The Shirpur Education Society's

R. C. Patel College of Engineering and Polytechnic, Shirpur

Department of Mechanical Engineering

NAME OF COURSE: - Strength of Materials

CODE OF COURSE: - 313308

SEMESTER: - SYME-3K

SUBJECT TEACHER: - Mr. Laxmikant Y.Borse

The Shirpur Education Society's

R. C. Patel College of Engineering and Polytechnic, Shirpur

QUESTION BANK

CHAPTER 4. Bending and Shear Stresses in Beams

Program Name: Mechanical Engineering
Program Code: ME3k
Name of Subject & Code : Strength of Materials (313308)
Semester : Third
Date & Time Slot:

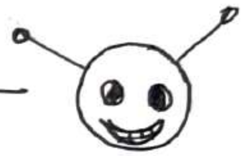
Q. NO.	QUESTION	DETAIL	MAPPING
1	Define section modulus and neutral axis.	S25 Q1d 2M	CO 4.1R
2	State any four assumptions in theory of pure bending.**	W24&25S26 Q1f &d 2M/4M	CO 4.2 U
3	Draw shear stress and bending stress distribution diagram for hollow rectangular beam section	W25 Q1f 2M	CO 4.5 A
4	Draw a bending stress distribution diagram for T section used as cantilever beam.	S26 Q1e 2M	CO 4.4 A
5	Sketch the shear distribution diagram for a rectangular beam of 600 × 200 mm (deep) subjected to a shear force of 20 kN.	W25 Q4b 4M	CO 4.5 A
6	A cantilever beam of rectangular metal cross section is 4 m in length carries an UDL of 5 kN/m. If permissible bending stress in the material is 5 N/mm^2, determine the size of the section. Assume depth to width ratio = 2	S25 Q2D 4M	CO 4.3 A
7	A simply supported beam of rectangular section 150 mm wide 300 mm deep is simply supported over a span of 4.0 m. It carries UDL 10 kN/m over entire span. Find the maximum and minimum bending stress induced in the section. Draw bending stress distribution diagram.	w24 Q4b 4M	CO 4.3 A
8	Draw shear stress distribution along cross section of circular beam for 300 mm diameter carrying 400 kN shear force. Also determine the ratio of maximum shear stress to average stress	w24 Q4c 4M	CO 4.4 A
9	A cantilever beam of rectangular section Support UDL of 5 kN/m. the span of beam is 3m .If the maximum bending stress is 100 N/mm^2, and depth of beam is 1.5 times the width ,Determine the size of beam	S26 Q4b 4M	CO 4.3A
10	A T-section flange 160 mm × 20 mm and web 180 mm × 20 mm is simply supported at both the ends. It carries two concentrated loads of 100 kN each acting 2 m from each support. Span of beam is 8 m. Determine maximum bending stress and draw bending stress distribution diagram. Also find bending stress layer 100 mm from bottom.	S25 Q5B 6M	CO 4.5 A
11	A timber beam 100 mm wide and 150 mm deep supports a udl over a span of 2 m. If the safe stress are 28 N/mm2 in bending and 2 N/mm2 in shear, calculate the maximum load which can be supported by the beam.	S25Q6B 6M	CO 4.4A
12	A rectangular beam section 300 mm wide and 500 mm deep is simply supported over a span of 4 m. It carries a full span uniformly distributed load of 10 kN/m. Find the maximum bending stress induced in the section. Draw the bending stress distribution diagram.	W25 Q5B 6M	CO 4.3 A

13	A cantilever beam of solid circular cross-section and 3 m long carries a concentrated load of 35 kN at its free end, if the maximum bending stress in tension or compression is not to exceed 125 N/mm2, determine the diameter of the beam	W25 Q6b 6M	CO 4.4 A
14	A beam section 100 mm × 200 mm is subjected to a shear force of 60 kN. Determine the shear stresses induced on a layer at 40 mm above N.A. and 20 mm below the N.A	W24 Q6b 6M	CO 4.5 A
15	A hollow rectangular section ,200 mm* 400 mm externally with uniform thickness of 40 mm carries a shearing force of 100 kN at section. Construct the shear stress distribution diagram giving all important values. Also calculate the ratio of max. to average shear stress.	S26 Q6b 6M	CO4.5 & 4.6A

4.1 BENDING STRESS:—

Bending moment at section tends to bend or deflect the beam and the internal stresses resist it's bending, when every c/s sets up full resistance to the bending moment, then the resistance offered by the internal stresses to the bending is called Bending stress.

4.2. ASSUMPTIONS IN PURE/SIMPLE BENDING:—



1. The material of beam is perfectly homogeneous and isotropic through out the section.

2. The material obeys Hooke's law.

3. The transverse sections, which were plane before bending, remains plane after bending also.

4. Each layer of beam is free to expand or contract.

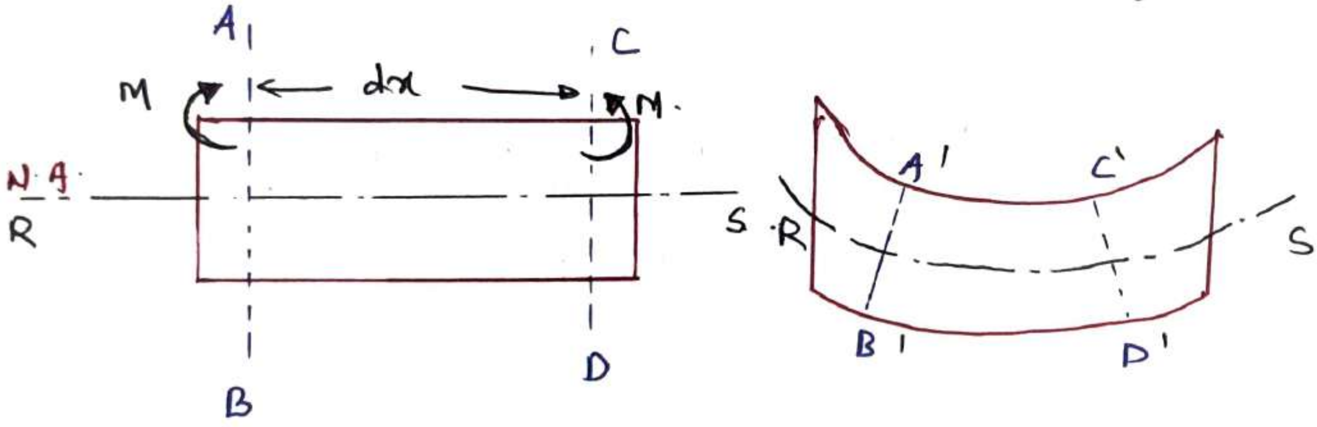
5. The value of Young's Modulus is same in the tension and compression.

6. The beam is in equilibrium i.e. there is no Resultant pull or push.

4.1

THEORY of SIMPLE BENDING / PURE BENDING :-

let us consider a simply supported beam subjected to bending moment as shown in fig. (a), there are two section AB and CD are perpendicular to the axis of beam RS, while bending moment whole beam will bend as shown in fig. (b)



consider small length of beam dx, then the curvature of beam is circular, As we can seen from the observation upper layer of beam possess compression and lower Layer possess tension while Bending Bending moment (M) applied. As the result of section of beam will be changed as A'B' and C'D'

4.1 PURE BENDING:—

When Beam is subjected to external loads, then the shear force offered by material is zero i.e. Bending moment is constant at every c/s of beam is called pure bending.

4.1 NEUTRAL AXIS:—

It is ~~Axis~~ an axis at where stress will be zero throughout the section. it means their is no compression nor tension. is called neutral axis.

4.) SECTION MODULUS:—

It is defined as the ratio of M.I about Neutral axis to the distance betn extreme layer of the beam from N.A.

- It is denoted by 'Z'

$$\therefore \quad Z = \frac{I}{Y}$$

Where,
$I = M.I = I_{xx}$
$Y = Y_{max}$

$$= \frac{mm^4}{mm}$$

$$Z = mm^3 \text{ (Unit)}$$

4.2

FLEXURAL FORMULA :— [Without derivation]

— It is also known as flexural equation or Bending equation.

As we know that,

Imp

$$\boxed{\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}}$$

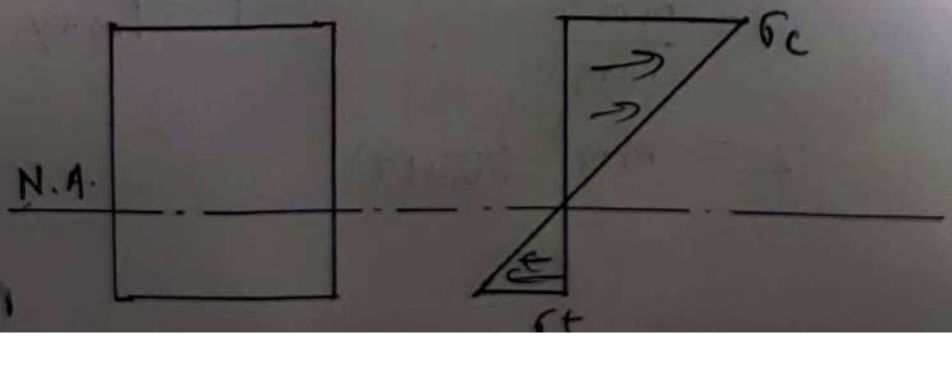
Here,
M = Max. B.M
I = I_{xx} = M.I
σ = Bending stress.
Y = distance from N.A.
R = Radius of curvature.

4.2.

