tan^{-1}(3)$

$= \tan^{-1}\left(\frac{3}{-1}\right) + \pi + \tan^{-1}(3)$

$= \tan^{-1}(-3) + \pi + \tan^{-1}(3)$

$= -\tan^{-1}(3) + \pi + \tan^{-1}(3)$ $\quad \because \tan^{-1}(-x) = -\tan^{-1}x$

$= \pi$

$=$ R.H.S.

Ex. show that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

Sol^n:- LHS $= \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right)$

Use prop. $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]$

Ex. prove that $2\tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{3}{4}\right)$

Sol^n:- $LHS = 2\tan^{-1}\left(\frac{1}{3}\right)$

$$= \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{3}\right)$$

Use prop. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

$$= \tan^{-1}\left[\frac{\frac{1}{3}+\frac{1}{3}}{1-\frac{1}{3}\cdot\frac{1}{3}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{3+3}{9}}{1-\frac{1}{9}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{6}{9}}{\frac{9-1}{9}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{6}{9}}{\frac{8}{9}}\right]$$

$$= \tan^{-1}\left[\frac{6}{9}\times\frac{9}{8}\right]$$

$$= \tan^{-1}\left(\frac{6}{8}\right)$$

$$= \tan^{-1}\left(\frac{3}{4}\right)$$

$= R.H.S.$

$$= \cos^{-1}\left[\frac{4}{5}\times\frac{12}{13} - \sqrt{1-\left(\frac{4}{5}\right)^2}\cdot\sqrt{1-\left(\frac{12}{13}\right)^2}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} - \sqrt{1-\frac{16}{25}}\cdot\sqrt{1-\frac{144}{169}}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} - \sqrt{\frac{25-16}{25}}\cdot\sqrt{\frac{169-144}{169}}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} - \sqrt{\frac{9}{25}}\cdot\sqrt{\frac{25}{169}}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} - \frac{3}{5}\times\frac{5}{13}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} - \frac{15}{65}\right]$$

$$= \cos^{-1}\left[\frac{48-15}{65}\right]$$

$$= \cos^{-1}\left(\frac{33}{65}\right)$$

$= R.H.S.$

Ex. Show that $\cos^{-1}\left(\frac{4}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{63}{65}\right)$

Sol^n:- LHS $= \cos^{-1}\left(\frac{4}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right)$

Use prop. $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left[xy + \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]$

$$= \cos^{-1}\left[\frac{4}{5}\times\frac{12}{13} + \sqrt{1-\left(\frac{4}{5}\right)^2}\cdot\sqrt{1-\left(\frac{12}{13}\right)^2}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} + \sqrt{1-\frac{16}{25}}\cdot\sqrt{1-\frac{144}{169}}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} + \sqrt{\frac{25-16}{25}}\cdot\sqrt{\frac{169-144}{169}}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} + \sqrt{\frac{9}{25}}\times\sqrt{\frac{25}{169}}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} + \frac{3}{5}\times\frac{5}{13}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} + \frac{15}{65}\right]$$

$$= \cos^{-1}\left[\frac{48+15}{65}\right]$$

$$= \cos^{-1}\left(\frac{63}{65}\right) = R.H.S.$$

Ex. prove that $\sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{77}{85}\right)$

Sol^n:- $LHS = \sin^{-1}\left(\frac{3}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right)$

Use prop. $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$

$$= \sin^{-1}\left[\frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2} + \frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\cdot\sqrt{1-\frac{64}{289}} + \frac{8}{17}\sqrt{1-\frac{9}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\cdot\sqrt{\frac{289-64}{289}} + \frac{8}{17}\sqrt{\frac{25-9}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\cdot\sqrt{\frac{225}{289}} + \frac{8}{17}\sqrt{\frac{16}{25}}\right]$$

$$= \sin^{-1}\left[\frac{3}{5}\times\frac{15}{17} + \frac{8}{17}\times\frac{4}{25}\right]$$

$$= \sin^{-1}\left[\frac{45}{85} + \frac{32}{85}\right]$$

$$= \sin^{-1}\left[\frac{45+32}{85}\right]$$

$$= \sin^{-1}\left(\frac{77}{85}\right)$$

$= R.H.S.$

Ex. prove that $\cos^{-1}\left(\frac{20}{29}\right) - \cos^{-1}\left(\frac{4}{5}\right) = \cos^{-1}\left(\frac{143}{145}\right)$

Sol^n:- $LHS = \cos^{-1}\left(\frac{20}{29}\right) - \cos^{-1}\left(\frac{4}{5}\right)$

