R.C.Patel College Of Engineering & Polytechnic, Shirpur

Department of Civil Engineering

Course Title- Strength of Material **Course Code** -313308

Programme Name - Civil Engineering **Semester**-Third

Unit	Title	COs	Learning hours	R Level	U Level	A Level	Total Marks
V	Direct and Bending Stresses	CO5	10	2	4	6	12

Syllabus:

5.1 Introduction to direct and eccentric loads, Eccentricity about one principal axis, nature of stresses.
5.2 Maximum and minimum stresses, resultant stress distribution diagram. Condition for 'No tension' condition(Problems on Column subjected to Eccentric load about one axis only.)
5.3 Limit of eccentricity, core of section for circular cross sections, middle third rule for rectangular section.
5.4 Introduction to compression members, effective length, radius of gyration, slenderness ratio, type of end conditions for columns.
5.5 Buckling (or Crippling) load for columns by Euler's Formula & Rankine's Formula with meaning of each term in it.(No numericals.)

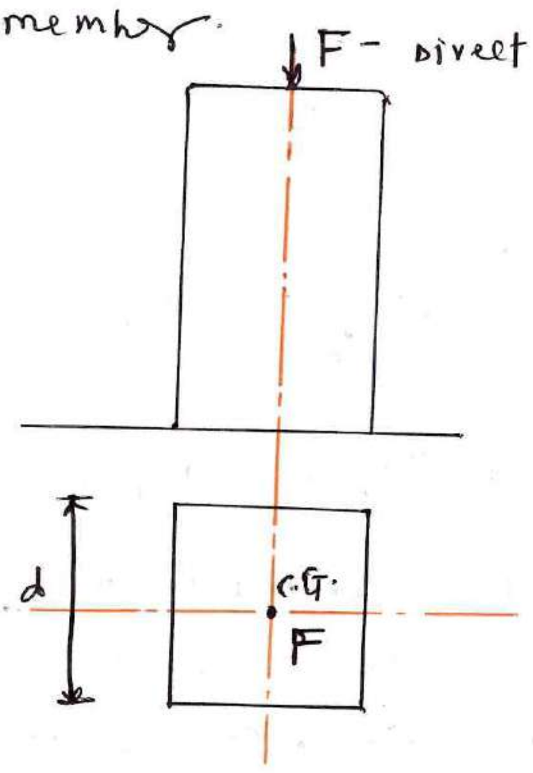
Subject Teacher
Mr.R.B.Patil

DIRECT LOAD / AXIAL LOAD :—

When a load is applied to a body whose line of action coincides with the axis of body is called direct load or axial load.

It produced uniform tensile / compressive stress into structural member.

i.e Applied load acts at centroidal axis of structural member.



ECCENTRIC LOAD :—

A load is Acted upon a compressive member whose line of action does not coincides with ~~axis~~ of centroidal axis of member is called eccentric load.

- It produce moment and bending stress and direct stress.

$\therefore \quad M = F \times e$

$\therefore$ Bending stress $= \sigma_b = \dfrac{M}{Z}$

Where, M = moment, Z = section Modulus.

e = eccentricity in mm.

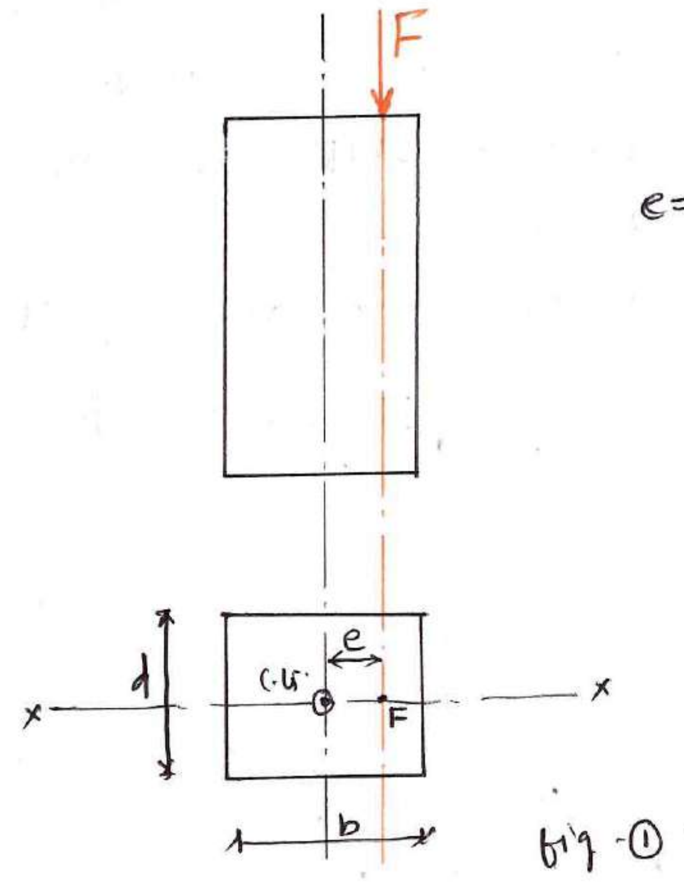


fig - ①

* ECCENTRICITY ABOUT PRINCIPAL AXIS :—

It occures when the line of action of load does not conincides with the centroidal axis of member. i.e X-axis or Y-axis.