BENDING STRESS DISTRIBUTION AND IT'S NATURE :—

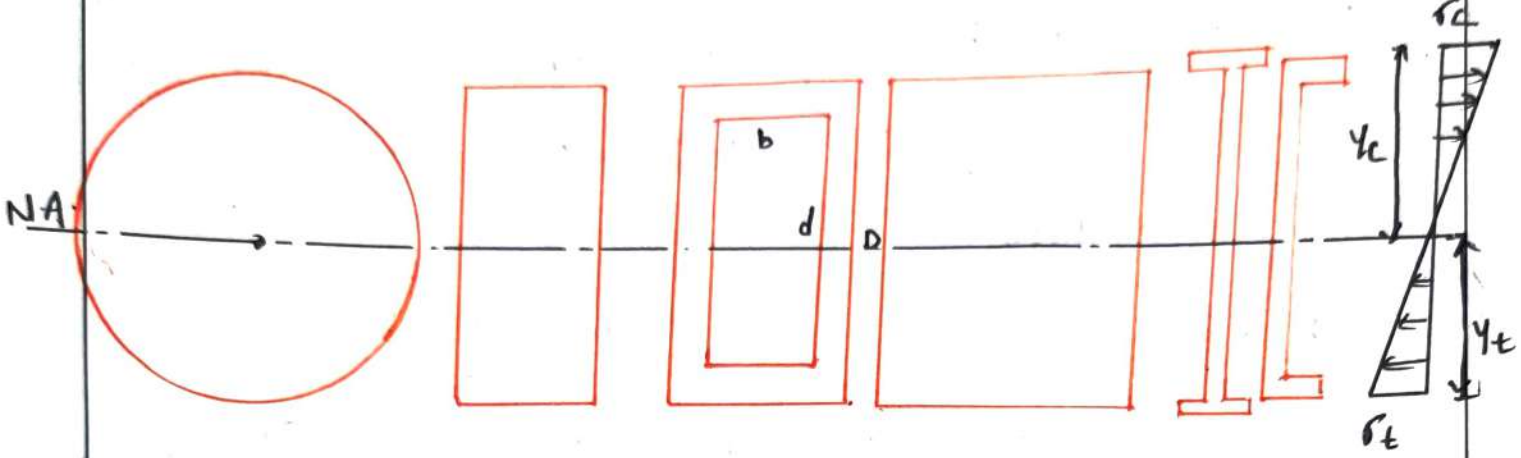
Consider a simply supported beam, there is no stress at it's N.A. But the compressive stress is developed into upper layer of beam and tensile stress developed below the N.A. i.e. lower layer of beam.

As we know, that the stress at point is directly proportional to it's distance from the N.A.



4.3

★ BENDING STRESS VARIATION DIAGRAM FOR SSB FOR SYMMETRICAL SECTIONS :-

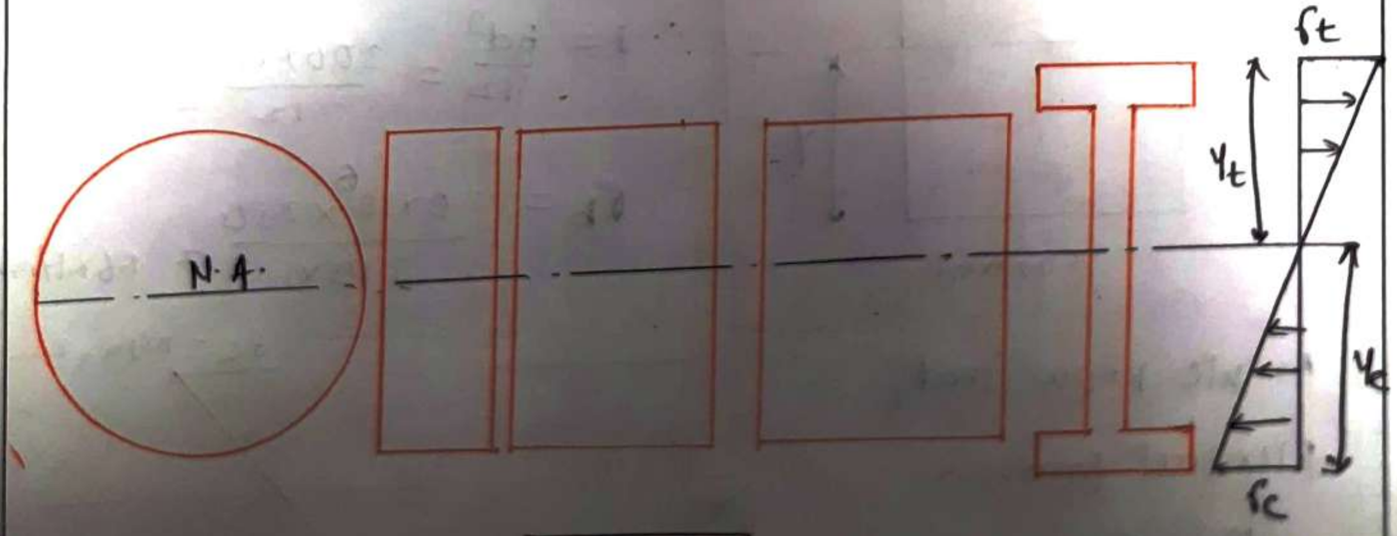


For symmetrical (SSB) bending stress is equally distributed through out the section.

$$\therefore \boxed{\sigma_c = \sigma_t}$$

4.3

★ BENDING STRESS VARIATION DIAGRAM FOR CANTILEVER FOR SYMMETRICAL SECTIONS :-

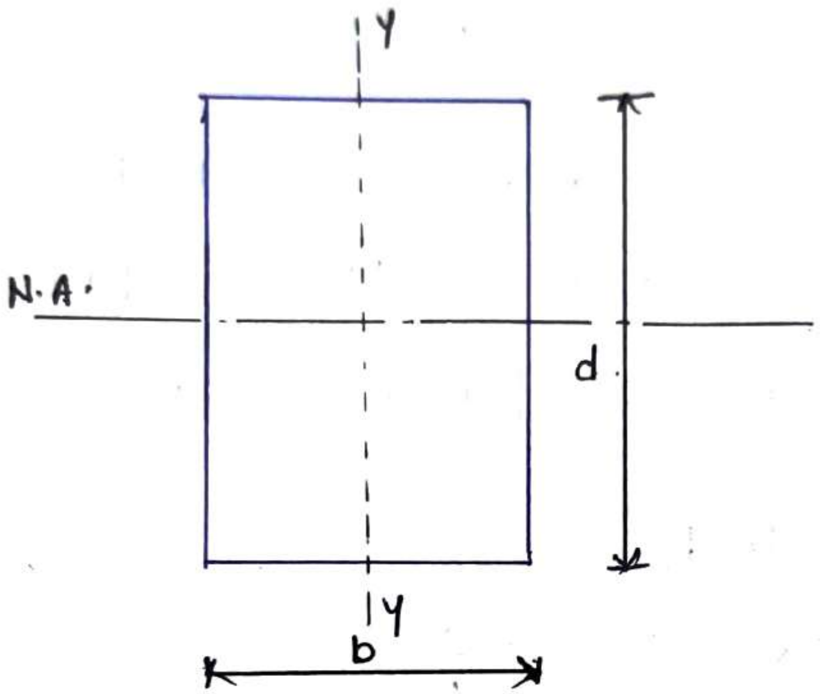


$$\boxed{\sigma_c = \sigma_t}$$

4.4

★ SHEAR STRESS EQUATION :—

$$\boxed{q = \frac{FA\bar{Y}}{Ib}}$$



Rectangular section of Beam.

Here,

q = shear stress (N/mm^2)

b = Width of beam section (mm)

F = Applied shear force

$A\bar{Y}$ = First moment of area.

4.4 RELATION BETWEEN AVG. SHEAR STRESS (q_{avg}) and MAXIMUM SHEAR STRESS :—

Avg. shear stress = $\dfrac{\text{shear force}}{\text{c/s area}}$.

Max. shear stress = Maximum shear stress induced at any section of beam due to applied shear force is called max. shear stress.

As we know that,

$$\text{shear stress} = \frac{F}{A}.$$

i) Rectangular section :—

$$\boxed{q_{max} = \frac{3}{2} q_{avg}.}$$

ii) square section :—

$$\boxed{q_{max} = \frac{3}{2} q_{avg}.}$$

iii) circular section :—

$$\boxed{q_{max} = \frac{4}{3} q_{avg}.}$$

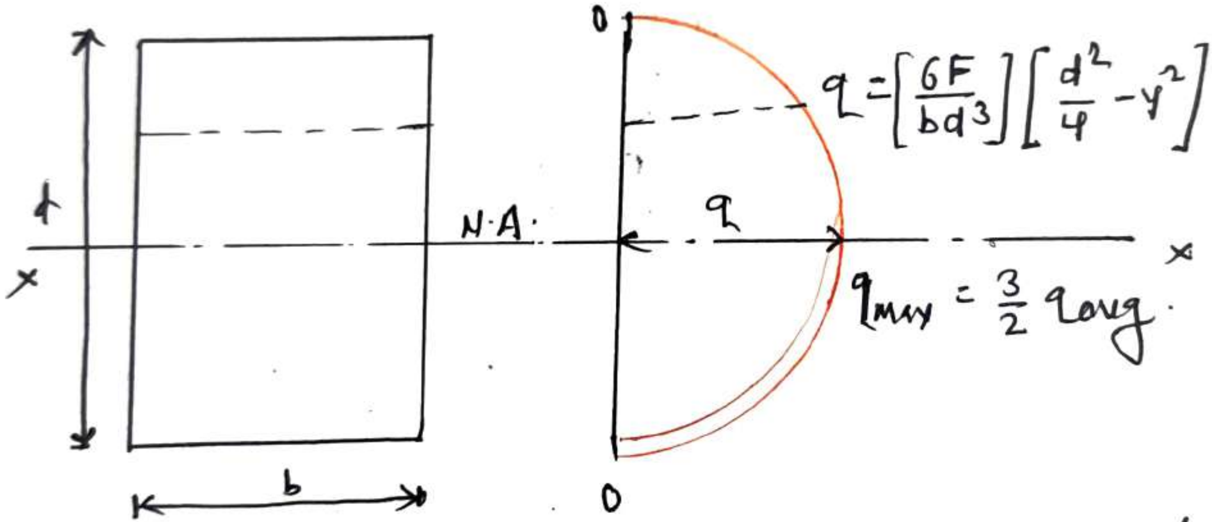
4.4.

SHEAR STRESS DISTRIBUTION DIAGRAM :-

If we applied the shear force to the beam it possess some stresses across it's section, at different point shear stress is induced, so to show that shear stresses values on graph. or a graphical representation of shear stresses values is called shear stress distribution dia.

4.5.