Use prop. $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\left[xy + \sqrt{1-x^2}\cdot\sqrt{1-y^2}\right]$

$$= \cos^{-1}\left[\frac{20}{29}\times\frac{4}{5} + \sqrt{1-\left(\frac{20}{29}\right)^2}\cdot\sqrt{1-\left(\frac{4}{5}\right)^2}\right]$$

$$= \cos^{-1}\left[\frac{80}{145} + \sqrt{1-\frac{400}{841}}\cdot\sqrt{1-\frac{16}{25}}\right]$$

$$= \cos^{-1}\left[\frac{80}{145} + \sqrt{\frac{841-400}{841}}\cdot\sqrt{\frac{25-16}{25}}\right]$$

$$= \cos^{-1}\left[\frac{80}{145} + \sqrt{\frac{441}{841}}\cdot\sqrt{\frac{9}{25}}\right]$$

$$= \cos^{-1}\left[\frac{80}{145} + \frac{21}{29}\times\frac{3}{5}\right]$$

$$= \cos^{-1}\left[\frac{80}{145} + \frac{63}{145}\right]$$

$$= \cos^{-1}\left[\frac{80+63}{145}\right]$$

$$= \cos^{-1}\left(\frac{143}{145}\right)$$

$= R.H.S.$

Ex. show that $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right) = \sin^{-1}\left(\frac{84}{85}\right)$

Sol^n:- $LHS = \sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{8}{17}\right)$

Use prop. $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\cdot\sqrt{1-x^2}\right]$

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{1-\left(\frac{8}{17}\right)^2} + \frac{8}{17}\sqrt{1-\left(\frac{4}{5}\right)^2}\right]$$

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{1-\frac{64}{289}} + \frac{8}{17}\sqrt{1-\frac{16}{25}}\right]$$

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{\frac{289-64}{289}} + \frac{8}{17}\sqrt{\frac{25-16}{25}}\right]$$

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{\frac{225}{289}} + \frac{8}{17}\sqrt{\frac{9}{25}}\right]$$

$$= \sin^{-1}\left[\frac{4}{5}\times\frac{15}{17} + \frac{8}{17}\times\frac{3}{5}\right]$$

$$= \sin^{-1}\left[\frac{60}{85} + \frac{24}{85}\right]$$

$$= \sin^{-1}\left[\frac{60+24}{85}\right]$$

$$= \sin^{-1}\left(\frac{84}{85}\right)$$

$= R.H.S.$

Dgf 29/5/2026

Subject :. Basic Mathematics (311302)
Question Bank
Unit - 2 - Trigonometry

2 - Marks Questions.

Q.1) Find the value of $\sin(15^\circ)$ using compound angles (Winter-2023)

Q.2) without using calculator, find value of $\cos(135^\circ)$ (Summer-2024)

Q.3) If $\sin A = 0.4$, find the value of $\sin 3A$ (winter-24)

Q.4) without using calculator, find value of $\cos 75^\circ$ (summer-2025)

Q.5) with out using calculator, find value of $\sin 210^\circ$ (winter-2025)

Q.6) prove that $\cos(\pi+\theta) = -\cos\theta$ (Summer-2026)

4 - Marks Questions

Q.7) If A and B are obtuse angle and $\sin A = \frac{5}{13}$ and $\cos B = \frac{-4}{5}$ then find $\sin(A+B)$ (Winter-2023)

Q.8) prove that $\frac{\sin 3A - \sin A}{\cos 3A + \cos A} = \tan A$ (Winter-23)

Q.9) prove that $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \cos^{-1}\left(\frac{84}{85}\right)$ (W-23)

Q.10) show that $\tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{1}{3}\right)$ (Summer-2024)

Q.11) without using calculator, find the value of
$\sin 150^\circ + \cos 300^\circ - \tan 315^\circ + \sec^2 360^\circ$ (summer-2024)

Q.12) prove that $\dfrac{\sin 4A + \sin 5A + \sin 6A}{\cos 4A + \cos 5A + \cos 6A} = \tan 5A$ (summer-2024)

Q.13) prove that $\sqrt{2+\sqrt{2+\cos 4\theta}} = 2\cos\theta$ (summer-2024)

Q.14) If $\tan x = \frac{5}{6}$ and $\tan y = \frac{1}{11}$ show that $x+y = \frac{\pi}{4}$ (winter-2024)

Q.15) prove that $\dfrac{\cos A}{1-\tan A} + \dfrac{\sin A}{1-\cot A} = \sin A + \cos A$ (W-24)

Q.16) without using calculator, prove that
$\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ = \frac{1}{16}$ (winter-2024)

Q.17) If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$ find $\tan(A+B)$ (summer-2025)

Q.18) prove that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$ (summer-2025)

Q.19) without using calculator, prove that
$\tan(210^\circ)\cdot\cot(390^\circ) - \tan(420^\circ)\cdot\cot(240^\circ) = 0$ (summer-2026)

Q.20) prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$ (summer-2026)

Dy/15/2022