From fig. ①

It is a rectangular column, having width 'b', and depth 'd'.

- Eccentric load is F at an eccentricity 'e'
- While applying eccentric load both direct and bending stresses are developed.

i.e Direct stress $= \frac{F}{A}$ $\{ A = b \times d$

$\therefore \sigma_D = \frac{F}{b \times d}$

From fig. it's compressive force on column.

$\therefore$ moment developed into the member is

$M = F \times e$

due to this moment Bending stress will be developed (σ_b).

As we know that; Flexural eqn.

$$\frac{M}{I} = \frac{\sigma_b}{Y}$$

$$\therefore \sigma_b = \frac{MY}{I} \qquad \{ \text{But } Z = \frac{I}{Y}$$

$$\therefore \boxed{\sigma_b = \frac{M}{Z}}$$

$\therefore$ Total stress or Resultant stress $= \sigma_b + \sigma_D$

$= \frac{M}{Z} \pm \frac{F}{A}$

$$\boxed{\sigma_{max} = \frac{F}{A} + \frac{M}{Z}} \qquad \boxed{\sigma_{min} = \frac{F}{A} - \frac{M}{Z}}$$

★ RESULTANT STRESS :—

Algebric sum of direct stress and bending stress developed due to Applied load and moment.

$\therefore$ Resultant stress = Direct stress $\pm$ Bending stress.

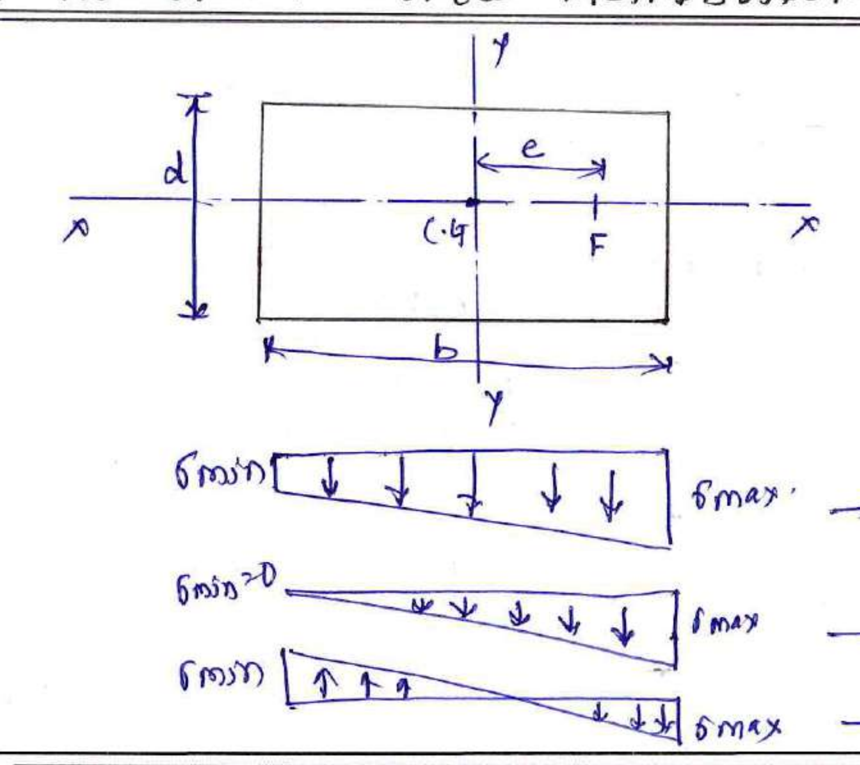
$$\boxed{\sigma_R = \sigma_D \pm \sigma_b}$$

From fig. ①

Resultant stress at base of column.