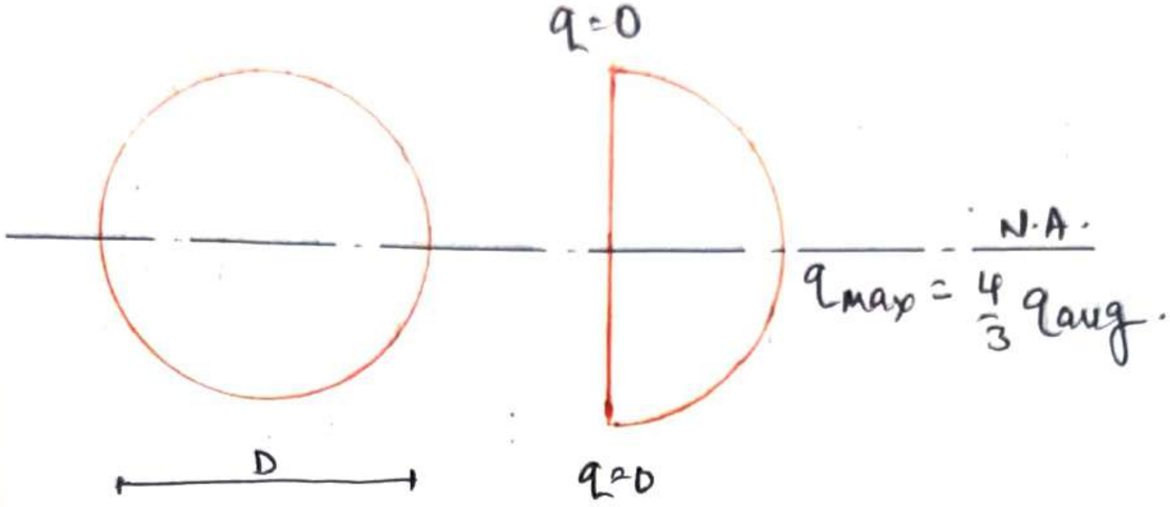
i) Rectangular section:-



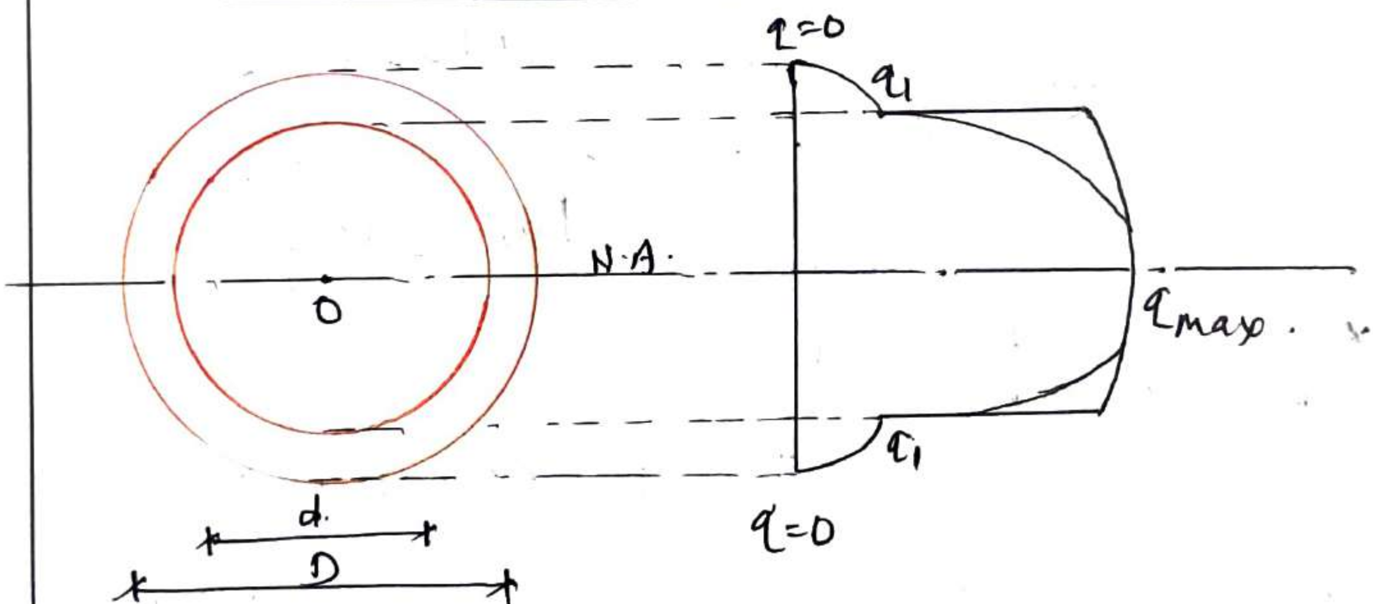
shear stress distribution dia

- From above dia, shear stress is zero at top and Bottom
- shear stress is maximum at N.A.
- parabolic in shape.

4.5
② Circular section:—



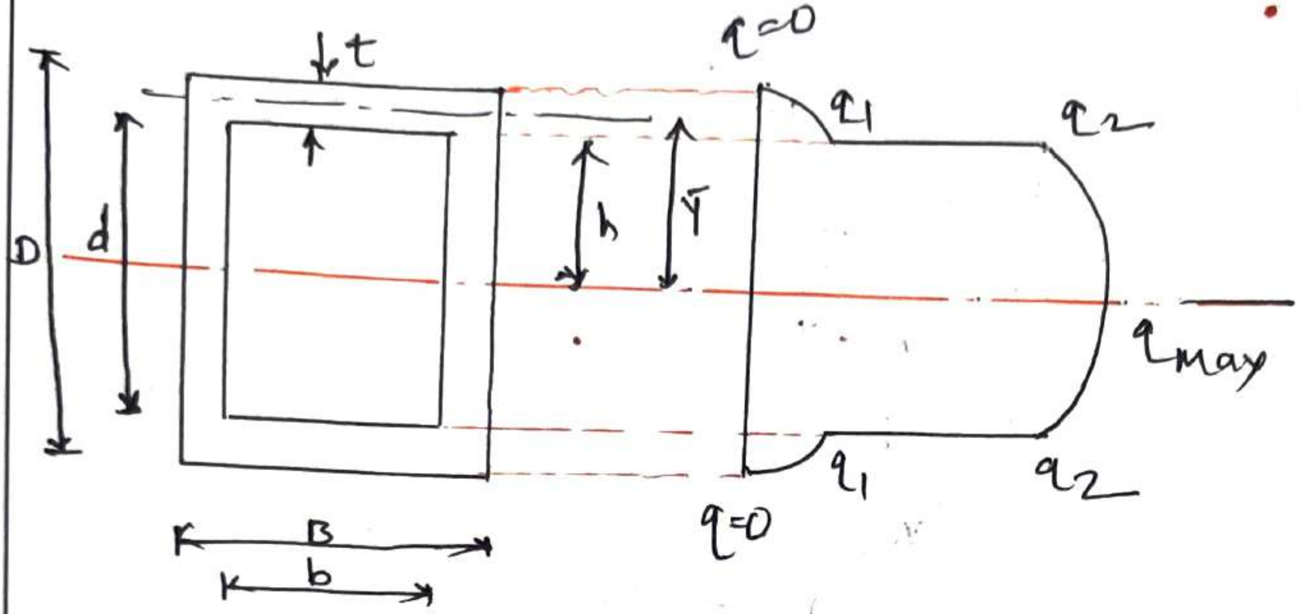
4.5
③ Hollow circular section:—



Here, d = Inner dia.
D = outer dia.

4.6

(4) Hollow Rectangular section:—



Here, $q=0$

and q_1 = shear stress at junction of web and flange

$\therefore q_1 = \frac{FA\bar{Y}}{IB}$ $\quad \left\{ \bar{Y} = h + \frac{t}{2} \right.$

Shear stress is increased upto q_2

$\therefore q_2 = \frac{B}{2b} \times q_1$

$$\boxed{\therefore q_{max} = q_2 + q_{add}}$$

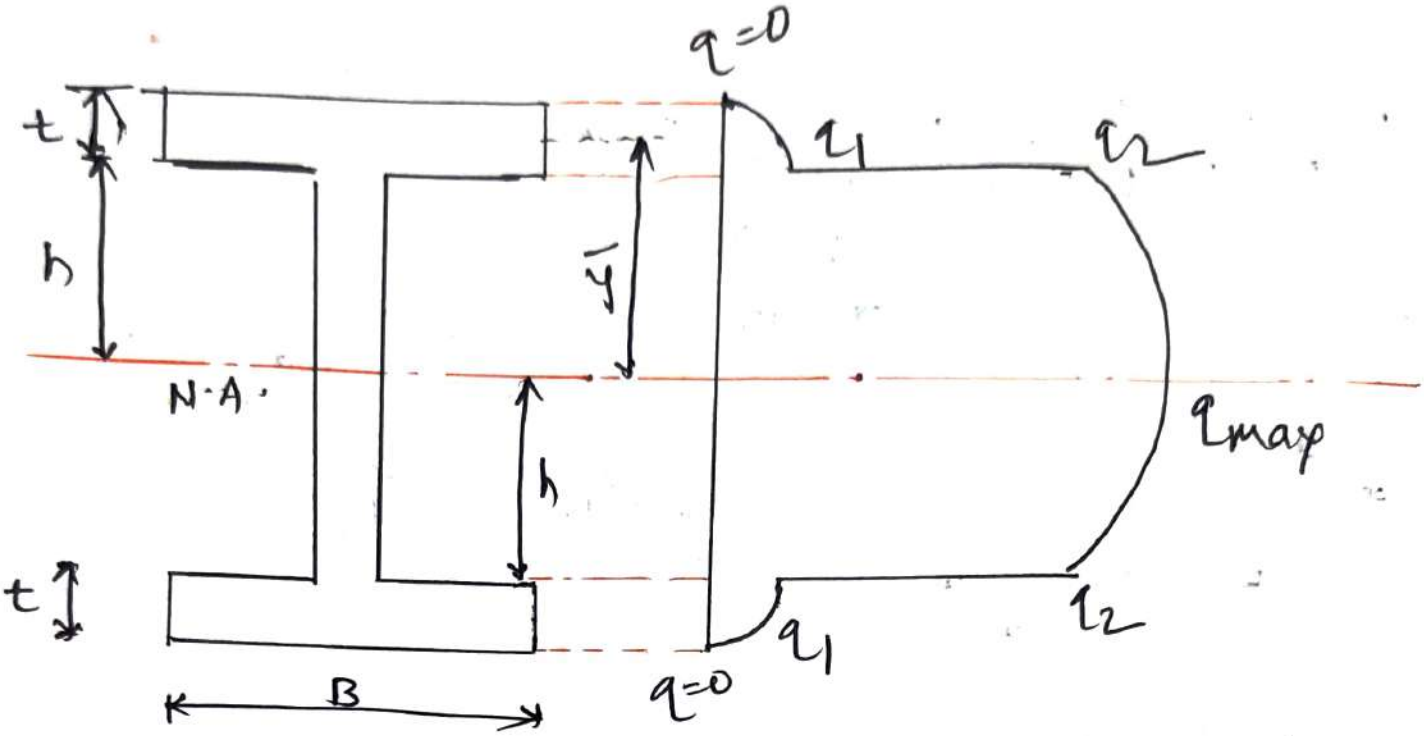
$\left\{ q_{add} = \frac{FA\bar{Y}}{Ib} \right.$

