The Shirpur Education Society's

R. C. Patel College of Engineering and Polytechnic, Shirpur

Department of Applied Sciences and Humanities

NAME OF COURSE: - Basic Mathematics

CODE OF COURSE: - 311302

SEMESTER: - FY CO/CW/EE/CE/ME- 1K

SUBJECT TEACHER: - Mr. Pankaj. G. Bhadane

Unit-1- Algebra

ch-1> Logarithm.

* **Logarithm :-** logarithm is nothing but a equivalent form of an exponential expression ie if $y = a^x$, $a>0$. $a \neq 1$ $a \in R$ then same expression can be expressed in equivalent logarithmic form as $x = \log_a^y$ and read as x is called logarithm of y to the base a.

For Examples:-

1> If $32 = 2^5$ then $5 = \log_2^{32}$

2> If $10 = 100^{\frac{1}{2}}$ then $\frac{1}{2} = \log_{100}^{10}$

* **Types of logarithm**

1> Natural logarithm :-

If the base of logarithm is a nepier's number e then it is called natural logarithm where $e = 2.7182$

For Examples: $\log_e^7$, $\log_e^2$ ... etc.

2> common logarithm :-

If the base of logarithm is 10 number then it is called common logarithm.

For examples:- $\log_{10}^3$, $\log_{10}^7$ ... etc.

* **Laws of Logarithm**

1> Logarithm of product

If m, n, a are positive real number and $a \neq 1$ then

$$\log_a(mn) = \log_a m + \log_a n$$

For e.g. $\log_5(2\times3) = \log_5 2 + \log_5 3$

2) Logarithm of Quotient

If m, n, a are positive real number and $a \neq 1$ then

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$$

For e.g. $\log_2\left(\frac{8}{7}\right) = \log_2 8 - \log_2 7$

3) logarithm of power

If m, a are positive real numbers, $a \neq 1$ and $n \in R$ then

$$\log_a m^n = n \log_a m$$

For eg. $\log_2 3^5 = 5\log_2 3$

4) Change of base Rule

If m, n, a are positive numbers $n \neq 1$ and $a \neq 1$ then

$$\log_n m = \frac{\log m}{\log n}$$

for e.g. $\log_2 8 = \frac{\log 8}{\log 2}$

Deduction, $\log_n m = \frac{1}{\log_m n}$

5) $a^{\log_a^x} = x$

For e.g.

6) $\log_a^a = 1$

7) $\log_a^1 = 0$ where $a \neq 1$

8) $\log \frac{1}{a} = -\log a$

Examples:. write the following term in logarithmic form.

1) $5^3 = 125$ logarithmic Term $\rightarrow$ $3 = \log_5^{125}$

2) $7^0 = 1$ Logarithmic Term $\rightarrow$ $0 = \log_7^1$

3) $5^{-2} = \frac{1}{25}$ Logarithmic Term $\rightarrow$ $-2 = \log_5\left(\frac{1}{25}\right)$

4) $x^y = z$ logarithmic Term $\rightarrow$ $y = \log_x^z$

Examples:. write the following terms in exponential form.

1) $\log_3^{27} = 3$ exponential term $\rightarrow$ $3^3 = 27$

2) $\log_{3\sqrt{2}}^{18} = 2$ exponential term $\rightarrow$ $(3\sqrt{2})^2 = 18$

3) $\log_{\sqrt{7}}^{343} = 6$ exponential term $\rightarrow$ $(\sqrt{7})^6 = 343$

4) $\log_4\left(\frac{1}{16}\right) = -2$ exponential term $\rightarrow$ $(4)^{-2} = \frac{1}{16}$

Examples: Find the value of $\log_{81}^{3}$

Sol^n:- $\log_{81}^{3}$

$= \dfrac{\log 3}{\log 81}$ --- from $\log_{n}^{m} = \dfrac{\log m}{\log n}$

$= \dfrac{\log 3}{\log 3^4}$

$= \dfrac{\cancel{\log 3}}{4 \cancel{\log 3}}$ -- from $\log m^n = n \log m$

$= \dfrac{1}{4}$

practice problems:-

1) Find $\log_{3}^{243}$ Ans:- 5

2) find $\log_{343}^{7}$ Ans: $\frac{1}{3}$

3) Find $\log_{3}^{27}$ Ans:- 3

Examples: Find x if $\log_{2}(x-3) = 3$

Sol^n:- $\log_{2}(x-3) = 3$

$x - 3 = 2^3$ --- from exponential form

$x - 3 = 8$

$x = 8 + 3$

$\boxed{x = 11}$

practice problems:-

1) Find x if $\log_3(x+4) = 4$ Ans: 77

Examples: solve $\log_2(x^2-6x+40) = 5$

Sol^n:- $\log_2(x^2-6x+40) = 5$

$x^2-6x+40 = 2^5$ --- from exponential form

$x^2-6x+40 = 32$

$x^2-6x+40-32 = 0$

$x^2-6x+8 = 0$ which is Quadratic eq^n

$(x-4)(x-2) = 0$

$x-4=0$ OR $x-2=0$

ie $\boxed{x=4}$ OR $\boxed{x=2}$

Practice problems:- solve $\log_{10}(x^2+6x+28) = 2$

Ans $\boxed{x=-12}$ OR $\boxed{x=6}$

Example:- find x if $\log_3(x-4)+\log_3(x-2) = 1$

Sol^n:- $\log_3(x-4) + \log_3(x-2) = 1$

$\log_3[(x-4)(x-2)] = 1$ use $\log_a m + \log_a n = \log_a(mn)$

$(x-4)(x-2) = 3^1$ -- from exponential form

$x^2-6x+8 = 3$

$x^2-6x+5 = 0$

$(x-5)(x-1) = 0$

$x-5=0$ OR $x-1=0$

$\boxed{x=5}$ OR $\boxed{x=1}$

practice problem :-

Find x if $\log_{10} x + \log_{10}(x-3) = 1$

Ans: $\boxed{x=5}$ OR $\boxed{x=-2}$

Example :. Find x if $\log_2[\log_3(\log_2 x)] = 1$

Sol^n :- $\log_2[\log_3(\log_2 x)] = 1$

$\log_3(\log_2 x) = 2^1$ --- exponential form

$\log_3(\log_2 x) = 2$

$\log_2 x = 3^2$ --- exponential form

$\log_2 x = 9$

$x = 2^9$ -- exponential form

$x = 512$

practice problem :.

Find x if $\log_2[\log_2(\log_3 x)] = 1$

Ans :. $\boxed{x=81}$

Examples:- Evaluate $25^{\log_5 8}$

Sol^n:- $25^{\log_5 8}$

$= \left[(5)^2\right]^{\log_5 8}$

$= 5^{2\log_5 8}$

$= 5^{\log_5 8^2}$ use $n\log m = \log m^n$

$= 5^{\log_5 64}$

$= 64$ .. use $a^{\log_a x} = x$

Examples: Evaluate $18^{\log_{3\sqrt{2}} 7}$

Sol^n:- $18^{\log_{3\sqrt{2}} 7}$

$= \left[(3\sqrt{2})^2\right]^{\log_{3\sqrt{2}} 7}$

$= (3\sqrt{2})^{2\log_{3\sqrt{2}} 7}$

$= (3\sqrt{2})^{\log_{3\sqrt{2}} 7^2}$ use $n\log m = \log m^n$

$= (3\sqrt{2})^{\log_{3\sqrt{2}} 49}$

$= 49$ -- use $a^{\log_a x} = x$

practice problems:-

1) Evaluate $625^{\log_5 7}$ Ans:. 2401

2) Evaluate $(12)^{\log_{2\sqrt{3}} 5}$ Ans:. 25.

Examples:- prove that $\dfrac{1}{\log_2 8} + \dfrac{1}{\log_{64} 8} + \dfrac{1}{\log_4 8} = 3$

Sol^n:- $L.H.S = \dfrac{1}{\log_2 8} + \dfrac{1}{\log_{64} 8} + \dfrac{1}{\log_4 8}$

$= \log_8 2 + \log_8 64 + \log_8 4$ $\quad \therefore \log_n m = \dfrac{1}{\log_m n}$

$= \log_8 (2 \times 64 \times 4)$ $\quad \therefore \log_a m + \log_a n + \log_a p = \log_a (mnp)$

$= \log_8 512$

$= \log_8 8^3$ $\quad \therefore 8^3 = 512$

$= 3 \log_8 8$ $\quad \therefore \log_a a = 1$

$= 3$

practice problems:.

1) prove that $\dfrac{1}{\log_3 6} + \dfrac{1}{\log_8 6} + \dfrac{1}{\log_9 6} = 3$.

2) prove that $\dfrac{1}{\log_a abc} + \dfrac{1}{\log_b abc} + \dfrac{1}{\log_c abc} = 1$

Unit - 1 . Algebra

ch-2) Determinant

Definition of Determinant :-

An arrangement of mn numbers in equal no. of rows and equal no. of columns enclosed between two vertical lines. is called determinant of order $m \times n$.