$$\sigma_{max} = \sigma_D + \sigma_b$$
$$\sigma_{min} = \sigma_D - \sigma_b$$

★ RESULTANT STRESS DISTRIBUTION DIAGRAM :—



Conditions

→ $\sigma_D > \sigma_b$

→ $\sigma_D = \sigma_b$

→ $\sigma_D < \sigma_b$

Remember

Condition	Nature	$\sigma_{min} / \sigma_{max}$.
$\sigma_o > \sigma_b$	comp. at base of column	$\boxed{\sigma_{min} = \sigma_o - \sigma_b}$ $\boxed{\sigma_{max} = \sigma_o + \sigma_b}$
$\sigma_o = \sigma_b$	comp. at base of column	$\sigma_{min} = \sigma_o - \sigma_b$ but $\sigma_o = \sigma_b$ $\therefore \boxed{\sigma_{min} = \sigma_o - \sigma_o = 0}$ $\boxed{\sigma_{max} = \sigma_o + \sigma_b}$
$\sigma_o < \sigma_b$	comp. at base of column	$\boxed{\sigma_{min} = \sigma_o - \sigma_b \text{ Tensile}}$ $\boxed{\sigma_{max} = \sigma_o + \sigma_b}$

* NO TENSION CONDITION

It means that there is no tension at the entire cross section but it possess only compression at base of any compressive member.

From defn,

$\sigma_{min} = 0$.

$\therefore \sigma_o - \sigma_b = 0$ $\quad \{ \sigma_o = \sigma_b$

* Problems on column subjected to eccentric load :-

WINTER -25

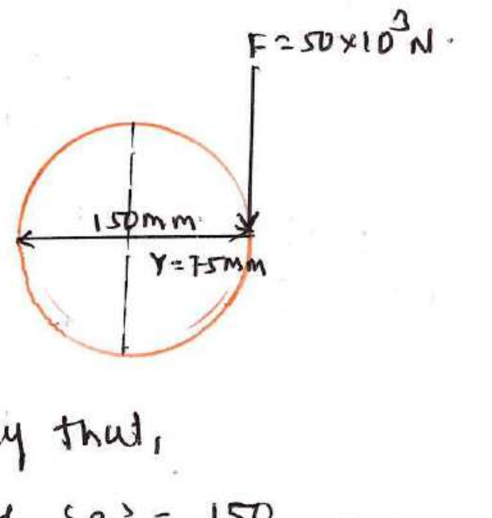
① A solid circular column of dia. 150 mm carries a vertical load of 50KN. at outer edge of column calculate σ_{max} and σ_{min}.

Ans:- Given data:-

A solid circular column 'd' = 150mm.

Load = F = 50KN = 50×10^3 N.

Soln:-



Here, we can say that,

eccentricity 'e' = $\frac{150}{2}$ = 75mm.

i) Direct stress 'σ_o' = $\frac{F}{A}$ $\quad \left\{ A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 150^2 = 17.67 \times 10^3 \text{ mm}^2 \right.$

$= \frac{50 \times 10^3}{17.67 \times 10^3}$

$\boxed{\sigma_o = 2.82 \text{ N/mm}^2}$

ii) Bending stress = 'σ_b' = $\frac{M}{Z}$ $\quad \left\{ \begin{array}{l} M = \text{Bending moment} \\ Z = \text{section modulus.} \end{array} \right.$

Ans:- Given data:-

External dia. D = 250 mm

Internal dia = 200 mm.

e = 400 mm.

$F = 20KN = 20\times10^3 N$.

soln:-

i) Direct stress 'σ_0'

$\therefore \sigma_0 = F/A$

$= \dfrac{20\times10^3}{17.67\times10^3}$ $\left\{ A = \frac{\pi}{4}(D^2-d^2) \quad A = \frac{\pi}{4}(250^2-200^2) \quad A = 17.67\times10^3\ mm^2 \right.$

$\boxed{\sigma_0 = 1.13\ N/mm^2}$

ii) Bending stress $= \dfrac{M}{Z}$

Here, M = Bending moment

$M = F\times e$

$= 20\times10^3\times400$

$= 8000\times10^3$

Z = section Modulus

$Z = \dfrac{I}{Y}$

$= \dfrac{\frac{\pi}{64}(D^4-d^4)}{\frac{D}{2}} = \dfrac{\pi(D^4-d^4)}{32D}$

$\therefore \cancel{\sigma_b = 8000\times10^3}$

$Z = \dfrac{\pi(250^4-200^4)}{32\times250}$

$Z = 9.05\times10^5$

$\sigma_b = \dfrac{800\times10^3}{9.05\times10^5}$

$\boxed{\sigma_b = 8.83\ N/mm^2}$

i.e., $\sigma_b > \sigma_0$

$\therefore \sigma_{max} = \sigma_0 + \sigma_b$

$= 1.13 + 8.83$

$\boxed{\sigma_{max} = 9.96\ N/mm^2}$

$\sigma_{min} = \sigma_0 - \sigma_b$

$= 1.13 - 8.83$

$\boxed{\sigma_{min} = -7.70\ N/mm^2}$

→ tensile

$\therefore B.M = F\times e$

$= 50\times10^3\times75$

$M = 3.75\times10^6\ N\text{-}mm.$

Now, $Z = \dfrac{\pi d^3}{32} = \dfrac{\pi\times150^3}{32}$

$Z = 3.31\times10^5$

$\therefore$ Bending stress $= \dfrac{3.75\times10^6}{3.31\times10^5}$

$\boxed{\sigma_b = 11.32\ N/mm^2}$

Now, to calculate,

$\sigma_{min} = \sigma_0 - \sigma_b$

$= 2.83 - 11.32$

$\sigma_{min} = -8.49\ N/mm^2$ (Tensile)