$q_{add} = \frac{FA\bar{Y}}{Ib}$

$\left\{ \begin{array}{l} A = 2[b \times h]^{*} \\ \bar{Y} = h/2 \text{ — from N.A.} \end{array} \right.$

45

(c) I-section:—



- shear stress at top and bottom of flanges should be zero.

$$\therefore q_1 = \frac{FA\bar{y}}{IB}$$

Here, $\bar{y} = h + \frac{t}{2}$

$A = B \times t$

and shear stress $q_2 = \frac{B}{b} \times q_1$

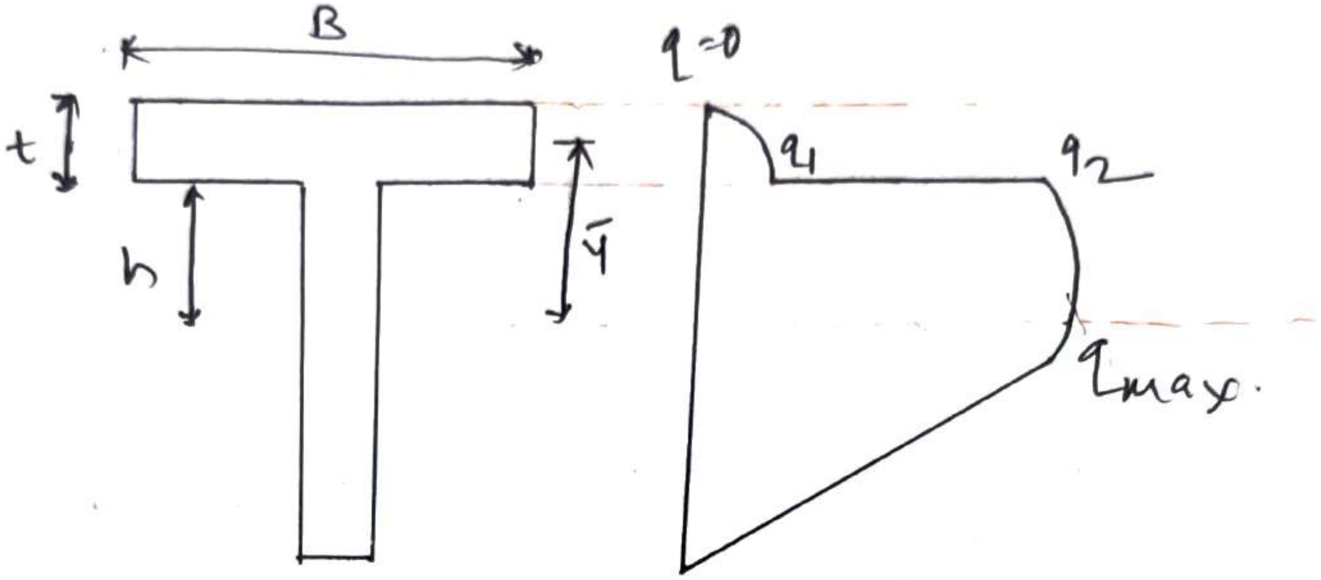
Now, maximum shear stress at N·A·

$$q_{max} = q_2 + q_{add}.$$

$$\left\{ q_{add} = \frac{FA\bar{y}}{Ib} \right.$$

$A = b \times h$, $\bar{y} = h/2$

T - section :-

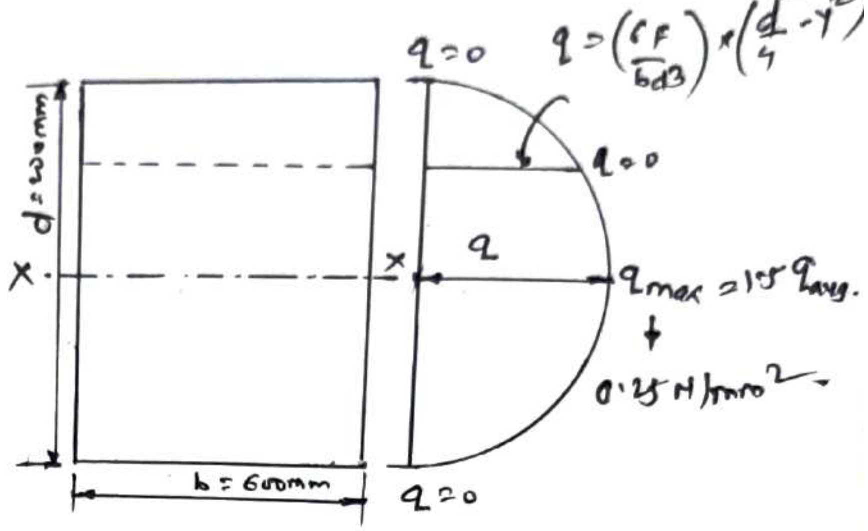


Topic :- 4.5 Shear Stress Distribution Diagram.

Q. 1) Sketch the shear stress distribution diagram for a rectangular beam 600 mm × 200 mm deep subjected to a shear force of 20 KN.

Given :-

1) b = width = 600 mm
2) d = depth = 200 mm
3) F = 20 KN



Sol^n :-

$y = \frac{d}{2} = \frac{200}{2} = 100\text{mm}$

$$\therefore q = \left(\frac{6 \times 20 \times 10^3}{600 \times (200)^3}\right) \times \left(\frac{(200)^2}{4} - (100)^2\right)$$

$$= (2.5 \times 10^{-5}) \times (0)$$

$$\boxed{q = 0}$$

We know that,

① $q_{avg} = \frac{F}{A} = \frac{20 \times 10^3}{200 \times 600} = 0.1667\ N/mm^2$

② $q_{max} = 1.5\, q_{avg} = 1.5 \times 0.1667$

$$\therefore \boxed{q_{max} = 0.25\ N/mm^2}$$

Topic :- 4.5 Shear Stress Distribution Diagram.

Q.5) Sketch the shear stress distribution diagram for a rectangular beam 600 mm × 200 mm deep subjected to a shear force of 20 kN.

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1) b = width = 600 mm
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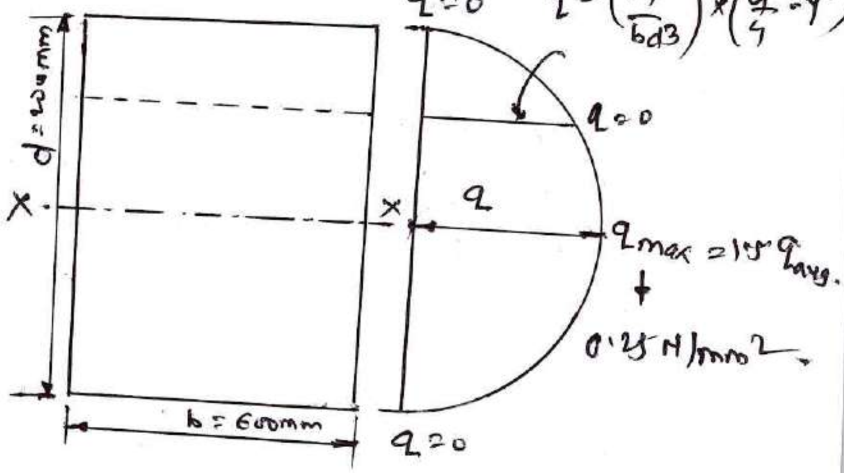

Sol^n:-

$$y = \frac{d}{2} = \frac{200}{2} = 100\,mm$$

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② $$q_{max} = 1.5\, q_{avg} = 1.5 \times 0.1667$$

$$\therefore \boxed{q_{max} = 0.25\ N/mm^2}$$

4.3 Q.B.⑥

A cantilever beam of rectangular metal cross-section is 4m in length carries udl of 5 kN/m. If permissible bending stress in the material is 5 N/mm². Determine the size of the section. Assume depth to width ratio = 2.

⇒ Given data:

① $L = 4m$

② $udl = W = 5 KN/m$

③ Bending stress, $(\sigma_b) = 5 N/mm^2$

④ depth to width ratio = 2

ie $\frac{d}{b} = 2$

$\therefore \boxed{d = 2b}$

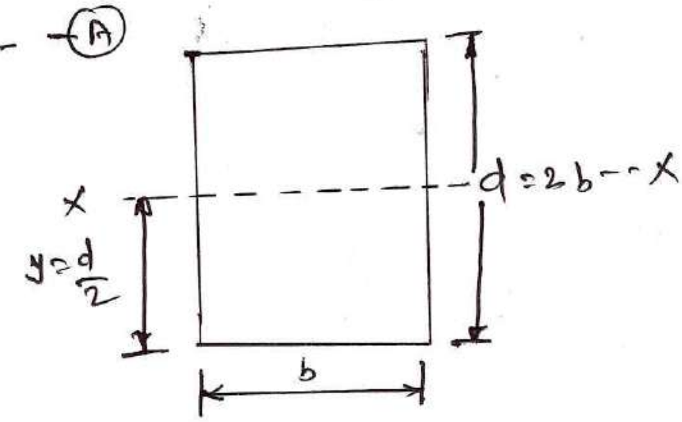
To find:

size of section ie ① $b = ?$

② $d = ?$

soln: By using flexural formula.

$$\frac{M}{I} = \frac{\sigma b}{y} \quad -(A)$$



Now,

I = M.I at x-x axis

$$I = I_{xx} = \frac{bd^3}{12}$$

$$= \frac{b \times (2b)^3}{12} = 0.667\, b^4$$

$\therefore y$ = Distance of external layer from x-x axis

$$\therefore y = \frac{d}{2} = \frac{2b}{2} = b$$

To find $M = ?$

$$M = \text{maximum B.M.} = \frac{Wl^2}{2} = \frac{5 \times (4)^2}{2}$$

$$M = 40 \text{ kNm} = 40 \times 10^6 \text{ N.mm}$$

put all values in eqn (A) becomes.

$$\frac{40\times10^6}{0.667\,b^4} = \frac{5}{b}$$

$$\therefore \frac{40\times10^6}{0.667\times5} = \frac{b^4}{b}$$

$$11.99\times10^6 = b^3$$

$$\therefore b = 228.90\,mm$$

$$\therefore d = 2\times b = 228.90\times2 = \underline{\underline{457.81\,mm}}$$

$$\boxed{\begin{array}{l} b = 228.90\,mm \\ \hline d = 457.81\,mm \end{array}}$$

Q.(7) A simply supported beam of rectangular section 150 mm wide 300 mm deep is simply supported over a span of 4.0m. It carries udl 10kN/m over entire span. Find the maximum and minimum bending stress induced in the section. Draw bending stress distribution diagram.