Def^n of order 2×2 Determinant.

An arrangement of four numbers a, b, c, d in two rows and two columns enclosed between two vertical bars is called a determinant of order 2×2. and it is denoted by Δ or D or $|A|$.

Thus we write,

$$|A| = \begin{array}{cc} \begin{vmatrix} a & b \\ c & d \end{vmatrix}_{2\times 2} & \begin{array}{l} R_1 \\ R_2 \end{array} \\ \begin{array}{cc} C_1 & C_2 \end{array} & \end{array}$$

Value of 2×2 Determinant

let $D = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$= \left(\text{Product of principal diagonal element}\right) - \left(\text{Product of Anti-diagonal element}\right)$

$= (a \times b) - (b \times c)$

$= ab - bc$

for examples,

$$D = \begin{vmatrix} 2 & 3 \\ -1 & 5 \end{vmatrix}$$

$= (2\times5) - (-1\times3)$

$= 10 - (-3)$

$= 10 + 3$

$= 13$

Def^n of order 3x3 Determinant

An arrangement of nine numbers a, b, c, d, e, f, g, h, i in theree numbers of rows and three no. of columns enclosed between two vertical bars is called determinant of order 3x3.

Thus we write,

$$D = |A| = \begin{array}{c|ccc|l} & a & b & c & \to R_1 \\ & d & e & f & \to R_2 \\ & g & h & i & \to R_3 \\ \hline & \downarrow & \downarrow & \downarrow & 3\times3 \\ & C_1 & C_2 & C_3 & \end{array}$$

Value of 3x3 Determinant

let $D = |A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

= a x minor of a − b x minor of b + c x minor of c

$$= a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - fh) - b(di - fg) + c(dh - eg)$$

for example,

$$D = \begin{vmatrix} 2 & 3 & 5 \\ 4 & 1 & 2 \\ 3 & 1 & 6 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & 2 \\ 1 & 6 \end{vmatrix} - 3\begin{vmatrix} 4 & 2 \\ 3 & 6 \end{vmatrix} + 5\begin{vmatrix} 4 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= 2(6-2) - 3(24-6) + 5(4-3)$$

$$= 2(4) - 3(18) + 5(1)$$

$$= 8 - 54 + 5$$

$$= -41$$

Examples on 2x2 & 3x3 Determinant.

Ex. 1) Find $\begin{vmatrix} 3 & -2 \\ 5 & -6 \end{vmatrix}$

Sol^n - $D = \begin{vmatrix} 3 & -2 \\ 5 & -6 \end{vmatrix}$

$$= (3 \times -6) - (-2 \times 5)$$

$$= -18 - (-10)$$

$$= -18 + 10$$

$$= -8$$

Ex. Find $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -3 \end{vmatrix}$

sol^n:-

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -3 \end{vmatrix}$$

$$= 1 \cdot \begin{vmatrix} 1 & -1 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 1 & -3 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= 1(-3+1) + 1(2\times -3+1) + 1(2-1)$$

$$= 1(-2) + 1(-5) + 1(1)$$

$$= -2 - 5 + 1$$

$$= -6$$

Ex. Find $\begin{vmatrix} 3 & -5 & -1 \\ 1 & 3 & 5 \\ -5 & 1 & 3 \end{vmatrix}$

sol^n:.

$$D = \begin{vmatrix} 3 & -5 & -1 \\ 1 & 3 & 5 \\ -5 & 1 & 3 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 3 & 5 \\ 1 & 3 \end{vmatrix} + 5 \begin{vmatrix} 1 & 5 \\ -5 & 3 \end{vmatrix} - 1 \begin{vmatrix} 1 & 3 \\ -5 & 1 \end{vmatrix}$$

$$= 3(9-5) + 5(3+25) - 1(1+15)$$

$$= 12 + 140 - 16 = 136.$$

ch-3) Matrices

Definition of Matrix :-

An arrangement of mn numbers in m no.of rows and n no.of columns enclosed between square bracket or rectangular bracket is called Matrix of order $m \times n$.

The matrix is denoted by capital letters A, B, C, ... etc.

Thus $$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}_{m \times n}.$$

Types of Matrices :-

1) Row Matrix - A matrix containing only one row and n no.of columns are called row matrix

for e.g. $A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$

2) Column Matrix - A matrix containing only one column and n no. of rows are called column matrix.

for eq. $A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

3) Square matrix - A matrix in which equal no. of rows and column are called square matrix.

e.g. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Matrices

Definition of Matrix:-

An arrangement of mn numbers in m no. of rows and n no. of columns enclosed between square bracket or rectangular bracket is called matrix of order $m \times n$.

The matrix is denoted by capital letters A, B, C, ... etc.

thus $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$

Types of matrices:-

1) Row matrix:- A matrix containing only one row and n no. of columns are called row matrix.

For e.g. $A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$

2) Column matrix:- A matrix containing only one column and n no. of rows are called column matrix.

For e.g. $A = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

3) Square matrix:- A matrix in which equal no. of rows and column are called square matrix.

e.g. $A = \begin{bmatrix} 1 & \cdots \\ 4 & \cdots \end{bmatrix}$

4) Diagonal Matrix - A square matrix in which all non-diagonal elements are zero is called diagonal matrix.

for e.g. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

5) scalar matrix - A diagonal matrix in which all diagonal elements are equal is called scalar matrix.

for e.g. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

6) Unit Matrix or Identity matrix - A scalar matrix having a diagonal element is one is called an unit or identity matrix.

for eq. $\cancel{A =}\ I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7) Upper triangular matrix - A square matrix whose every element below the diagonal is zero, is called an upper triangular matrix.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

8) Lower triangular matrix - A square matrix whose every element above the diagonal is zero

[illegible] matrix A in which [illegible] non diagonal element [illegible] is called [illegible]

[illegible] A [illegible]

5. [illegible] diagonal matrix in which [illegible] diagonal elements are equal is called [illegible] matrix.

[illegible]

[illegible] diagonal [illegible] matrix.

[illegible] B [illegible]

7. [illegible] elements [illegible] the diagonal is [illegible] called [illegible] triangular matrix.

[illegible] A [illegible]

is called lower triangular matrix.

for e.g. $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$

9) Null or zero Matrix – Any matrix in which all the elements are zero is called a zero or null matrix.

for e.g $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

10) Transpose of Matrix - If A is any matrix, then the matrix obtained from A by interchanging the rows and column is called the transpose of Matrix.

It is written as A^T or A'

for eg. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 6 \end{bmatrix}$

11) Symmetric Matrix - A square matrix A is said to be symmetric if $A^T = A$

for e.g. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ OR $\begin{bmatrix} 1 & 7 & 2 \\ 7 & 6 & 0 \\ 2 & 0 & 8 \end{bmatrix}$

12) Skew symmetric matrix - A square matrix is said to be skew symmetric matrix if $A^T = -A$

for e.g. $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{bmatrix}$ OR $\begin{bmatrix} 0 & +1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$

14) Singular matrix - A square matrix A is said to be singular matrix if $|A| = 0$

for e.g $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ ie $|A| = 0$

15) Non-singular matrix - A square matrix 'A' is said to be non-singular matrix if $|A| \neq 0$

for e.g. $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ $|A| = -2 \neq 0$.

* Algebra of Matrices -

1) Equality of matrix -

TWO matrices A & B are said to be equal, if their order is same and element in the corresponding positions of two matrices are equal.

for e.g. $A = \begin{bmatrix} 4 & 5 \\ -2 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 \\ -2 & 6 \end{bmatrix}$ then $A = B$.

2) Addition of matrices -

If two matrices are of the same order then only, they can be added. then by adding their corresponding elements we get matrix.

for e.g. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$

then $A + B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 9 & 11 \end{bmatrix}$

3) substraction of matrices -

If two matrices are of the same order then only, they can be substract, then by substracting their corresponding elements we get matrix.

for e.g. $A = \begin{bmatrix} 7 & 8 \\ 9 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix}$ then

$$A-B = \begin{bmatrix} 7 & 8 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 3 \end{bmatrix}$$

4) Scalar multiplication of matrix -

A be the matrix and K be a number (scalar) then the scalar multiple of A by K is denoted by KA and it obtained by multiplying every element of the matrix A by K.

for e.g. $K \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} Ka & Kb \\ Kc & Kd \end{bmatrix}$ K = scalar.

* Simple examples on Algebra of matrices.