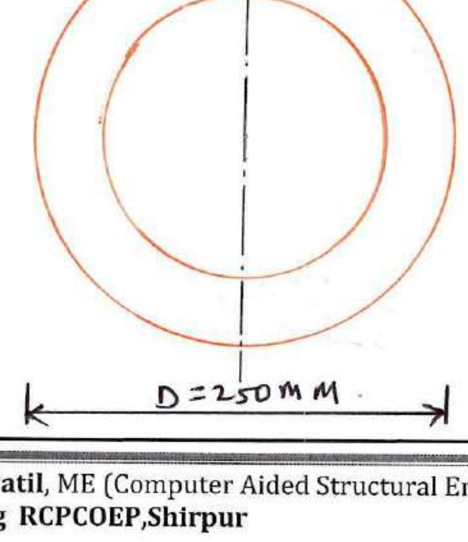
$\sigma_{max} = \sigma_0 + \sigma_b$

$= 2.83 + 11.32$

$\sigma_{max} = 14.15\ N/mm^2$

$\therefore \sigma_{min} < 0$

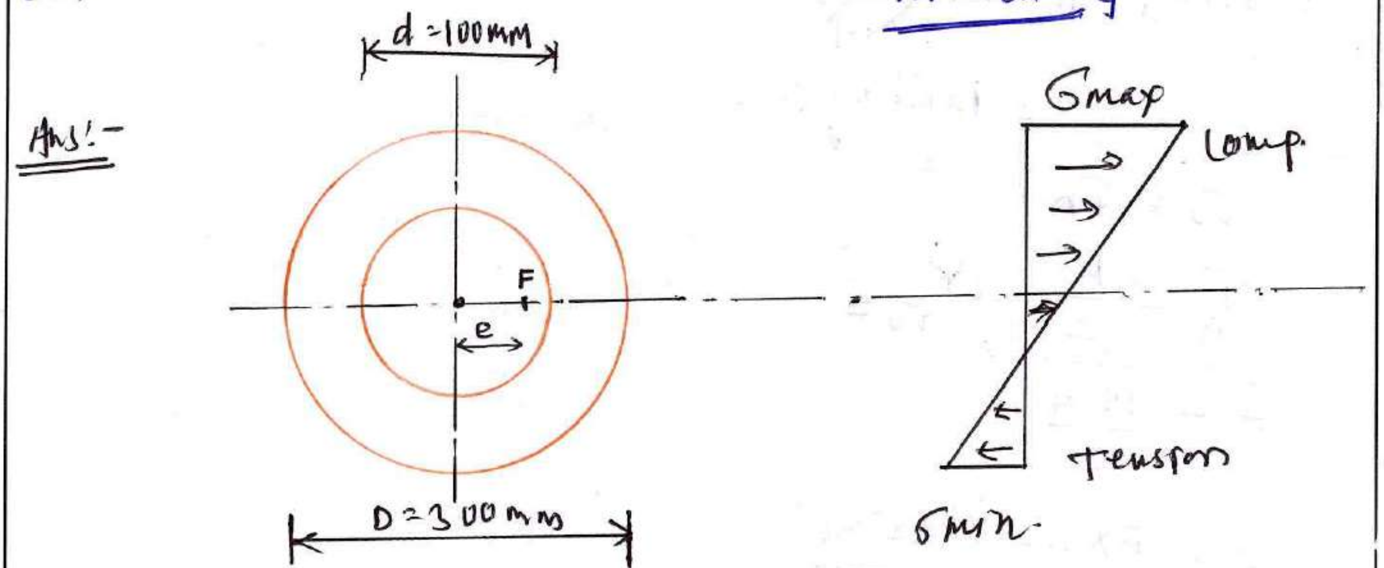
② A C.I. hollow circular column has external dia. of 250mm and internal dia 200mm, It is subjected to vertical load of 20KN at a distance of 400mm from the vertical axis of column calculate σmax and σmin. at the base of column.   WINTER-25



③ Determine limit of eccentricity for hollow circular section having D = 300 mm and d = 100 mm also draw stress distribution dia.

—WINTER-24

Ans:-



Soln:- given data:-

D = 300 mm.

d = 100 mm.

As we know that,

No tension condition.

$\sigma_0 = \sigma_b$.