⇒ Given: (1) span = l = 4m

(2) udl = w = 10kN/m

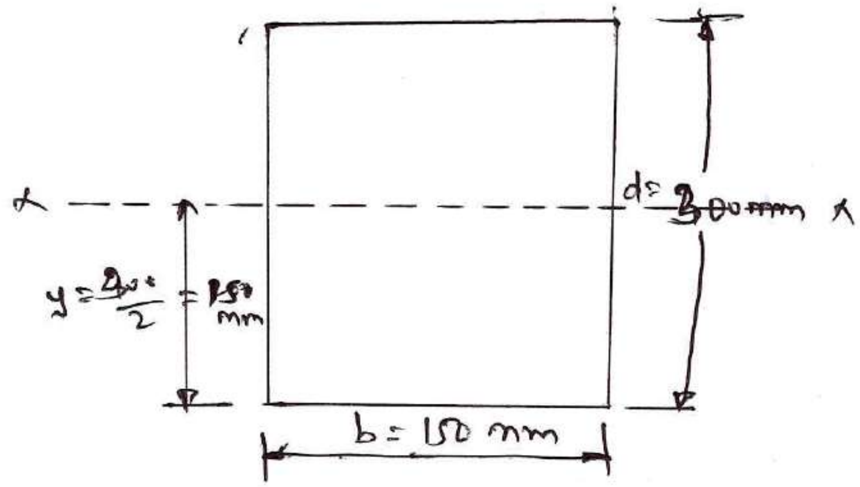
(3) width; b = 150mm

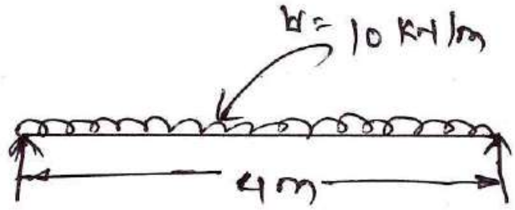
(4) depth; d = 300mm.

To find: maximum bending stress, σ_b = ?

1) σ_{bmax} = ?

2) σ_{bmin} = ?





Soln:- by using flexural formula

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\therefore \sigma_b = \frac{M \times y}{I} \quad \text{—— (A)}$$

$$M = \text{Bending moment} = \frac{wl^2}{8} =$$

$$M = \frac{10 \times (4)^2}{8} = 20 \text{ kNm}$$

$$\therefore M = 20 \times 10^6 \text{ Nmm}$$

Now, $I = I_{xx} = \frac{bd^3}{12} = \frac{150 \times (300)^3}{12}$

$$I = I_{xx} = 337.5 \times 10^6 \text{ mm}^4$$

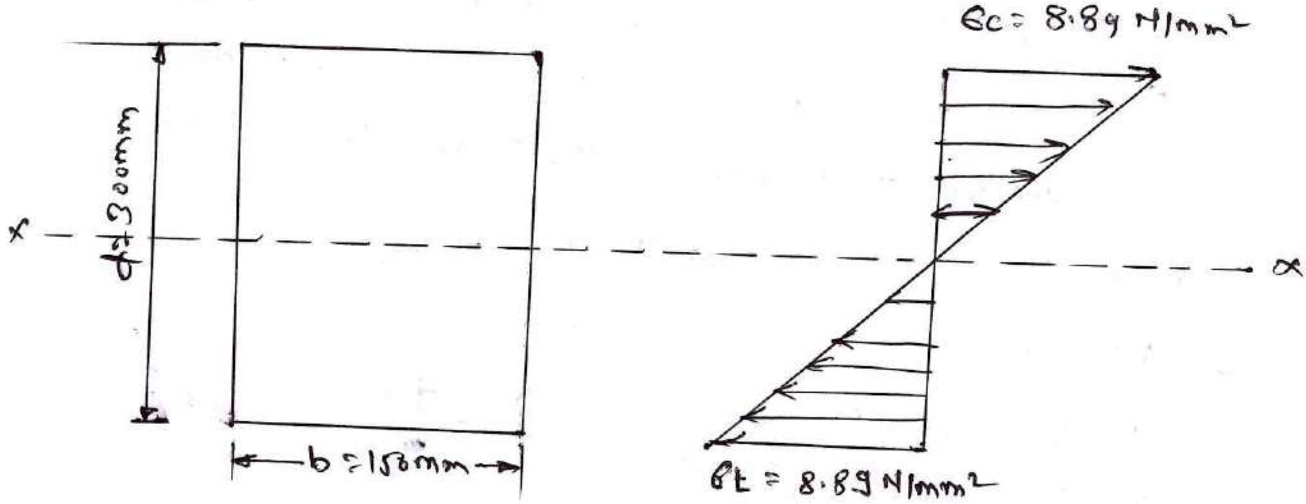
& $y = \frac{d}{2} = \frac{300}{2} = 150 \text{ mm}$

from eqn (A) becomes.

$$\sigma_b = \frac{20 \times 10^6 \times 150}{337.5 \times 10^6} = \underline{\underline{8.88}} \text{ N/mm}^2$$

Ans.

② Bending stress distribution diagram.



Cross-section of beam

Bending stress distribution diagram.

9.8) Draw shear stress distribution along cross-section of circular beam for 300 mm diameter carrying 400 kN shear force. Also determine the ratio of maximum shear stress to average stress.

→ Given:- ① $D = 300mm$

② $F = 400 kN = 400 \times 10^3 N$

$\therefore q_{avg} = \frac{F}{A} = \frac{\text{shear force}}{\text{Cross-section Area}}$

$\therefore A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (300)^2 =$

$A = 70686 \ mm^2$

$\therefore q_{avg} = \frac{400 \times 10^3}{70686} = 5.658 \ N/mm^2$

$\therefore q_{max} = \frac{4}{3} q_{avg} = 7.545 \ N/mm^2$

② $\boxed{\frac{q_{max}}{q_{avg}} = 4/3}$

Q.9) A cantilever beam of rectangular section support udl of 5 kN/m the span of beam 3m, If the maximum bending stress is 100 N/mm^2, and depth of beam is 1.5 times the width, Determine the size of beam.

⇒ Given:

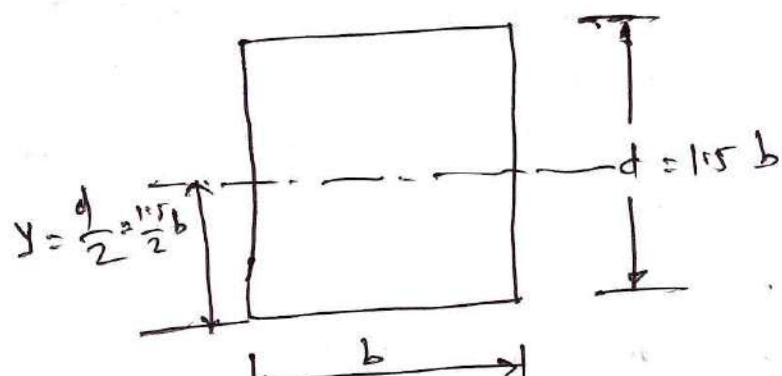
① L = span = 3m

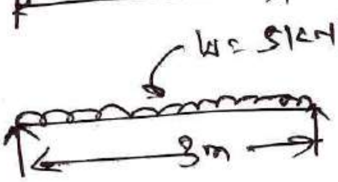
② W = udl = 5 kN/m

③ σ_{bmax} = 100 N/mm^2

④ $d = 1.5\,b$

To find: $b = ?$
$d = ?$





Applying flexural formula,

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{— ①}$$

∴ M = maximum bending moment = $\frac{Wl^2}{8}$

① ∴ $M = \frac{5 \times (3)^2}{8} = 5.625 \times 10^6$ N.mm

② I = M.I. of $I_{xx} = \frac{bd^3}{12} = \frac{b \times (1.5b)^3}{12}$

$I_{xx} = 0.281\,b^4$.

(3) $y = \frac{d}{2} = \frac{1.5b}{2} = 0.75b$

Put all values in eqn (1) becomes.

$$\frac{5.625 \times 10^6}{0.281\, b^4} = \frac{100}{0.75b}$$

$$\therefore \frac{5.625 \times 10^6 \times 0.75}{100 \times 0.281} = b^3$$

$$b = 53.148 \text{ mm}$$

& $d = 1.5 \times b = 53.148 \times 1.5 = 79.723$ mm

$\therefore$ $\boxed{b = 53.148 \text{ mm}}$
$\boxed{d = 79.723 \text{ mm}}$

Q.10) A T-section flange 160mm x 20mm and web 180mm x 20mm is simply supported at both ends. It carries two concentrated loads of 100 KN each acting 2m from each support span of beam is 8m. Determine maximum bending stress and draw bending stress distribution diagram. Also find bending stress layer 100mm from bottom.

$\Rightarrow$ Given data. (1) T-section

a) Flange = 160mm x 20mm

b) Web = 180mm x 20mm

c) Two points loads on a simply supported beam 100 kN

d) Span = L = 8 m

① To find $\sigma_{max} = ?$

② Find bending stress at 100 mm from bottom = ?