Ex. If $A = \begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ find $2A-3B$

sol^n:-

$$2A-3B = 2\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 6 \\ -2 & 2 \end{bmatrix} - \begin{bmatrix} 6 & -3 \\ 9 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 9 \\ -11 & -4 \end{bmatrix}$$

Ex. if $A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$, $C = \begin{bmatrix} 7 & 11 \\ -8 & 9 \end{bmatrix}$

show that $3A + B = C$

$$LHS = 3A + B = 3\begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 6 \\ -9 & 12 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 11 \\ -8 & 9 \end{bmatrix}$$

$$= C$$

$$= R.H.S.$$

Ex. find X if $\begin{bmatrix} 4 & 5 \\ -3 & 6 \end{bmatrix} + X = \begin{bmatrix} 10 & -1 \\ 0 & -5 \end{bmatrix}$

Sol^n:. $\begin{bmatrix} 4 & 5 \\ -3 & 6 \end{bmatrix} + X = \begin{bmatrix} 10 & -1 \\ 0 & -5 \end{bmatrix}$

$$X = \begin{bmatrix} 10 & -1 \\ 0 & -5 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ -3 & 6 \end{bmatrix}$$

$$X = \begin{bmatrix} 6 & -6 \\ 3 & -11 \end{bmatrix}$$

Ex. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}$ find $2A+3B-4I$

where I is the identity matrix of order two.

Sol^n:. $2A+3B-4I = 2\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} + 3\begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$2A + 3B - 4I = \begin{bmatrix} 4 & 6 \\ 8 & 14 \end{bmatrix} + \begin{bmatrix} 3 & 9 \\ 12 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 15 \\ 20 & 28 \end{bmatrix}.$$

Ex. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 \\ 3 & -2 \end{bmatrix}$ $C = \begin{bmatrix} 3 & -1 \\ 0 & 6 \end{bmatrix}$

find $3A + 4B - 2C$.

sol^n:- $3A + 4B - 2C = 3\begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix} + 4\begin{bmatrix} 4 & 5 \\ 3 & -2 \end{bmatrix} - 2\begin{bmatrix} 3 & -1 \\ 0 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 6 & -9 \\ 9 & 12 \end{bmatrix} + \begin{bmatrix} 16 & 20 \\ 12 & -8 \end{bmatrix} - \begin{bmatrix} 6 & -2 \\ 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 13 \\ 21 & -8 \end{bmatrix}$$

Ex. If $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$ find the matrix 'x' such that $2A + X = 3B$

sol^n:- Given $2A + X = 3B$

$$X = 3B - 2A$$

$$X = 3\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} - 2\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -6 \\ -3 & 12 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 8 & 6 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & -4 \\ -11 & 6 \end{bmatrix}$$

Ex. If $A = \begin{bmatrix} 1 & 3 \\ 7 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 7 \\ 9 & 0 \end{bmatrix}$

Show that $A+B = B+A$.

Sol^n:- let $LHS = A+B$

$$= \begin{bmatrix} 1 & 3 \\ 7 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 7 \\ 9 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 10 \\ 16 & 8 \end{bmatrix} \quad — (1)$$

$RHS = B+A$

$$= \begin{bmatrix} 2 & 7 \\ 9 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 7 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 10 \\ 16 & 8 \end{bmatrix} \quad — (2)$$

from (1) & (2), we get

$$A+B = B+A.$$

Ex. If $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ find 'X'

such that $2X + 3A - 4B = I$

Sol^n:. Given $2X + 3A - 4B = I$

$$2X = I - 3A + 4B$$

$$2X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 3\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 9 & -3 \\ 6 & 12 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ -12 & 0 \end{bmatrix}$$

$$2X = \begin{bmatrix} -4 & 11 \\ -18 & -11 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -4 & 11 \\ -18 & -11 \end{bmatrix}$$

Ex. If $\begin{bmatrix} 3 & -6 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ find a, b, c, d

sol^n:- Given $\begin{bmatrix} 3 & -6 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 5 & -3 \\ 2 & 3 \end{bmatrix}_{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

By equality of matrices,

$a = 5$, $b = -3$, $c = 2$, $d = 3$.

Ex. find the value of x and y satisfying equations.

$$\begin{bmatrix} 1 & x & 0 \\ y & 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ 6 & 5 & 2 \end{bmatrix}$$

sol^n:- Given $\begin{bmatrix} 1 & x & 0 \\ y & 2 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 1 & 2 \\ 4 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ 6 & 5 & 2 \end{bmatrix}$

$$\begin{bmatrix} 4 & 1+x & 2 \\ y+4 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 2 \\ 6 & 5 & 2 \end{bmatrix}$$

By equality of matrices,

$1 + x = 2$ $\quad$ $y + 4 = 6$

$\therefore \boxed{x = 1}$ $\quad$ $\therefore \boxed{y = 2}$

Ex. if $A = \begin{bmatrix} x & 2 & -5 \\ 3 & 1 & 2y \end{bmatrix}$, $B = \begin{bmatrix} 2y+5 & 6 & -15 \\ 9 & 3 & -6 \end{bmatrix}$

and if $3A = B$ find x and y.

sol^n:- Given $3A = B$

$$3\begin{bmatrix} x & 2 & -5 \\ 3 & 1 & 2y \end{bmatrix} = \begin{bmatrix} 2y+5 & 6 & -15 \\ 9 & 3 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 3x & 6 & -15 \\ 9 & 3 & 6y \end{bmatrix} = \begin{bmatrix} 2y+5 & 6 & -15 \\ 9 & 3 & -6 \end{bmatrix}$$

By equality of matrices,

$3x = 2y+5$ —— ①

$6y = -6$

$y = \frac{-6}{6}$

$\boxed{y = -1}$

Put $y = -1$ in eq^n ①

$3x = -2+5$

$3x = 3$

$x = \frac{3}{3}$

$\boxed{x = 1}$

$\therefore \boxed{x = 1}, \boxed{y = -1}$

* Multiplication of Matrices :-

The product of two matrices A and B is possible only if number of columns in A is equal to the number of rows in B.

* Examples on multiplication of matrices.

Ex. If $A = \begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 3 \\ 4 & 5 & -2 \end{bmatrix}$ find AB.

sol^n:- $AB = \begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 3 \\ 4 & 5 & -2 \end{bmatrix}$

$= \begin{bmatrix} 3-4 & -3-5 & 9+2 \\ 2+0 & -2+0 & 6+0 \\ 1-8 & -1-10 & 3+4 \end{bmatrix}$

$AB = \begin{bmatrix} -1 & -8 & 11 \\ -7 & -11 & 7 \end{bmatrix}$

Ex. If $A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 0 & 4 \end{bmatrix}$ find AB

sol^n:- $AB = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} -3+0 & 6-8 \\ -4+0 & 8+4 \end{bmatrix}$

$= \begin{bmatrix} -3 & -2 \\ -4 & 12 \end{bmatrix}$

Ex. if $A = \begin{bmatrix} 1 & -5 \\ 6 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ find matrix $AB - 2I$

sol^n:- find AB

$$AB = \begin{bmatrix} 1 & -5 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 \\ 6 & -4 \end{bmatrix}$$

Now $AB - 2I = \begin{bmatrix} 1 & 5 \\ 6 & -4 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 1 & 5 \\ 6 & -4 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 5 \\ 6 & -6 \end{bmatrix}$$

Ex. If $A = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 6 \\ -4 & -12 \end{bmatrix}$ show that AB is null matrix.

sol^n:- $AB = \begin{bmatrix} 4 & 2 \\ 8 & 4 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -4 & -12 \end{bmatrix}$

$$= \begin{bmatrix} 8-8 & 24-24 \\ 16-16 & 48-48 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ AB is null matrix

Ex. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ find $A^2 - 9A + 14I$

where I is unit matrix of order two.

sol^n:. $A^2 - 9A + 14I$

$$= \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} - 9\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} + 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16+6 & 12+15 \\ 8+10 & 6+25 \end{bmatrix} - \begin{bmatrix} 36 & 27 \\ 18 & 45 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} - \begin{bmatrix} 36 & 27 \\ 18 & 45 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{Null matrix.}$$

Ex. If $A = \begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 2 & -5 \\ 1 & 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 6 & -7 & 0 \\ -1 & 2 & 5 \\ 1 & 0 & 3 \end{bmatrix}$

prove that $(AB)C = A(BC)$

sol^n:.

$$AB = \begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} 4 & 2 & -5 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -11 \\ -13 & -6 & 12 \end{bmatrix}$$

$$\text{LHS } (AB)C = \begin{bmatrix} 2 & 2 & -11 \\ -13 & -6 & 12 \end{bmatrix}\begin{bmatrix} 6 & -7 & 0 \\ -1 & 2 & 5 \\ 1 & 0 & 3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -1 & -10 & -23 \\ -60 & 79 & 6 \end{bmatrix} \quad \text{--- (1)}$$

$$BC = \begin{bmatrix} 4 & 2 & -5 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 6 & -7 & 0 \\ -1 & 2 & 5 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 17 & -24 & -5 \\ 9 & -7 & 9 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & -2 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} 17 & -24 & -5 \\ 9 & -7 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -10 & -23 \\ -60 & 79 & 6 \end{bmatrix} \quad \text{--- (2)}$$

from (1) & (2), we get.