$\therefore \frac{F}{A} = \frac{M}{Z}$

$\therefore \frac{F}{A} = \frac{F \times e}{Z}$

$\therefore \frac{1}{A} = \frac{e}{Z}$

$\boxed{e = \frac{Z}{A}}$

Here, $A = \frac{\pi}{4}(D^2 - d^2)$

$= \frac{\pi}{4}[300^2 - 100^2]$

$A = 62.83 \times 10^3 \ mm^2$

$\therefore Z = \frac{I_{xx}}{y_{max}} = \frac{\frac{\pi}{64}(D^4 - d^4)}{D/2}$

$= \frac{\frac{\pi}{64}[300^4 - 100^4]}{150}$

$Z = 2.62 \times 10^6$

$\therefore e = \frac{2.62 \times 10^6}{62.83 \times 10^3}$

$\boxed{e = 41.66 \ MM}$

How to find σmax and σmin.

$\therefore$ Direct stress $= \cancel{\sigma_0} \times \frac{F}{A}$

= Bending stress.

$\therefore \sigma_0 = \sigma_b$

$\frac{F}{A} = \frac{M}{Z}$ or $\sigma_0 =$

$F = \frac{M \cdot A}{Z}$

$F = \frac{F \times 41.66 \times 62.83 \times 10^3}{2.62 \times 10^6}$

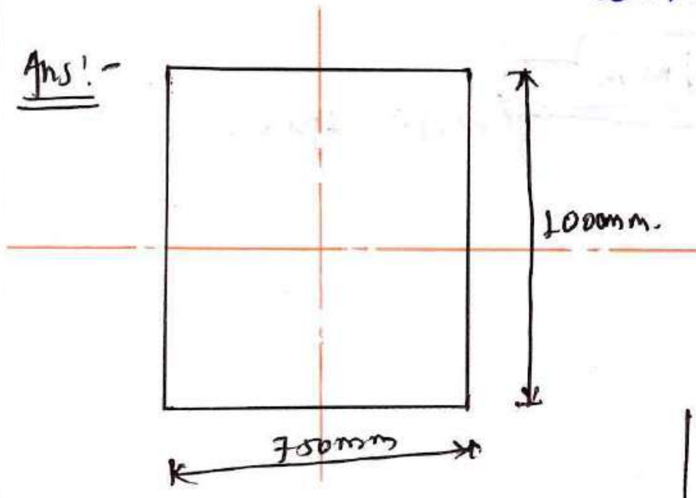
$\therefore F = 0.999 F$

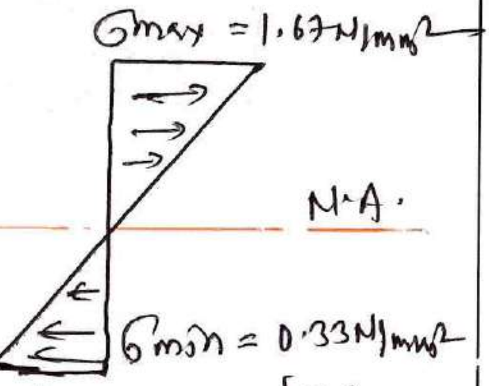
$F = F$

SUMMER-26
6M

(4) A Rectangular pier 750 mm × 1000 mm is subjected to a comp. load of 500 kN. with an eccentricity of 250 mm along the axis bisecting 1000 mm side. Determine Resultant stresses at the base cross section of the pier. Also draw Resultant stress distribution dia. showing all stresses.

Ans! -





Given data! -

$F = 500 kN = 500 \times 10^3 N$.

$e = 250 mm$.

size $= (750 \times 1000) mm$.

i) Direct stress: σ_0

$\sigma_0 = \frac{F}{A} = \frac{500 \times 10^3}{750 \times 1000}$

$\boxed{\sigma_0 = 0.667 N/mm^2}$

ii) Bending stress (σ_b)

$\sigma_b = \frac{M}{Z}$

Here, $M = F \cdot e$

$= 500 \times 10^3 \times 250$

$M = 125 \times 10^6 N\text{-}mm$.

$Z = \frac{I}{Y} = \frac{\frac{bd^3}{12}}{\frac{D}{2}} = \frac{\frac{750 \times 1000^3}{12}}{\frac{1000}{2}}$

$Z = 125 \times 10^6$

$\therefore \sigma_b = \frac{125 \times 10^6}{125 \times 10^6} = 1 N/mm^2$

Resultant stresses ! -

$\sigma_{max} = \sigma_0 + \sigma_b$

$= 0.667 + 1$

$\boxed{\sigma_{max} = 1.667 N/mm^2}$.

$\sigma_{min} = \sigma_0 - \sigma_b$

$= 0.667 - 1$

$\boxed{\sigma_{min} = -0.33 N/mm^2}$

Tensile stress

LIMIT OF ECCENTRICITY :-

If the applied load on column at maximum distance from principal or centroidal axis of column without producing tensile stress.

i.e. There is no tension condition.