Soln: By using flexural formula.

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\therefore \frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M \times y}{I} \quad \text{--- ①}$$

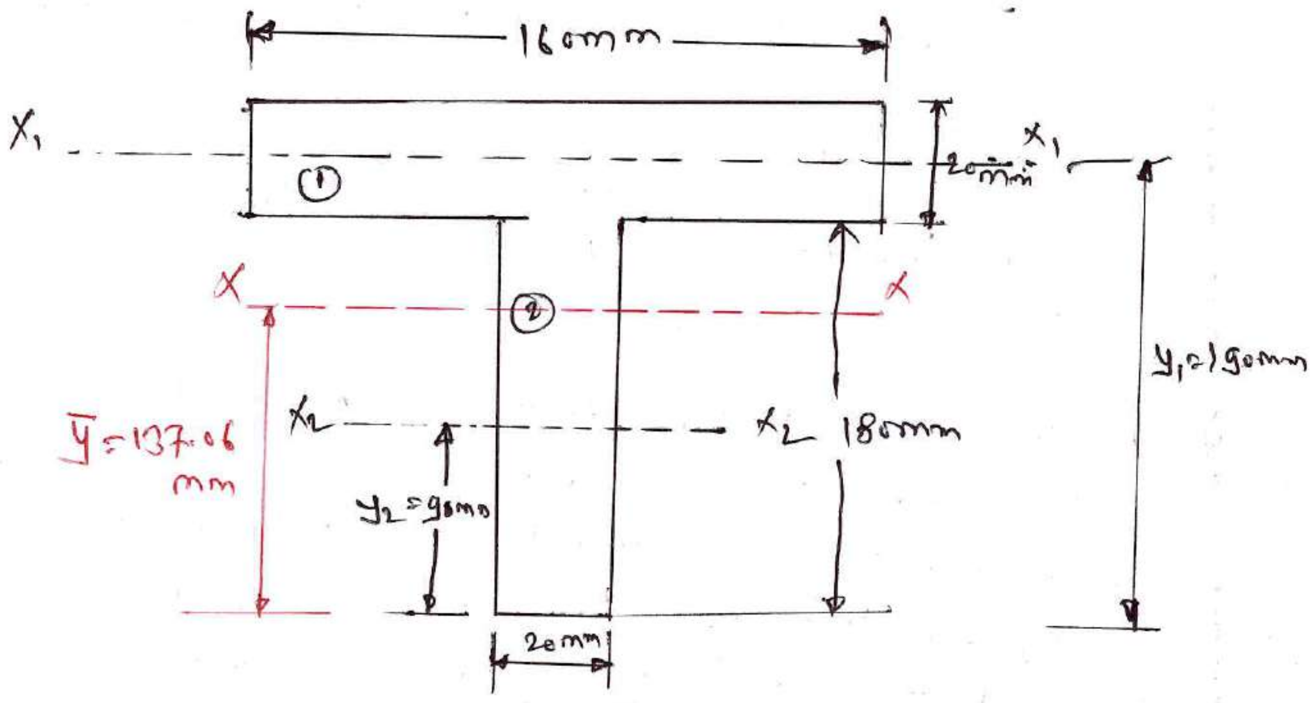


fig. (a)

To find $I_{xx} = ?$

Split the fig. (a) into two parts ① & ② respectively.

For part ① $b_1 = 160$mm , $d_1 = 20$ mm

" ② $b_2 = 20$mm , $d_2 = 180$mm.

By using parallel axis theorem,

$$I = I_{xx} = I_{xx_1} + I_{xx_2}$$

$$I_{xx} = \left(\frac{bd^3}{12} + Ah^2\right)_{(1)} + \left(\frac{bd^3}{12} + Ah^2\right)_{(2)}$$

$$= \left(\frac{b_1 d_1^3}{12} + A_1 h_1^2\right) + \left(\frac{b_2 d_2^3}{12} + A_2 h_2^2\right) \quad \text{--- ②}$$

$\therefore A_1 = b_1 \times d_1 = 160 \times 20 = 3200\ mm^2$

$A_2 = b_2 \times d_2 = 20 \times 180 = 3600\ mm^2$

Now to find h_1 & h_2

We have to find y_1, y_2 & $\bar{y}$

$$y_1 = \frac{20}{2} + 180 = 190\ mm$$

$$y_2 = \frac{180}{2} = 90\ mm$$

$$\therefore \bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{3200 \times 190 + 3600 \times 90}{3200 + 3600}$$

$\bar{y} = 137.06\ mm$

$\therefore h_1$ = Distance betn two axes X_1X_1 & XX

$\therefore h_1 = y_1 - \bar{y} = 190 - 137.06 = 52.94\ mm$

h_2 = Distance betn two axes XX & X_2X_2

$\therefore h_2 = \bar{y} - y_2 = 137.06 - 90 = 47.06\ mm$

from eqn (2) becomes,

$$I_{XX} = \left[\frac{160 \times (20)^3}{12} + 3200 \times (52.94)^2\right] + \left[\frac{20 \times (180)^3}{12} + 3600 \times (47.06)^2\right]$$

$I_{XX} = 9.075 \times 10^6 + 17.69 \times 10^6$

$$\boxed{I_{XX} = 26.77 \times 10^6\ mm^4}$$

Now, we have to find $y = ?$

$\bar{y} = y_t = 137.06$ mm

$y_c = 200 - \bar{y} = 200 - 137.06 = 62.94$ mm

We have find $M = ?$

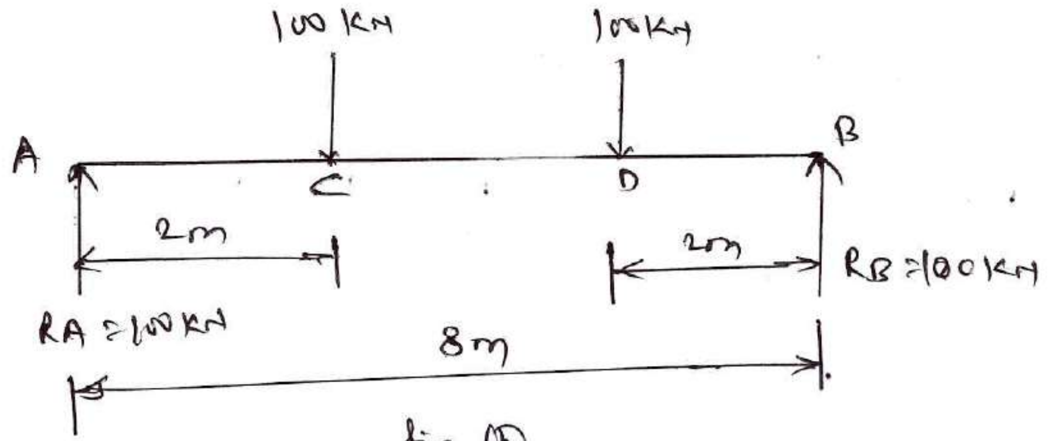


fig. (5).

Due to symmetry of member

$R_A = R_B = 100$ kN

B.M. at

C = B.M. at D = M max.

$= M = 100 \times 2 = 200$ kN.m

$M = 200 \times 10^6$ N.mm.

from eqn (1) becomes.

$\sigma_b = \frac{M}{I} \times y =$

For compression in top layer.

$\sigma_{bc} = \frac{M}{I} \times y_c = \frac{200 \times 10^6}{26.77 \times 10^6} \times 62.94$

$\sigma_{bc} = 470.23$ N/mm

For tension in bottom layer $\sigma_{bt} = \frac{M}{I} \times y_t$

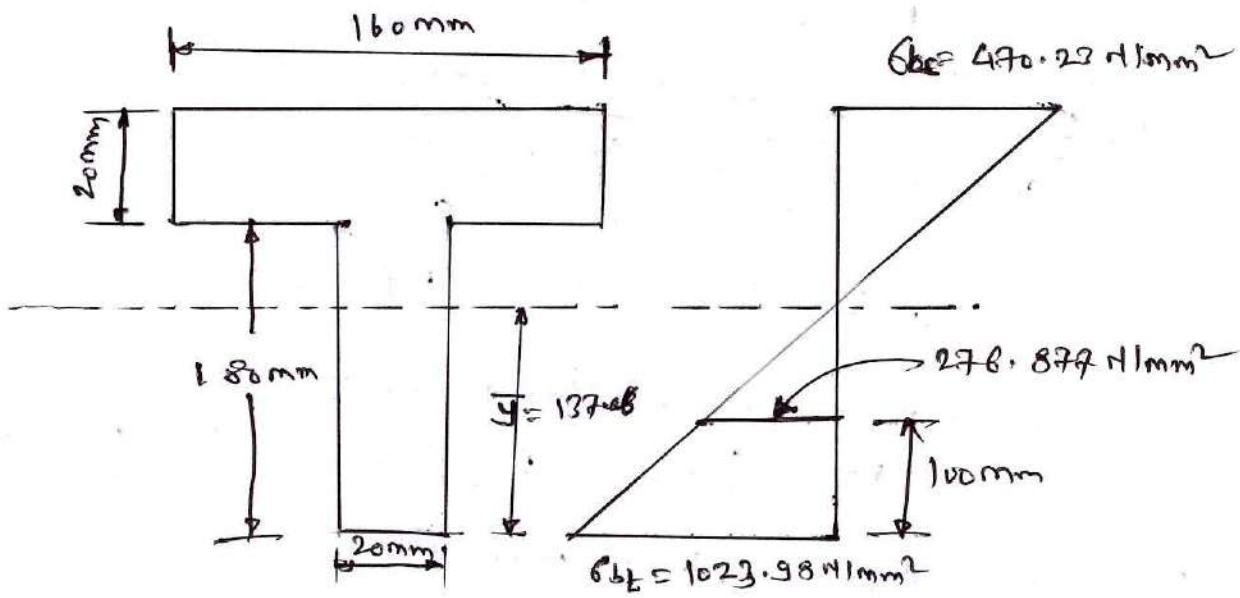
$\sigma_{bt} = \frac{200 \times 10^6}{26.77 \times 10^6} \times 137.06 = 1023.98$ N/mm^2