$(AB)C = A(BC)$ hence prove.

Ex. If $A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$ find A^2

Sol^n:. $A^2 = A \cdot A = \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 4 & 2 & 4 \\ 4 & 4 & 2 \end{bmatrix}$

$$= \begin{bmatrix} 4+16+16 & 8+8+16 & 8+16+8 \\ 8+8+16 & 16+4+16 & 16+8+8 \\ 8+16+8 & 16+8+8 & 16+16+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix}$$

Ex. Find x and y if

$$\left\{ 4 \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 & -1 \\ 2 & -3 & 4 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

sol^n:-

$$\left\{ 4 \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 & -1 \\ 2 & -3 & 4 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 4 & 8 & 0 \\ 8 & -4 & 12 \end{bmatrix} - \begin{bmatrix} 2 & 6 & -2 \\ 4 & -6 & 8 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 2 \\ 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

By equality of matrices, we get

$x = 2$, $y = 4$.

Ex. if $\left\{ 3 \begin{bmatrix} 3 & 1 \\ 4 & 0 \\ 3 & -3 \end{bmatrix} - 2 \begin{bmatrix} 0 & 2 \\ -2 & 3 \\ -5 & 4 \end{bmatrix} \right\} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

find x, y, z.

sol^n

$$\left\{ 3 \begin{bmatrix} 3 & 1 \\ 4 & 0 \\ 3 & -3 \end{bmatrix} - 2 \begin{bmatrix} 0 & 2 \\ -2 & 3 \\ -5 & 4 \end{bmatrix} \right\} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\left\{\begin{bmatrix} 9 & 3 \\ 12 & 0 \\ 9 & -9 \end{bmatrix} - \begin{bmatrix} 0 & 4 \\ -4 & 6 \\ -10 & 8 \end{bmatrix}\right\} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 9 & -1 \\ 16 & -6 \\ 19 & -17 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} -11 \\ -28 \\ -53 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

By equality of matrices, we get

$x = -11, \quad y = -28, \quad z = -53.$

Ex. find x, y, z if $\left\{\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + 2\begin{bmatrix} 3 & 0 & 2 \\ 1 & 4 & 5 \\ 2 & 1 & 0 \end{bmatrix}\right\} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Sol^n:- $\left\{\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 4 \\ 2 & 8 & 10 \\ 4 & 2 & 0 \end{bmatrix}\right\} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\begin{bmatrix} 7 & 3 & 6 \\ 4 & 8 & 11 \\ 7 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 31 \\ 53 \\ 19 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

By equality of matrices $x = 31, y = 53, z = 19.$

* Examples on Transpose of Matrices.

Ex. if $A = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 6 \\ -3 & 4 \end{bmatrix}$ find $(AB)^T$

sol^n:- Given $A = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 6 \\ -3 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6 & 6+8 \\ 10-9 & 30+12 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 14 \\ 1 & 42 \end{bmatrix}$$

Transpose of AB,

$$(AB)^T = \begin{bmatrix} -4 & 1 \\ 14 & 42 \end{bmatrix}.$$

Ex. if $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$

verify that $(A+B)^T = A^T + B^T$.

sol^n:- Given matrices, $A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} -1 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$

$$A+B = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 5 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 & 3 \\ 5 & 8 & 0 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 1 & 5 \\ 5 & 8 \\ 3 & 0 \end{bmatrix} \quad — (1)$$

NOW $A^T = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & 0 \end{bmatrix}$ & $B^T = \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 4 & 0 \end{bmatrix}$

$$A^T + B^T = \begin{bmatrix} 2 & 4 \\ 3 & 5 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 \\ 5 & 8 \\ 3 & 0 \end{bmatrix} \quad — (2)$$

from (1) & (2), we get

$$(A+B)^T = A^T + B^T$$

Ex. if $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

verify that $(AB)^T = B^T \cdot A^T$.

sol^n:-

$$AB = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 11 & 5 \\ 16 & 6 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 11 & 16 \\ 5 & 6 \end{bmatrix} \quad — (1)$$

$$B^T \cdot A^T = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 16 \\ 5 & 6 \end{bmatrix} \quad — (2)$$

from (1) & (2), we get $(AB)^T = B^T \cdot A^T$.

Ex. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

verify that $(AB)' = B'.A'$.

Sol^n:-

$$AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 5 & 3 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{bmatrix} \quad \text{— (1)}$$

$$B'.A' = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 2 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+0 & 3+0+0 & 4+10+0 \\ 0+2-1 & 0+0+2 & 0+5+0 \\ 0+0-3 & 0+0+6 & 0+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{bmatrix} \quad \text{— (2)}$$

from (1) & (2) we get,

$$(AB)' = B'.A'.$$

Ex. If $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 7 \\ -5 & 6 \\ -4 & 4 \end{bmatrix}$

then show that $(AB)' = B' \cdot A'$.

sol^n:. Given $A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 7 \\ -5 & 6 \\ -4 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} -3 & 7 \\ -5 & 6 \\ -4 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} -17 & 28 \\ -19 & 23 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} -17 & -19 \\ 28 & 23 \end{bmatrix} \quad \text{--- } (1)$$

$$B' \cdot A' = \begin{bmatrix} -3 & -5 & -4 \\ 7 & 6 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -6-15+4 & -3+0-16 \\ 14+18-4 & 7+0+16 \end{bmatrix}$$

$$= \begin{bmatrix} -17 & -19 \\ 28 & 23 \end{bmatrix} \quad \text{--- } (2)$$

from (1) & (2), we get.

$(AB)' = B' \cdot A'$.

hence prove.

Examples on singular & non-singular matrix.

Ex. prove that the matrix $\begin{bmatrix} 1 & 4 \\ 6 & 9 \end{bmatrix}$ is non-singular matrix.

Sol^n:. Given $A = \begin{bmatrix} 1 & 4 \\ 6 & 9 \end{bmatrix}$

find $|A|$,

$$|A| = \begin{vmatrix} 1 & 4 \\ 6 & 9 \end{vmatrix}$$

$$= 9 - 24$$

$$= -15$$

$|A| \neq 0$.

$\therefore$ A is non-singular matrix.

Ex. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

Show that AB is non-singular matrix.

Sol^n:- Given $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 7 \\ 6 & 15 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 2 & 7 \\ 6 & 15 \end{vmatrix}$$

$$= 30 - 42$$

$$|AB| = -12 \neq 0$$

$\therefore$ AB is non-singulae matrix.

Ex. if $A = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$

Show that the matrix AB is non-singular matrix.

Sol^n:-

$$AB = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & -1 \\ 7 & 10 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} 1 & -1 \\ 7 & 10 \end{vmatrix}$$

$$= 10 + 7$$

$$|AB| = 17 \neq 0$$

$\therefore$ AB is non-singular matrix.

* Adjoint of a square matrix:-

The transpose of cofactor matrix of given matrix is called Adjoint of matrix.

* Steps to find adjoint of a matrix.

If A is a given matrix then

1) find co-factor of each element of the matrix A.
2) make a co-factor of matrix.
3) find the transpose of co-factor matrix.

Examples on Adjoint of matrix.

Ex. If $A = \begin{bmatrix} 6 & 5 \\ 2 & 1 \end{bmatrix}$ find Adj A.

Sol^n:- $A = \begin{bmatrix} 6 & 5 \\ 2 & 1 \end{bmatrix}$

cofactor of 6 = 1

cofactor of 5 = -2

cofactor of 2 = -5

co-factor of 1 = 6

co-factor of matrix $= \begin{bmatrix} 1 & -2 \\ -5 & 6 \end{bmatrix}$

Adj A = Transpose of cofactor matrix.