$\therefore \sigma_0 \geq \sigma_b$.

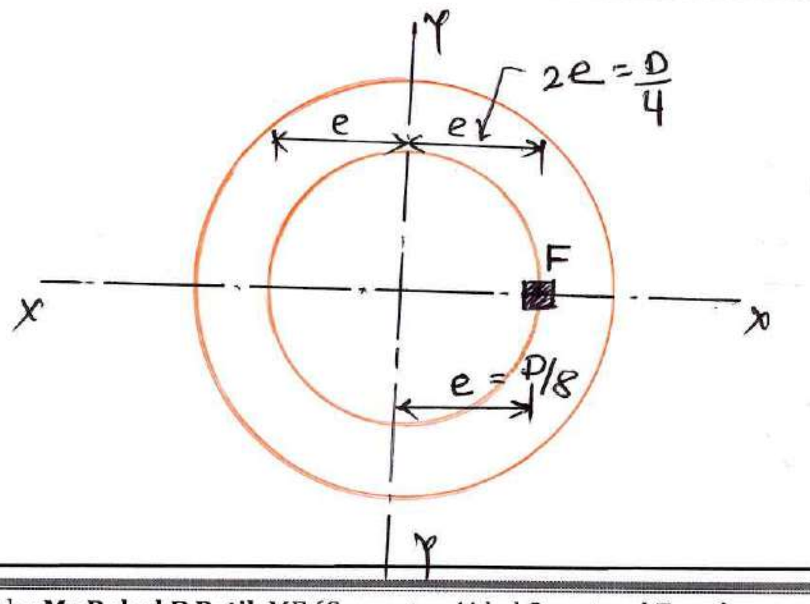
$$\frac{P}{A} \geq \frac{M}{Z}$$

$$\frac{P}{A} \geq \frac{P \cdot e}{Z}$$

$$\therefore e \leq \frac{Z}{A}$$

i.e $e < \frac{Z}{A}$ or $e = \frac{Z}{A}$

CORE OF SECTION FOR CIRCULAR CROSS SECTIONS :-



A circle of external dia 'D' and Internal dia 'd' under the compressed load 'F' acts at x-axis.

As we know, that

No tension condition

$$\sigma_0 = \sigma_b$$

$$\frac{F}{A} = \frac{M}{Z}$$

$$\therefore \frac{F}{A} = \frac{F \cdot e}{Z}$$

$$e = \frac{Z}{A} \quad \text{—(1)}$$

Here, $A = \frac{\pi}{4} D^2$

$$Z = \frac{I_{xx} = I_{yy}}{y_{max}}$$

$$= \frac{\frac{\pi}{64} D^4}{\frac{D}{2}} = \frac{\pi}{32} D^3$$

eq^n (1) $\Rightarrow$

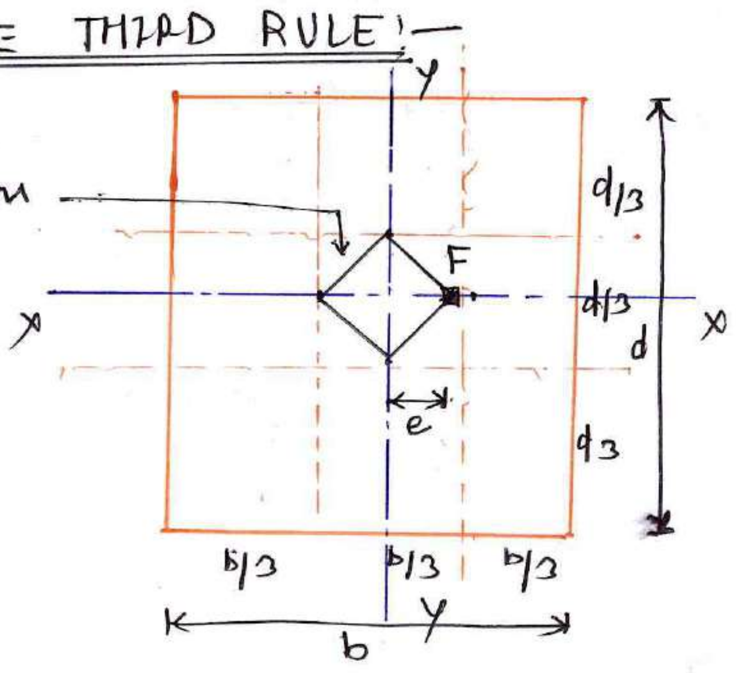
$$e = \frac{\frac{\cancel{\pi}}{\cancel{32}\,8} D^{\cancel{3}}}{\frac{\cancel{\pi}}{\cancel{4}\,1} D^{\cancel{2}}}$$