Bending stress at layer 100 mm from the bottom

$$\therefore \sigma_{bt} = \frac{M}{I} \times y$$

$$= \frac{200 \times 10^6}{21.77 \times 10^6} \times (137.06 - 100).$$

$$\sigma_{bt} = 276.877 \text{ N/mm}^2.$$



Cross-section of T. beam

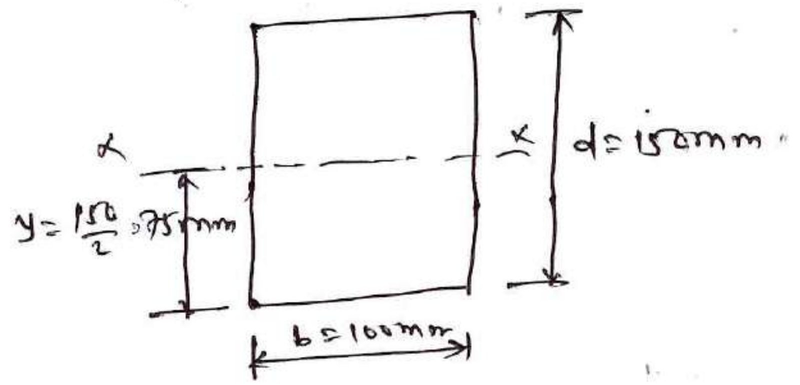
Bending stress distribution diagram

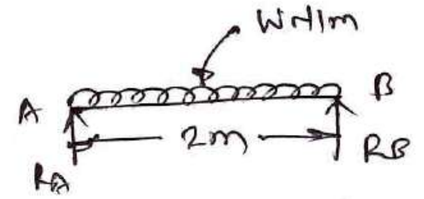
Q. 11) A timber beam 100 mm wide and 150mm deep supported a udl over a span 2m. If safe stress are 28 N/mm² in bending and 2 N/mm² in shear, calculate the maximum load which can be supported by the beam.

⇒ Given data:-

1) width = b = 100mm
2) Depth = d = 150mm
3) Span = L = 2m = 2000mm.
4) Safe bending stress = σ_b = 28 N/mm²
5) Safe shear stress = τ = 2 N/mm².

The beam is simply supported and carries udl (w) over span.





Soln:

① Load based on bending stress (σ_b)

section modulus (Z) $= \dfrac{bd^2}{6}$

$$\therefore Z = \frac{100 \times (150)^2}{6} = 375000 \text{ mm}^3$$

maximum permissible bending moment

$$M = \sigma_b \times Z = 28 \times 375000 = 10.5 \times 10^6 \text{ N-mm}$$

For simply supported beam with udl.

$$M_{max} = \frac{wL^2}{8} = \frac{w \times (2000)^2}{8} = 10.5 \times 10^6$$

$$\therefore w = \frac{10.5 \times 10^6 \times 8}{(2000)^2} = 21 \text{ N/mm}$$

$$w = 21 \text{ N/mm}$$

Total load

$$W_b = w \times L = 21 \times 2 = 42 \text{ kN}$$

② Load Based on shear stress. (τ)

For rectangular section

$$\tau_{max} = \frac{3F}{2A} \quad \cdots \{ A = b \times d$$

$$2 = \frac{3 \times F}{2 \times 100 \times 150}$$

$$F = \frac{2 \times 2 \times 100 \times 150}{3} = 20000\,N$$

$$\boxed{F = 20\,kN}$$ —— shear force

for simply supported beam

$$F = \frac{W}{2}$$

$$\therefore W_t = 2F$$

$$W_t = 2 \times 20 = 40\,kN$$

Corresponding udl :

$$\therefore w = \frac{40}{2} = 20\,kN/m$$

(B) <u>maximum safe load</u>

(1) Load from bending $(W_b) = 42\,kN$

(2) Load from shearing $(W_t) = 40\,kN$

Q.12 A rectangular beam section 300mm wide & 500mm deep is simply supported over a span of 4m. It carries a full span udl of 10 kN/m, find the maximum bending stress induced in the section. Draw the bending stress distribution diagram.

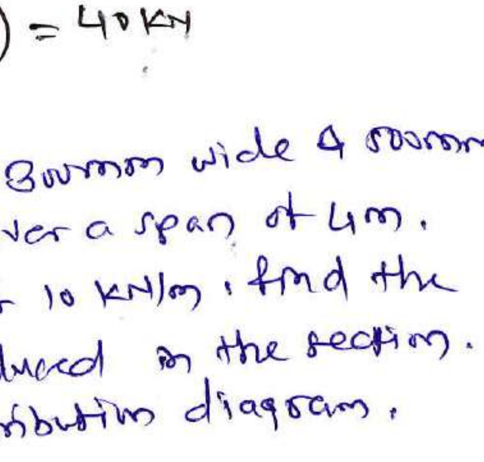
⇒ <u>Given</u> :⇒

(1) b = width = 300 mm

(2) d = depth = 500 mm

(3) span = L = 4m

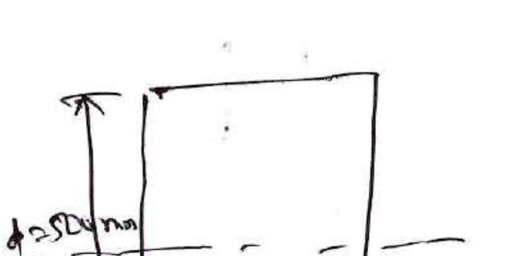
(4) udl = w = 10 kN/m



<u>To find</u> :

(1) $6_{max} = ?$

(2) Draw bending stress distribution diagram.



(1) maximum Bending moment for simply supported beam carrying udl over entire span

$$M_{max} = \frac{WL^2}{8} = \frac{10 \times (4000)^2}{8}$$

$$\boxed{M_{max} = 20 \times 10^6 \text{ N mm.}}$$

(2) section modulus (z)

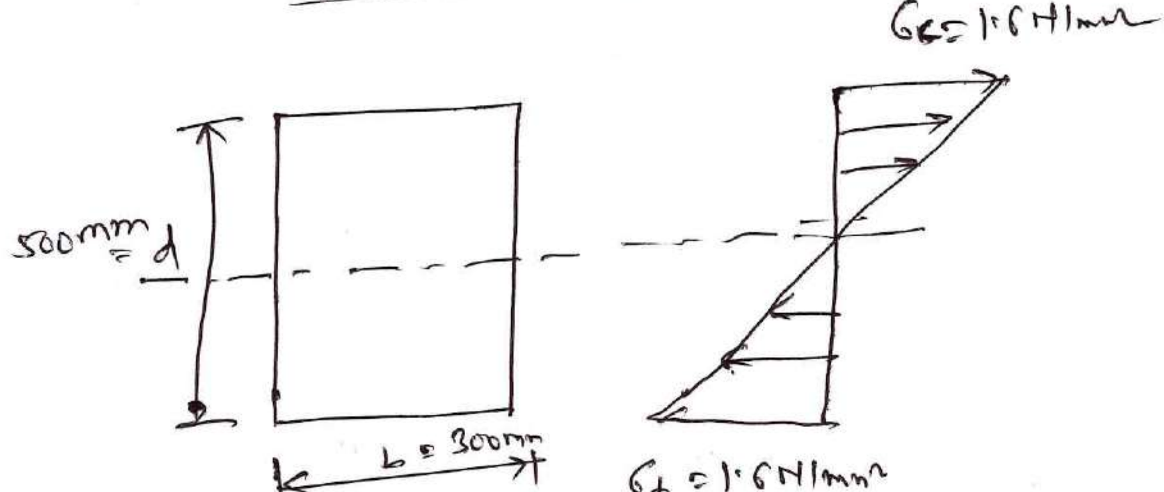
$$z = \frac{bd^2}{6} = \frac{300 \times (500)^2}{6}$$

$$\boxed{Z = 12.5 \times 10^6 \text{ mm}^3}$$

(3) maximum Bending stress (σ_b)

$$\sigma_b = \frac{M}{Z} = \frac{20 \times 10^6}{12.5 \times 10^6} = 1.6 \text{ N/mm}^2$$

$$\boxed{\sigma_b = 1.6 \text{ N/mm}^2}$$



Rectangular cross-section fig (c) maximum stress distribution diagram.

Q.13 A cantilever beam of solid circular-cross-section and 3m long carries a concentrated load 35KN at its free end, if maximum bending stress in tension or compression is not exceed 125 N/mm², determine diameter of the beam.

⇒ Given :- 1) $L = 3m = 3000 mm$

2) $W = 35KN = 35 \times 10^3 N$

3) Allowable bending stress $\sigma_b = 125 N/mm^2$

To find :- ① diameter = d = ?

Soln :- 1) Maximum Bending moment :- (M_{max})

$$M_{max} = WL = 35 \times 10^3 \times 3000$$

$$\boxed{M_{max} = 105 \times 10^6 N\cdot mm}$$

② section modulus of solid circular section (Z)

$$Z = \frac{\pi d^3}{32}$$

Using bending equation

$$\sigma_b = \frac{M}{Z}$$

$$125 = \frac{105 \times 10^6}{\frac{\pi d^3}{32}}$$

$$d^3 = \frac{105 \times 10^6 \times 32}{125 \times \pi} = 8.5562 \times 10^6$$

$$\therefore \boxed{d = 204.5 mm}$$

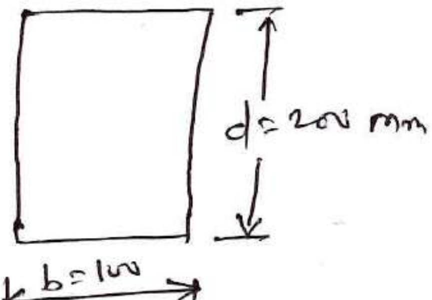
Q.14 A beam section 100mm × 200mm is subjected to a Shear force of 60 kN. Determine the shear stresses induced on a layer at 40mm above N.A. & 20mm below the N.A.

⇒ Given data:-

① b = width = 100 mm

② d = depth = 200 mm

③ $F = 60\,kN = 60\times10^3\,N$



To find:-

① Shear stresses induced on 40mm above N.A. and 20 mm below N.A.

ie $y_{above} = 40\,mm$ - from N.A.

$y_{below} = 20\,mm$ - from N.A.