$\text{Adj } A = \begin{bmatrix} 1 & -5 \\ -2 & 6 \end{bmatrix}$

Ex. find Adjoint of matrix A if $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$

Sol^n:- find co-factors,

$$C_{11} = + \begin{vmatrix} 4 & 5 \\ -6 & -7 \end{vmatrix} = +(-28+30) = 2$$

$$C_{12} = - \begin{vmatrix} 3 & 5 \\ 0 & -7 \end{vmatrix} = -(-21-0) = 21$$

$$C_{13} = + \begin{vmatrix} 3 & 4 \\ 0 & -6 \end{vmatrix} = +(-18-0) = -18$$

$$C_{21} = -\begin{vmatrix} 0 & -1 \\ -6 & -7 \end{vmatrix} = -(0-6) = 6$$

$$C_{22} = +\begin{vmatrix} 1 & -1 \\ 0 & -7 \end{vmatrix} = +(-7-0) = -7$$

$$C_{23} = -\begin{vmatrix} 1 & 0 \\ 0 & -6 \end{vmatrix} = -(-6-0) = 6$$

$$C_{31} = +\begin{vmatrix} 0 & -1 \\ 4 & 5 \end{vmatrix} = +(0+4) = 4$$

$$C_{32} = -\begin{vmatrix} 1 & -1 \\ 3 & 5 \end{vmatrix} = -(5+3) = -8$$

$$C_{33} = +\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = +(4-0) = 4$$

$$\text{Co-factor of matrix} = \begin{bmatrix} 2 & 21 & -18 \\ 6 & -7 & 6 \\ 4 & -8 & 4 \end{bmatrix}$$

Adj A = Transpose of co-factor matrix.

$$\text{Adj } A = \begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$$

Ex. find Adj A if $A = \begin{bmatrix} 2 & -1 & -3 \\ 3 & -4 & -2 \\ 5 & 2 & 4 \end{bmatrix}$

Sol: find co-factors.

$$C_{11} = + \begin{vmatrix} -4 & -2 \\ 2 & 4 \end{vmatrix} = +(-16+4) = -12$$

$$C_{12} = - \begin{vmatrix} 3 & -2 \\ 5 & 4 \end{vmatrix} = -(12+10) = -22$$

$$C_{13} = + \begin{vmatrix} 3 & -4 \\ 5 & 2 \end{vmatrix} = +(6+20) = 26$$

$$C_{21} = - \begin{vmatrix} -1 & -3 \\ 2 & 4 \end{vmatrix} = -(-4+6) = -2$$

$$C_{22} = + \begin{vmatrix} 2 & -3 \\ 5 & 4 \end{vmatrix} = +(8+15) = 23$$

$$C_{23} = - \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = -(4+5) = -9$$

$$C_{31} = + \begin{vmatrix} -1 & -3 \\ -4 & -2 \end{vmatrix} = +(2-12) = -10$$

$$C_{32} = - \begin{vmatrix} 2 & -3 \\ 3 & -2 \end{vmatrix} = -(-4+9) = -5$$

$$C_{33} = + \begin{vmatrix} 2 & -1 \\ 3 & -4 \end{vmatrix} = +(-8+3) = -5$$

$$\text{co-factor matrix} = \begin{bmatrix} -12 & -22 & 26 \\ -2 & 23 & -9 \\ -10 & -5 & -5 \end{bmatrix}$$

Adj A = Transpose of co-factor matrix.

$$\text{Adj } A = \begin{bmatrix} -12 & -2 & -10 \\ -22 & 23 & -5 \\ 26 & -9 & -5 \end{bmatrix}$$

* Inverse of matrix :-

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A \quad \text{where } |A| \neq 0$$

Examples on inverse of matrix.

Ex. If $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ find A^{-1}.

Sol^n:- $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} = (8-9) = -1 \neq 0.$$

find co-factors.

$C_{11} = +2$

$C_{12} = -3$

$C_{21} = -3$

$C_{22} = 4$

$$\text{Co-factor matrix} = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$$

Adj A = Transpose of co-factor matrix

$$\text{Adj } A = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A$$

$$= \frac{1}{-1} \cdot \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

Ex. find inverse of matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

Sol^n: Given $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= 1(3-0) - 2(-1+0) - 2(2-0)$$

$$= 1(3) - 2(-1) - 2(2)$$

$$= 3 + 2 - 4$$

$$|A| = 1 \neq 0$$

$\therefore A^{-1}$ exist.

find co-factor's.

$$C_{11} = + \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} = +(3-0) = 3$$

$$C_{12} = - \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = -(-1-0) = 1$$

$$C_{13} = + \begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} = +(2-0) = 2$$

$$C_{21} = - \begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = -(2-4) = 2$$

$$C_{22} = + \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = +(1-0) = 1$$

$$C_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = -(-2-0) = 2$$

$$C_{31} = +\begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} = +(0+6) = 6$$

$$C_{32} = -\begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} = -(0-2) = 2$$

$$C_{33} = +\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = +(3+2) = 5$$

$$\text{Co-factor matrix} = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$$

Adj A = Transpose of co-factor matrix.

$$\text{Adj}\,A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj}\,A$$

$$= \frac{1}{1} \cdot \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

* Solution of simultaneous equation by matrix inversion Method.

let the simultaneous eq^{ns}.

$$a_1 x + b_1 y + c_1 z = d_1$$

$$a_2 x + b_2 y + c_2 z = d_2$$

$$a_3 x + b_3 y + c_3 z = d_3$$

write eq^n in Matrix form

$$AX = B$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

we have $AX = B$ then

$$\boxed{X = A^{-1} \cdot B}$$

Examples on matrix inversion method.

Ex. Solve eq^n by matrix inversion method.

$$x + 3y + 2z = 6$$

$$3x - 2y + 5z = 5$$

$$2x - 3y + 6z = 7$$

sol^n:- Given eq^{ns} are

$$x+3y+2z=6$$
$$3x-2y+5z=5$$
$$2x-3y+6z=7$$

eq^{ns} written in matrix form $AX=B$

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

where $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$.

first we find A^{-1}.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 3 & 2 \\ 3 & -2 & 5 \\ 2 & -3 & 6 \end{vmatrix}$$

$$= 1(-12+15) - 3(18-10) + 2(-9+4)$$
$$= 1(3) - 3(8) + 2(-5)$$
$$= 3 - 24 - 10$$
$$= -31$$

$$\boxed{|A| = -31}$$

find co-factors,

$$C_{11} = + \begin{vmatrix} -2 & 5 \\ -3 & 6 \end{vmatrix} = + (-12+15) = 3$$

$$C_{12} = - \begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} = - (18-10) = -8$$

$$C_{13} = + \begin{vmatrix} 3 & -2 \\ 2 & -3 \end{vmatrix} = + (-9+4) = -5$$

$$C_{21} = - \begin{vmatrix} 3 & 2 \\ -3 & 6 \end{vmatrix} = - (18+6) = -24$$

$$C_{22} = + \begin{vmatrix} 1 & 2 \\ 2 & 6 \end{vmatrix} = + (6-4) = 2$$

$$C_{23} = - \begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix} = - (-3-6) = 9$$

$$C_{31} = + \begin{vmatrix} 3 & 2 \\ -2 & 5 \end{vmatrix} = + (15+4) = 19$$

$$C_{32} = - \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = - (5-6) = 1$$

$$C_{33} = + \begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} = + (-2-9) = -11$$

$$\text{co-facter matrix} = \begin{bmatrix} 3 & -8 & -5 \\ -24 & 2 & 9 \\ 19 & 1 & -11 \end{bmatrix}$$

Adj A = Transpose of co-factor matrix.

$$\text{Adj } A = \begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A$$

$$= \frac{-1}{31} \cdot \begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{31} \begin{bmatrix} 3 & -24 & 19 \\ -8 & 2 & 1 \\ -5 & 9 & -11 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{31} \begin{bmatrix} 18-120+133 \\ -48+10+7 \\ -30+45-77 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{31} \begin{bmatrix} 31 \\ -31 \\ -62 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$$

By equality of matrix.

$\boxed{x = -1}$, $\boxed{y = 1}$, $\boxed{z = 2}$

Ch-4) Partial Fraction.

Fraction :- A number of the form $\frac{P}{Q}$ where $Q \neq 0$ is called a fraction.

Rational fraction :- The two polynomials in x say $P(x)$ & $Q(x)$ then the ratio $\frac{P(x)}{Q(x)}$ is called Rational fraction.

Types of fraction :-

1) Proper fraction — If the degree of fraction $Q(x)$ is greater than degree of fraction $P(x)$ then the fraction $\frac{P(x)}{Q(x)}$ is called proper fraction.

2) Improper fraction — If the degree of fraction $Q(x)$ is less than or equal to the degree of fraction $P(x)$ then the fraction $\frac{P(x)}{Q(x)}$ is called Improper fraction.

* Partial fraction :-

The procedure of expressing the rational fraction into the sum of the simple fractions is called Partial fraction. e.g. $\frac{x+18}{(x-3)(2x+1)} = \frac{3}{x-3} - \frac{5}{2x+1}$;

There are Three cases to solve the partial fraction.

Case-1) When denominator contains non-repeated linear factors.

ie $\frac{P(x)}{(x+\alpha)(x+\beta)} = \frac{A}{x+\alpha} + \frac{B}{x+\beta}$.