$$\boxed{e = \frac{D}{8}}$$

$$2e = 2 \times \frac{D}{8}$$

$$\boxed{2e = \frac{D}{4}}$$

★ MIDDLE THIRD RULE:-



As we know that,
NO tension condition,

$$\sigma_d = \sigma_b$$

$$\frac{F}{A} = \frac{M}{Z}$$

$$\frac{F}{A} = \frac{F \cdot e}{Z}$$

$$e = \frac{Z}{A} \quad \text{--- (1)}$$

Now,

$$Z = \frac{I_{yy}}{y} \quad \left\{ \text{load acts on x-axis} \right.$$

$$= \frac{\frac{db^3}{12}}{b/2}$$

$$Z = \frac{db^2}{6}$$

eq^n (1) ⇒

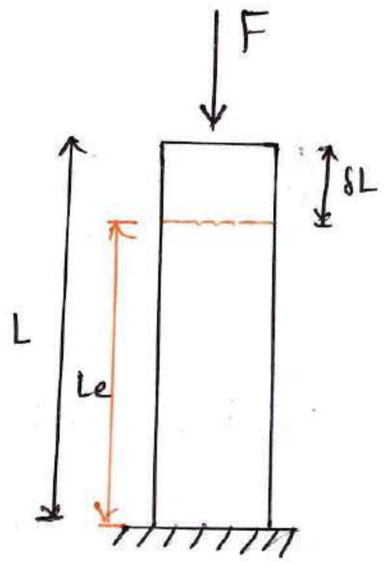
$$e = \frac{\frac{\cancel{d}b^{\cancel{2}}}{6}}{\cancel{b} \times \cancel{d}}$$

$$\boxed{e = \frac{b}{6}}$$

$$\boxed{2e = \frac{b}{3}} \text{ and } \boxed{2e = \frac{d}{3}}$$

★ COMPRESSION MEMBER:—

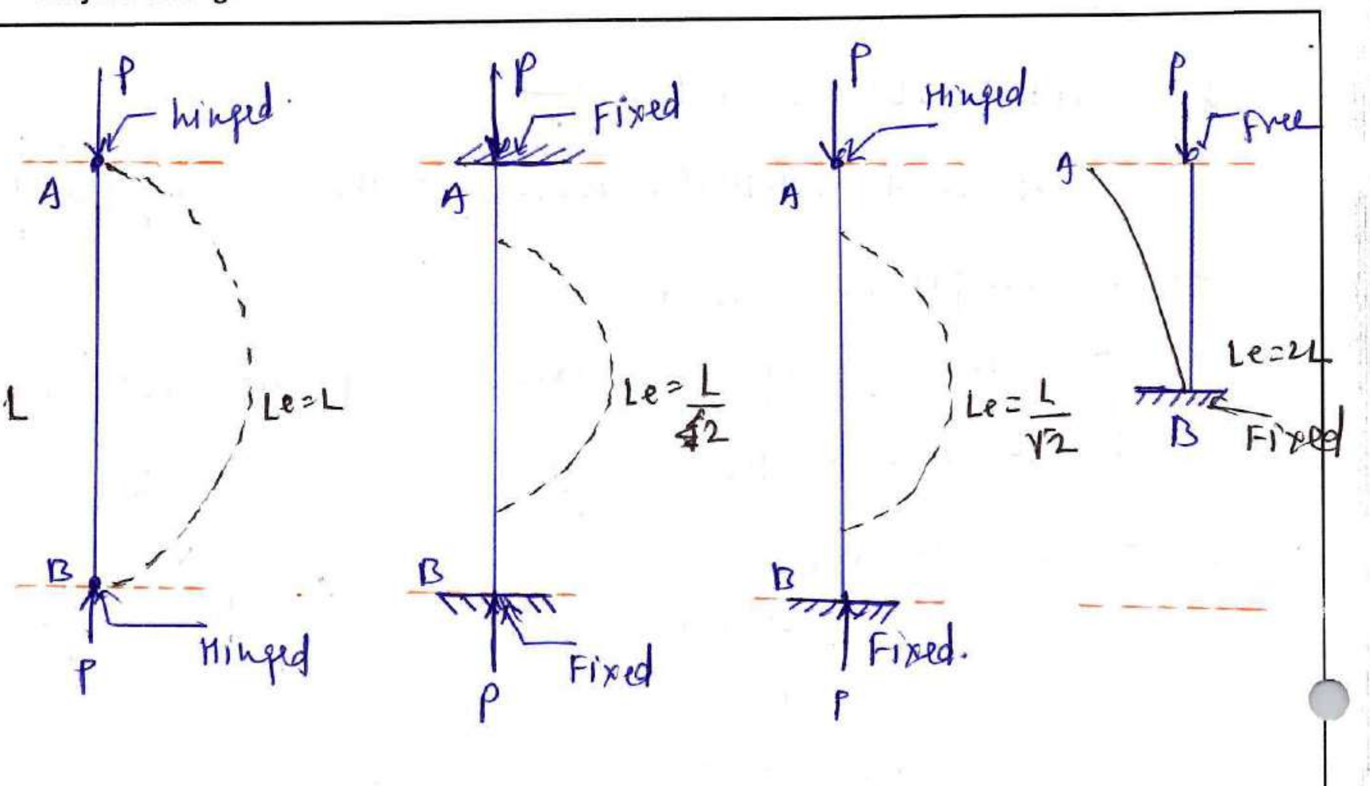
- It is a structural member they carry only axial compressive load.
- While applying axial compressive load which tends to reduce it's length.
- For example
  - i) Column
  - ii) piers
  - iii) strut