Soln:

① $q_{avg} = \frac{F}{A} = \frac{60\times10^3}{100\times200} = 3\,N/mm^2$

② $q_{max} = 1.5\,q_{avg} = 3\times1.5 = 4.5\,N/mm^2$

③ To find shear stress at 40mm above N.A.

$q_{40} = \frac{6F}{bd^3}\left(\frac{d^2}{4}-y^2\right) = \frac{6\times60\times10^3}{100\times(200)^3}\left[\frac{(200)^2}{4}-(40)^2\right]$

$q_{40} = 4.5\times10^{-4}\,[10000-1600]$

$\boxed{q_{40} = 3.78\,N/mm^2}$

(2) Shear stress at 20mm below N.A.

$$y = 20$$

$$q_{20} = 1.5\, q_{avg} \left(1 - \frac{4y^2}{d^2}\right) \quad \text{—(A)}$$

OR

$$q_{20} = \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

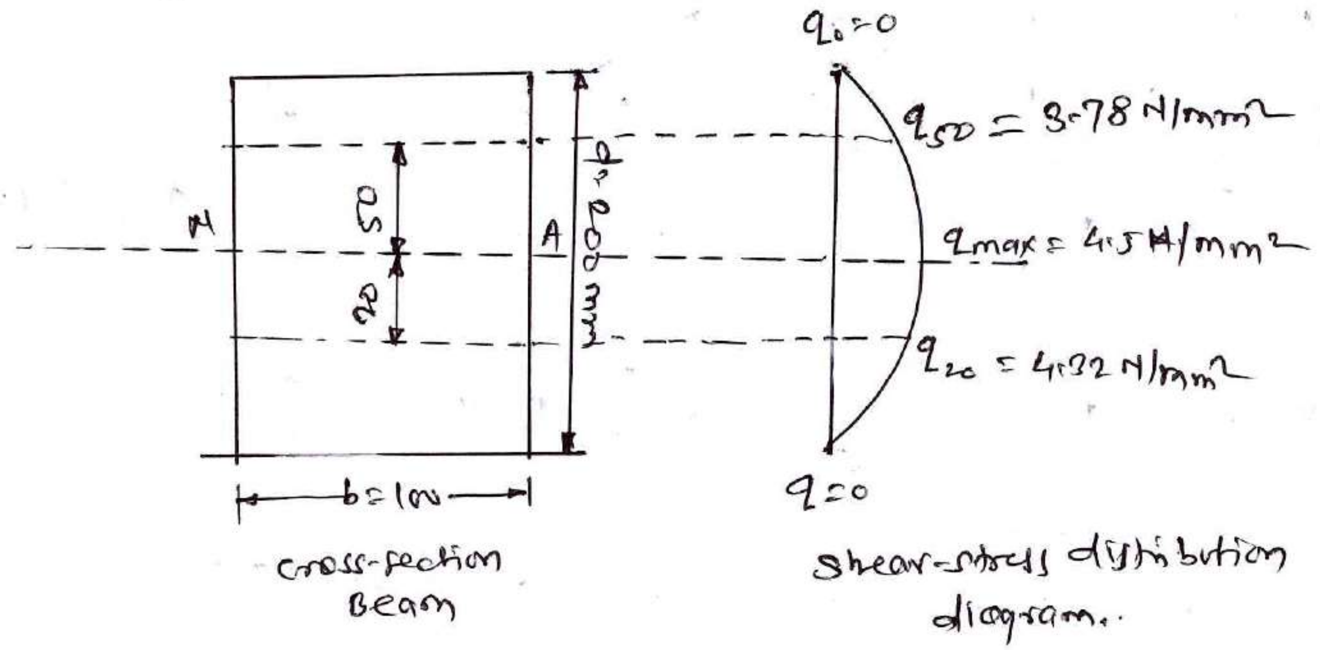
We used formula (A)

$$\therefore q_{20} = 1.5 \times 3 \left[1 - \frac{4 \times (20)^2}{(200)^2}\right]$$

$$= 4.5(1 - 0.04)$$

$$\boxed{q_{20} = 4.32\ N/mm^2}$$

(3) Shear stress distribution diagram.



Cross-section Beam

Shear-stress distribution diagram.

Topic 5.6

Q.15) A Hollow rectangular section, 200mm X 400mm externally with uniform thickness of 40mm carries a shearing force of 100KN at section. Construct the shear stress distribution diagram giving all important values. Also calculate the ratio max. to average shear stress.

→ Given:

(1) $B = 200mm$

(2) $D = 400mm$

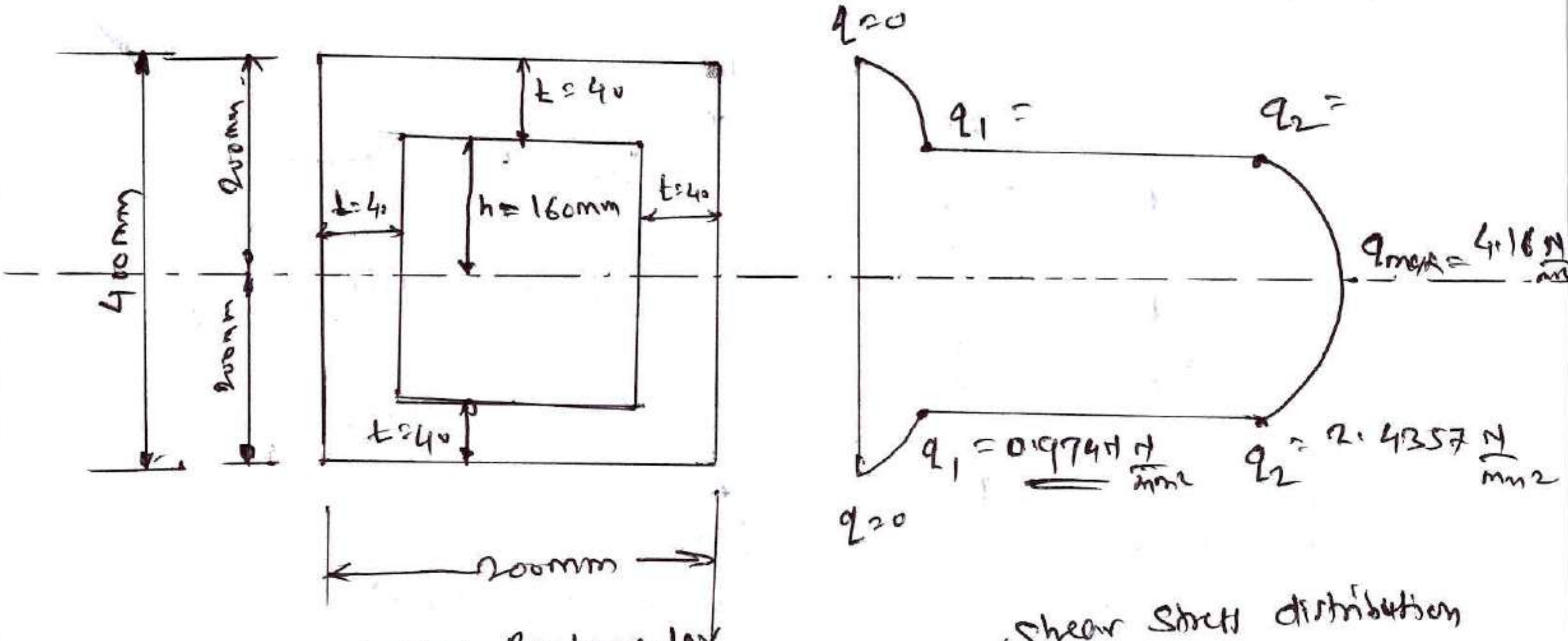
(3) $t = 40mm$

(4) $F = 100\,KN = 100 \times 10^3\,N$

∴ Innear diameation $d = 400 - 2 \times 40 = 320mm$

$b = 200 - 2 \times 40 = 120mm$

To find (1) draw shear stress distribution diagram & find the ratio of $\frac{q_{max}}{q_{mm}} = ?$



Hollow Rectangular Cross-section

Shear Stress distribution diagram.

1. Shear stress of external layer $q = 0$

2. Shear stress of the juction of flange and web

$$q_1 = \frac{FA\bar{y}}{IB} \quad \text{——— (1)}$$

Here,

$F = 100\,KN = 100 \times 10^3\,N$; $\bar{y} = 200 - \frac{40}{2} = 180\,mm$ from N.A.

$A = 200 \times 40 = 8000\,mm^2$), $B = 200\,mm$,

Now, $I = I_{xx} = \frac{1}{12}\left[BD^3 - bd^3\right]$

$$I_{xx} = \frac{1}{12}\left[200 \times (400)^3 - 120 \times (320)^3\right]$$

$$= 738.9867 \times 10^6\,mm^4$$

from eqn (1) becomes.

$$q_1 = \frac{100 \times 10^3 \times 8000 \times 180}{738.9867 \times 10^6 \times 200} = \underline{\underline{0.9743\,N/mm^2}}$$

$$\boxed{q_1 = 0.9743\,N/mm^2}$$

3. To find $q_2 = ?$

$$\therefore q_2 = q_1 \times \frac{B}{2b} = q_1 \times \frac{\text{width of flange}}{\text{width of two web}}$$

$$= 0.9743 \times \frac{200}{2 \times 40} = \underline{\underline{2.4357\,N/mm^2}}$$

$$\boxed{q_2 = 2.4357\,N/mm^2}$$

4. $q_{max} = q_2 + q_{add}$

$$\therefore q_{add} = \frac{FA\bar{y}}{IB} = \frac{FA\bar{y}}{I(2b)}$$

Here, $F = 100 \times 10^3\,N/mm^2$

$A = 2 \times h \times t = 2 \times 160 \times 40 = 12800\,mm^2$

$\bar{y} = \frac{160}{2} = 80\,mm$ from N.A.

$\therefore I = I_{xx} = 738.9867 \times 10^6 \ mm^4$

$$\therefore q_{add} = \frac{100 \times 10^3 \times 12800 \times 80}{738.9867 \times 10^6 \times 2 \times 40} = \underline{\underline{1.732 \ N/mm^2}}$$

$$\boxed{q_{add} = 1.732 \ N/mm^2}$$

$$\therefore q_{max} = q_2 + q_{add}$$

$$= 2.4357 + 1.732$$

$$\therefore \boxed{q_{max} = 4.1678 \ N/mm^2}$$