Case-2) When denominator contains repeated linear factors

ie $\dfrac{p(x)}{(x+\beta)(x+\alpha)^2} = \dfrac{A}{x+\beta} + \dfrac{B}{x+\alpha} + \dfrac{C}{(x+\alpha)^2}$

Case-3) When denominator contains non-repeated irreducible quadratic factors.

ie $\dfrac{p(x)}{(x+\alpha)(x^2+\beta)} = \dfrac{A}{x+\alpha} + \dfrac{Bx+C}{x^2+\beta}$ OR

$$\frac{p(x)}{(x+\alpha)(ax^2+bx+c)} = \frac{A}{x+\alpha} + \frac{Bx+C}{ax^2+bx+c}$$

* Examples on Case-1)

Ex. Resolve into the partial fractions $\dfrac{1}{(x-3)(x+2)}$

Sol^n:- let $\dfrac{1}{(x-3)(x+2)} = \dfrac{A}{x-3} + \dfrac{B}{x+2}$ —— (1)

$$\frac{1}{\cancel{(x-3)(x+2)}} = \frac{A(x+2)+B(x-3)}{\cancel{(x-3)(x+2)}}$$

$1 = A(x+2) + B(x-3)$ —— (2)

Put $x = 3$ in (2)

$1 = A(3+2) + 0$

$1 = 5A$

$\boxed{A = \dfrac{1}{5}}$

put $x = -2$ in (2)

$1 = 0 + B(-2-3)$

$1 = -5B$

$\boxed{B = \dfrac{-1}{5}}$

put value of $A = \frac{1}{5}$, $B = -\frac{1}{5}$ in eq^n (1) we get

$$\frac{1}{(x-3)(x+2)} = \frac{1/5}{x-3} - \frac{1/5}{x+2}$$

$$= \frac{1}{5}\left[\frac{1}{x-3} - \frac{1}{x+2}\right]$$

Ex. Resolve into partial fractions. $\frac{1}{x^2 - x}$

Sol^n:. let $\frac{1}{x^2 - x} = \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$ — (1)

$$\frac{1}{\cancel{x(x-1)}} = \frac{A(x-1) + Bx}{\cancel{x(x-1)}}$$

$1 = A(x-1) + Bx$ ——— (2)

put $x = 0$ in eq^n (2)

$1 = A(0-1) + 0$

$1 = -A$

$\boxed{A = -1}$

put $x = 1$ in eq^n (2)

$1 = 0 + B(1)$

$1 = B$

$\boxed{B = 1}$

put value of A, B in (1)

$$\frac{1}{x^2 - x} = \frac{1}{x(x-1)} = \frac{-1}{x} + \frac{1}{x-1}$$

Ex. Resolve into partial fraction $\frac{1}{x^2-1}$

sol^n:. let $\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$ --- (1)

$$\frac{1}{\cancel{(x+1)(x-1)}} = \frac{A(x-1)+B(x+1)}{\cancel{(x+1)(x-1)}}$$

$$1 = A(x-1) + B(x+1) \quad \text{--- (2)}$$

put $x = -1$ in eq^n (2)

$$1 = A(-1-1) + 0$$

$$1 = -2A$$

$$\boxed{A = \frac{-1}{2}}$$

put $x = 1$ in eq^n (2)

$$1 = 0 + B(1+1)$$

$$1 = 2B$$

$$\boxed{B = \frac{1}{2}}$$

Put $A = \frac{-1}{2}$, $B = \frac{1}{2}$ in eq^n (1)

$$\frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)} = \frac{-1/2}{x+1} + \frac{1/2}{x-1}$$

$$= \frac{1}{2}\left[\frac{-1}{x+1} + \frac{1}{x-1}\right]$$

Ex. Resolve into partial fraction $\frac{2x+3}{x^2-2x-3}$

sol^n:. $\frac{2x+3}{x^2-2x-3} = \frac{2x+3}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$ --- (1)

$$\frac{2x+3}{\cancel{(x-3)(x+1)}} = \frac{A(x+1)+B(x-3)}{\cancel{(x-3)(x+1)}}$$

$$2x+3 = A(x+1) + B(x-3) \quad --- (2)$$

put $x = 3$ in eqⁿ (2)

$$6+3 = A(3+1) + 0$$

$$9 = 4A$$

$$\boxed{A = \frac{9}{4}}$$

put $x = -1$ in eqⁿ (2)

$$-2+3 = 0 + B(-1-3)$$

$$1 = -4B$$

$$\boxed{B = \frac{-1}{4}}$$

put value of $A = \frac{9}{4}$, $B = \frac{-1}{4}$ in eqⁿ (1)

$$\frac{2x+3}{(x-3)(x+1)} = \frac{9/4}{x-3} - \frac{1/4}{x+1}$$

$$= \frac{1}{4}\left[\frac{9}{x-3} - \frac{1}{x+1}\right]$$

Ex. Resolve into partial fraction $\frac{x+18}{(x-3)(2x+1)}$

Solⁿ: $\frac{x+18}{(x-3)(2x+1)} = \frac{A}{x-3} + \frac{B}{2x+1} \quad --- (1)$

$$\frac{x+18}{\cancel{(x-3)(2x+1)}} = \frac{A(2x+1)+B(x-3)}{\cancel{(x-3)(2x+1)}}$$

$$x+18 = A(2x+1) + B(x-3) \quad --- (2)$$

put $x = 3$ in eqⁿ (2)

$3 + 18 = A(6+1) + 0$

$21 = 7A$

$A = \frac{21}{7}$

$\boxed{A = 3}$

put $x = -\frac{1}{2}$ in eqⁿ (2)

$-\frac{1}{2} + 18 = 0 + B\left(-\frac{1}{2} - 3\right)$

$\frac{35}{2} = \frac{-7}{2} B$

$B = \frac{35}{2} \times \frac{-2}{7}$

$\boxed{B = -5}$

put value of $A = 3$ & $B = -5$ in eqⁿ (1)

$$\frac{x+18}{(x-3)(2x+1)} = \frac{3}{x-3} - \frac{5}{2x+1}$$

Ex. Resolve into the partial fraction

$$\frac{x^2+4x+1}{(x-1)(x+1)(x+3)}$$

solⁿ: $$\frac{x^2+4x+1}{(x-1)(x+1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+3} \quad - (1)$$

$$\frac{x^2+4x+1}{(x-1)\cancel{(x+1)(x+3)}} = \frac{A(x+1)(x+3) + B(x-1)(x+3) + C(x-1)(x+1)}{\cancel{(x-1)(x+1)}(x+3)}$$

$$x^2+4x+1 = A(x+1)(x+3)+B(x-1)(x+3)+C(x-1)(x+1) \quad \cdots (2)$$

put $x=1$ in eq^n (2)

$$1^2+4+1 = A(2)(4)+0+0$$

$$6 = 8A$$

$$A = \frac{6}{8}$$

$$\boxed{A = \frac{3}{4}}$$

put $x=-1$ in eq^n (2)

$$(-1)^2+4(-1)+1 = 0+B(-2)(2)+0$$

$$-2 = -4B$$

$$B = \frac{\cancel{4}}{\cancel{2}}\ \frac{2}{4}$$

$$\boxed{B = \frac{1}{2}} \quad \therefore \boxed{B = \frac{1}{2}}$$

put $x=-3$ in eq^n (2)

$$(-3)^2+4(-3)+1 = 0+0+C(-4)(-2)$$

$$9-12+1 = 8C$$

$$-2 = 8C$$

$$C = \frac{-2}{8}$$

$$\boxed{C = -\frac{1}{4}}$$

put value of A, B, C in eq^n (1)

$$\frac{x^2+4x+1}{(x-1)(x+1)(x+3)} = \frac{3/4}{x-1} + \frac{1/2}{x+1} - \frac{1/4}{x+3}.$$

Ex. Resolve into the partial fraction $\frac{x-5}{x^3+x^2-6x}$

sol^n:- $\frac{x-5}{x^3+x^2-6x} = \frac{x-5}{x(x^2+x-6)}$

$$\frac{x-5}{x(x+3)(x-2)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2} \quad --- (1)$$

$$x-5 = A(x+3)(x-2) + B(x)(x-2) + C(x)(x+3) \quad — (2)$$

put $x = 0$ in eq^n (2)

$-5 = A(3)(-2) + 0 + 0$

$-5 = -6A$

$\boxed{A = \frac{5}{6}}$

put $x = -3$ in eq^n (2)

$-3-5 = 0 + B(-3)(-3-2)$

$-8 = 15B$

$\boxed{B = \frac{-8}{15}}$

put $x = 2$ in eq^n (2)

$2-5 = 0+0+C(2)(2+3)$

$-3 = 10C$

$\boxed{C = \frac{-3}{10}}$

put value of A, B, C in eq^n (1)

$$\frac{x-5}{x(x+3)(x-2)} = \frac{5/6}{x} - \frac{8/15}{x+3} - \frac{3/10}{x-2}$$

problem Reducible to case-1 after suitable substitution.