★ EFFECTIVE LENGTH:—

When both end of column is hinged, then the distance betn two sucussive supports of zero moment is called effective length.

Effective length of column for various End conditions:—



i) Both ends are fixed $= Le = \frac{L}{2}$

ii) Both ends are hinged $= Le = L$

iii) one end fixed and one end is hinged $= Le = \frac{L}{\sqrt{2}}$

iv) one end free and other end is fixed $= Le = 2L$

★ Slenderness Ratio:—

It is the ratio of effective length of column to the minimum radius of gyration of that column.

$$\lambda = \frac{Le}{K_{min}}$$

RANKINE'S FORMULA :-

- Rankine's formula is used for all length of column. i.e. (long as well as short)

$$\therefore \boxed{P_R = \frac{\sigma_c \cdot A}{1 + \alpha \left[\frac{Le}{K_{min}} \right]^2}}$$

where,

P_R = Rankine's crippling load.
Le = effective length.
K_{min} = minimum radius of gyration.
σ_c = crushing stress.
A = Area of cross-section.
α = Rankine's constant.

Material	Rankine's constant.
cast iron	1/1600
Mild steel	1/7500.
wrought Iron	1/7000

★ EULER'S FORMULA :-

$$\boxed{P_E = \frac{\pi^2 E I_{min}}{Le^2}}$$

Where,

P_E = Euler's buckling load.
E = Young's Modulus
I_{min} = Radius of gyration.
Le = effective length of column.

Assumptions :-

i) column must be perfectly straight.
ii) Material is homogeneous and isotropic.
iii) Axial load passes through centroidal axis.
iv) It must obeys hooke's law.
v) self wt must be neglected.
vi) It is used for ~~tot~~ long column. ($\lambda > 100$)

Rahul
27/6/26

AAY
27/06/26

Unit - V
Direct and Bending Stresses
Question Bank

1. State the condition of no tension at the base of column.
2. Define Slenderness ratio and Limit of eccentricity
3. State Euler's formula and Rankine's formula giving meaning of symbols used in it. State the effective length of column for various end conditions.
4. State the middle third rule.
5. State the Rankin's formula with meaning of each term used in it
6. Explain core of section for rectangular section
7. A solid circular column of diameter 150 mm carries a vertical load of 50 kN at outer edge of the column. Calculate 6 max and 6 min.
8. Draw a neat sketch to show core of rectangular section of (B × D) dimensions.
9. Determine the limit of eccentricity for a hollow circular section having D = 300 mm and d =100 mm also draw stress distribution diagram.
10. A rectangular column is 200mm and 100mm thick is subjected to a load of 180 kN at eccentricity of 100mm in plane bisecting the thickness. Draw the combined stress distribution diagram showing their values
11. A rectangular column 150 mm wide and 100 mm thick carries a load of 150 kN at an eccentricity of 50 mm in the plane bisecting the thickness. Find maximum and minimum intensities of stress in the section
12. A solid circular column of diameter 150 mm carries a vertical load of 50 kN at outer edge of the column. Calculate 6 max and 6 min
13. A C.I. hollow circular column has external diameter of 250 mm and internal diameter of 200 mm. It is subjected to a vertical load of 20 kN at a distance of 400 mm from the vertical axis of the column. Calculate the maximum and minimum stress of the base of column.
14. A rectangular pier 750 mm x 1000 mm is subjected to a compressive load of 500 kN with an eccentricity of 250 mm along the axis bisecting 1000 mm side. Determine the resultant stresses at the base cross section of the pier. Also draw resultant stress distribution diagram showing all stresses
15. A hollow circular section of 250mm as external diameter and 25mm thickness is 4m long and used as a column. One end of the column is fixed and other end is hinged. Calculate the safe load the column can carry. Use Euler's formula, Take E = 2×105N/mm2, Factory of safety = 2.5