Ex. Resolve into partial fraction $\dfrac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)}$

sol^n:- put $\tan\theta = t$ then

$\dfrac{t+1}{(t+2)(t+3)}$ is a proper fraction.

$$\frac{t+1}{(t+2)(t+3)} = \frac{A}{t+2} + \frac{B}{t+3} \quad ---(1)$$

~~put~~ $t+1 = A(t+3) + B(t+2) \quad --(2)$

put $t=-2$ in eq^n (2)

$-2+1 = A(-2+3) + 0$

$-1 = A$

$\boxed{A=-1}$

Put $t=-3$ in eq^n (2)

$-3+1 = 0 + B(-3+2)$

$-2 = -B$

$\boxed{B=+2}$

put value of $A=-1$ & $B=-2$ in eq^n (1)

$$\frac{t+1}{(t+2)(t+3)} = \frac{-1}{t+2} + \frac{2}{t+3}$$

But $t=\tan\theta$

$$\frac{\tan\theta+1}{(\tan\theta+2)(\tan\theta+3)} = \frac{-1}{(\tan\theta+2)} + \frac{2}{(\tan\theta+3)}$$

Ex. Resolve into partial fraction $\frac{\sin\theta+1}{(\sin\theta+2)(\sin\theta+3)}$

sol^n:- Given fraction $\frac{\sin\theta+1}{(\sin\theta+2)(\sin\theta+3)}$

put $\sin\theta = t$ then

$$\frac{t+1}{(t+2)(t+3)} = \frac{A}{t+2} + \frac{B}{t+3} \quad ---(1)$$

$$t+1 = A(t+3) + B(t+2) \quad ---(2)$$

put $t=-2$ in eq^n (2)

$$-2+1 = A(-2+3) + 0$$

$$-1 = A$$

$$\boxed{A=-1}$$

Put $t=-3$ in eq^n (2)

$$-3+1 = 0 + B(-3+2)$$

$$-2 = -B$$

$$\boxed{B=2}$$

put value of $A=-1$ & $B=2$ in eq^n (1)

$$\frac{t+1}{(t+2)(t+3)} = \frac{-1}{t+2} + \frac{2}{t+3}$$

Put $t = \sin\theta$ then

$$\frac{\sin\theta+1}{(\sin\theta+2)(\sin\theta+3)} = \frac{-1}{\sin\theta+2} + \frac{2}{\sin\theta+3}.$$

Ex. Resolve into the partial fraction $\frac{e^x+1}{(e^x+2)(e^x+3)}$

sol^n:- Given fraction $\frac{e^x+1}{(e^x+2)(e^x+3)}$

put $e^x = t$

$$\frac{t+1}{(t+2)(t+3)} = \frac{A}{t+2} + \frac{B}{t+3} \quad --- (1)$$

$$t+1 = A(t+3) + B(t+2) \quad --- (2)$$

put $t = -2$ in eq^n (2)

$-2+1 = A(-2+3) + 0$

$-1 = A$

$\boxed{A=-1}$

put $t = -3$ in eq^n (2)

$-3+1 = 0 + B(-3+2)$

$-2 = -B$

$\boxed{B=2}$

Put value of $A=-1$ & $B=2$ in eq^n (1)

$$\frac{t+1}{(t+2)(t+3)} = \frac{-1}{t+2} + \frac{2}{t+3}$$

Put $t = e^x$ then

$$\frac{e^x+1}{(e^x+2)(e^x+3)} = \frac{-1}{e^x+2} + \frac{2}{e^x+3}$$

Examples on case-2)

Ex. Resolve into partial fraction. $\frac{9}{(x-1)(x+2)^2}$

sol^n:. $\frac{9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ --- (1)

$9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$ --- (2)

put $x = 1$ in eq^n (2)

$9 = A(1+2)^2 + 0 + 0$

$9 = 9A$

$A = \frac{9}{9}$

$\boxed{A = 1}$

put $x = -2$ in eq^n (2)

$9 = 0 + 0 + C(-2-1)$

$9 = -3C$

$C = \frac{9}{-3}$

$\boxed{C = -3}$

put $x = 0$ in eq^n (2)

$9 = 4A - 2B - C$

put A & C in above eq^n

$9 = 4(1) - 2B + 3$

$2B = 7 - 9$

$2B = -2$

$B = \frac{-2}{2}$ $\boxed{B = -1}$

put value of A, B, C in eq^n (1) we get.

$$\frac{9}{(x-1)(x+2)^2} = \frac{1}{x-1} + \frac{-1}{x+2} - \frac{3}{(x+2)^2}$$

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Ex. Resolve into the partial fraction $\frac{x^2-2x+7}{(x+1)(x-1)^2}$

Sol^n:- $$\frac{x^2-2x+7}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \quad --(1)$$

$$x^2-2x+7 = A(x-1)^2 + B(x+1)(x-1) + C(x+1) \quad --(2)$$

put $x=-1$ in eq^n (2)

$$(-1)^2+2+7 = 4A+0+0$$

$$10 = 4A$$

$$\boxed{A = \frac{10}{4}}$$

put $x = 1$ in eq^n (2)

$$1^2-2(1)+7 = 0+0+2C$$

$$6 = 2C$$

$$C = \frac{6}{2}$$

$$\boxed{C=3}$$

put $x=0$ in eq^n (2)

$$7 = A-B+C$$

$$7 = \frac{10}{4} - B + 3$$

$$B = \frac{10}{4} - \frac{4}{1}$$

$$B = \frac{-6}{4} = \frac{-3}{2} \qquad \boxed{B = \frac{-3}{2}}$$

put value of A, B, C in eqⁿ (1)

$$\frac{x^2-2x+7}{(x+1)(x-1)^2} = \frac{5/2}{x+1} - \frac{3/2}{x-1} + \frac{3}{(x-1)^2}$$

Ex. Resolve into partial fraction $\frac{2x+1}{x^2(x+1)}$

Solⁿ :- $\frac{2x+1}{x^2(x+1)} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2}$ --- (1)

$2x+1 = A(x^2) + B(x)(x+1) + C(x+1)$ --- (2)

put $x = -1$ in eqⁿ (2)

$-2+1 = A$

$-1 = A$

$\boxed{A = -1}$

put $x = 0$ in eqⁿ (2)

$1 = 0+0+C$

$\boxed{C = 1}$

put $x = +1$ in eqⁿ (2)

$3 = A + 2B + 2C$

$3 = -1 + 2B + 2$

$2B = 2$

$\boxed{B = 1}$

Put value of A, B, C in eqⁿ (1) we get.

$$\frac{2x+1}{x^2(x+1)} = \frac{-1}{x+1} + \frac{1}{x} + \frac{1}{x^2}$$

Ex. Resolve into partial Fraction $\frac{3x+2}{(x+1)(x^2-1)}$

Sol^n:- $\frac{3x+2}{(x+1)(x^2-1)} = \frac{3x+2}{(x+1)(x+1)(x-1)} = \frac{3x+2}{(x+1)^2(x-1)}$

$\frac{3x+2}{(x-1)(x+1)^2} = \cancel{B}\cdot\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ --- (1)

$3x+2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$ --- (2)

put $x=1$ in eq^n (2), we get

$5 = 4A + 0 + 0$

$\boxed{A = \frac{5}{4}}$

put $x=-1$ in eq^n (2), we get

$-1 = 0 + 0 - 2C$

$-1 = -2C$

$\boxed{C = \frac{1}{2}}$

put $x = 0$ in eq^n (2), we get

$2 = A - B - C$

$2 = \frac{5}{4} - B - \frac{1}{2}$

$\boxed{B = \frac{-5}{4}}$

put value of A, B, C in eq^n (1). we get

$\frac{3x+2}{(x+1)(x+1)^2} = \frac{5/4}{x+1} - \frac{5/4}{x+1} + \frac{1/2}{(x+1)^2}$

Examples on case-3.

Ex. Resolve into partial fraction $\dfrac{x^2+23x}{(x+3)(x^2+1)}$

sol^n: $\dfrac{x^2+23x}{(x+3)(x^2+1)} = \dfrac{A}{x+3} + \dfrac{Bx+C}{x^2+1}$ --- ①

$x^2+23x = A(x^2+1) + (Bx+C)(x+3)$ --- ②

put $x=-3$ in eq^n ②

$(-3)^2 + 23(-3) = A[(-3)^2+1] + [B(-3)+C](-3+3)$

$-60 = 10A + 0$

$A = \dfrac{-60}{10}$

$\boxed{A=-6}$

put $x=0$ in eq^n ② we get.

$0 = A+3C$

$0 = -6+3C$

$3C = 6$

$C = \dfrac{6}{3}$

$\boxed{C=2}$

put $x=1$ in eq^n ② we get

$1^2+23(1) = 2A+4B+4C$

$24 = 2(-6)+4B+2(4)$

$24 = -12+4B+8$

$4B = 28$

$B = \dfrac{28}{4}$

$\boxed{B=7}$

put value of A, B, C in eq^n ① we get

$$\frac{x^2+23x}{(x+3)(x^2+1)} = \frac{-6}{x+3} + \frac{7x+2}{x^2+1}$$

Ex. Resolve into partial Fraction $\frac{2x+1}{(x-1)(x^2+1)}$

Sol^n:- $$\frac{2x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} \quad ---①$$

$$2x+1 = A(x^2+1) + (Bx+C)(x-1) \quad ---②$$

put $x=1$ in eq^n ② we get

$3 = 2A + 0$

$\boxed{A = \frac{3}{2}}$

put $x=0$ in eq^n ② we get

$1 = A - C$

$1 = \frac{3}{2} - C$

$C = \frac{3}{2} - 1$

$\boxed{C = \frac{1}{2}}$

put $x=-1$ in eq^n ② we get

$-1 = 2A + 2B - 2C$

$-1 = 2\left(\frac{3}{2}\right) + 2B - 2\left(\frac{1}{2}\right)$

$2B = -3$

$\boxed{B = -\frac{3}{2}}$

put value of A, B, C in eq^n ① we get

$$\frac{2x+1}{(x-1)(x^2+1)} = \frac{3/2}{x-1} + \frac{-\frac{3}{2}x + \frac{1}{2}}{x^2+1}$$

Ex. Resolve into partial Fraction $\frac{x^4}{x^3+1}$

sol^n:- Given fraction $\frac{x^4}{x^3+1}$ is a improper fraction.

we first convert improper fraction into proper fraction by division method.

Divide Numerator by denominator.

$$\begin{array}{r|l} & x \leftarrow \text{Quotient} \\ \hline x^3+1 & x^4 \\ & \underline{-x^4 \mp x} \\ & -x \leftarrow \text{Remainder} \end{array}$$

$\swarrow$ Divisor

Now Ratio $= Q + \frac{R}{D}$

$$\frac{x^4}{x^3+1} = x - \underbrace{\frac{x}{x^3+1}}_{\text{Proper Fraction.}} \quad \text{---} (*)$$

Now $\frac{x}{x^3+1} = \frac{x}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$ .. (1)

$x = A(x^2-x+1) + (Bx+C)(x+1)$ --- (2)

put $x=-1$ in eq^n (2) we get

$-1 = 3A + 0$

$\boxed{A = -1/3}$

put $x=0$ in eq^n (2), we get

$0 = A + C$

$0 = -\frac{1}{3} + C$

$\boxed{C = \frac{1}{3}}$

put $x = 1$ in eqⁿ ② we get,

$$1 = A + 2B + 2C$$

$$1 = \frac{-1}{3} + 2B + \frac{2}{3}$$

$$2B = 1 - \frac{1}{3}$$

$$2B = \frac{2}{3}$$

$$\boxed{B = \frac{1}{3}}$$

put value of A, B, C in eqⁿ ①

$$\frac{x}{x^3+1} = \frac{-1/3}{x+1} + \frac{\frac{1}{3}x + \frac{1}{3}}{x^2-x+1}$$

eqⁿ (*) becomes,

$$\frac{x^4}{x^3+1} = x - \left[\frac{-1/3}{x+1} + \frac{1/3\,x + 1/3}{x^2-x+1}\right]$$

* practice problems

1) Resolve into partial fraction $\frac{x+5}{(2x-1)(x+4)}$ $A = \frac{11}{9}, B = \frac{-1}{9}$

2) Resolve into partial fraction $\frac{5x-1}{x^2-5x-6}$ $A = \frac{29}{7}, B = \frac{6}{7}$

3) Resolve into partial fraction $\frac{x+6}{x(x+1)(x+4)}$ $A = \frac{3}{2}, B = \frac{-5}{3}, C = \frac{1}{6}$

4) Resolve into Partial Fraction $\frac{3x-1}{x^3-3x^2+2x}$ $A = \frac{-1}{2}, B = \frac{5}{2}, C = -2$

5) Resolve into partial fraction $\frac{2x+7}{(x+4)(x+7)}$ $A = \frac{-1}{3}, B = \frac{7}{3}$

Ex.6) Resolve partial fraction $\frac{x^2+5x+7}{(x-1)(x+2)(x+4)}$ $A=\frac{13}{15}$, $B=\frac{-1}{6}$, $C=\frac{3}{10}$.

Ex.7) Resolve partial fraction $\frac{\log x}{(\log x-2)(\log x-3)}$ $A=2$, $B=3$

Ex.8) Resolve partial fraction $\frac{x}{(x+1)(x-2)^2}$ $A=\frac{2}{3}$, $B=\frac{1}{9}$, $C=\frac{-1}{9}$

Ex.9) Resolve into partial fraction. $\frac{x^2-8x+9}{(x+1)(x-2)^2}$ $A=2$, $B=-$ $C=-1$

10) Resolve partial fraction $\frac{1-x-x^2}{(1-2x)(1-x)^2}$ $A=1$, $B=-1$, $C=1$

11) Resolve into partial Fraction $\frac{4x^2-5x-1}{(x-2)(x-1)^2}$ $A=3$, $B=-1$ $C=2$

12) Resolve into partial Fraction $\frac{2x-3}{(x+1)(x^2+4)}$ $A=-1$, $B=1$ $C=1$

13) Resolve partial Fraction $\frac{x}{x^3-1}$ $A=\frac{1}{3}$, $B=-\frac{1}{3}$, $C=\frac{1}{3}$

14) Resolve partial Fraction $\frac{3x-2}{(x+2)(x^2+4)}$ $A=-1$, $B=1$, $C=1$

15) Resolve partial Fraction $\frac{x}{x^3+1}$ $A=-\frac{1}{3}$, $B=\frac{1}{3}$, $C=\frac{1}{3}$

16) Resolve partial Fraction $\frac{2x^2-11x+5}{(x-3)(x^2+2x-5)}$ $A=-1$, $B=$ $C=0$

17) Resolve into partial Fraction $\frac{x^2+1}{x^2-1}$ $A=-1$, $B=1$

18) Resolve partial Fraction $\frac{x^3+x}{x^2-9}$ $A=5$, $B=5$.

20/5/2022

Sub:- Basic Mathematics (311302)

Question Bank

unit-1- Algebra.

2-Marks Questions.

Q.1) Find the value of x if $\log(x^2-5x+11)=1$

(winter-2023)

Q.2) Find the value of $\log\left(\frac{2}{3}\right)+\log\left(\frac{4}{5}\right)-\log\left(\frac{8}{15}\right)$

(Summer-2024)

Q.3) Find 'x' if $\log_2(x+2)=3$ (winter-2024)

Q.4) Find 'x' if $\log_3(x+5)=4$ (summer-2025)

Q.5) Find the value of $\log\left(\frac{5}{6}\right)+\log\left(\frac{12}{35}\right)+\log\left(\frac{7}{2}\right)$

(winter-2025)

Q.6) Find the value of x, if $\log_3 x-\log_3(x-2)=3$

(Summer-2026)

4-Marks Questions.

Q.7) If $A=\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ then find the matrix 'X' such that $2X+3A-4B=I$ where I is identity matrix of order 2 (W-23)

Q.8) If $A=\begin{bmatrix} -2 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 0 & 2 \end{bmatrix}$ whether AB is singular or non-singular matrix?

(Winter-2023)

Q.9) Resolve into partial fraction $\dfrac{3x-2}{(x+2)(x^2+4)}$

(winter-2023)

Q.10) If $P = \begin{bmatrix} 1 & 2 & -3 \\ 3 & -1 & 2 \\ -2 & 1 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, then

Find Matrix R such that $P+Q+R=0$

(summer-2024)

Q.11) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix}$ Show that

AB is singular or non-~~sig~~ singular matrix.

(summer-2024)

Q.12) Resolve into the partial fraction

$\dfrac{x^2-2x+3}{(x+2)(x^2+1)}$ (Summer-2024)

Q.13) If $A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

verify that $(AB)^T = B^T \cdot A^T$ (winter-2024)

Q.14) Resolve into partial fraction $\dfrac{x}{x^2-x-2}$ (W-24)

Q.15) Simplify $\dfrac{1}{\log_5 10} + \dfrac{1}{\log_{20} 10}$ (winter-2024)

6-Marks Questions.

Q.16) solve the equations by Matrix inversion Method
$x+y+z=3$, $3x-2y+3z=4$, $5x+5y+z=11$ (W-23)

Q.17) Using Matrix inversion method, solve the following system of equations:
$x+y+z=6$, $3x-y+3z=10$, $5x+5y-4z=3$ (S-24)

Q.18) solve the equations by Matrix inversion method
$3x+y+2z=3$, $2x-3y-z=-3$, $x+2y+z=4$ (W-24)

Q.19) Solve the following equations by matrix inversion method. (Summer-2025)
$x+3y+2z=12$, $x+4y+4z=15$, $x+3y+4z=13$

Q.20) Using matrix inversion method, solve the following equations. (winter-2025)
$x+y+z=3$, $x+2y+3z=4$, $x+4y+9z=6$

20/5/2026