R.C.Patel College Of Engineering & Polytechnic, Shirpur
Department of Civil Engineering

Course Title- Strength of Material
Programme Name -Civil Engineering

Course Code -313308
Semester-Third

Unit	Title	COs	Learning hours	R Level	U Level	A Level	Total Marks
IV	**Bending and Shear Stresses in beams**	**CO4**	**10**	**2**	**4**	**6**	**12**

Syllabus:

4.1 Theory of pure bending, assumptions in pure bending, Concept of Neutral Axis and section modulus.

4.2 Flexural Equation (without derivation) with meaning of each term used in equation, bending stresses and their nature, bending stress distribution diagram.

4.3 Bending stress variation diagram across depth of given cross section for cantilever and simply supported beams for symmetrical sections only.

4.4 Shear stress equation (without derivation), meaning of each term used in equation, relation between maximum and average shear stress for square, rectangular and circular section (numerical), shear stress distribution diagram.

4.5 Shear stress distribution diagram for square, rectangular, circle, hollow square, hollow rectangular, hollow circle, T- section & symmetrical I- section only. (no numericals)

4.6 Use of shear stress equation for determination of shear stresses in hollow rectangular section.

Subject Teacher
Mr.R.B.Patil

BENDING STRESS :-

Bending moment at section tends to bend or deflect the beam and the internal stresses resist it's bending, when every c/s sets up full resistance to the bending moment, then the resistance offered by the internal stresses to the bending is called Bending stress.

ASSUMPTIONS IN PURE / SIMPLE BENDING :-

1. The Material of beam is perfectly homogeneous and isotropic through out the section.

2. The Material obeys Hooke's law.

3. The transverse sections, which were plane before bending, remains plane after bending also.

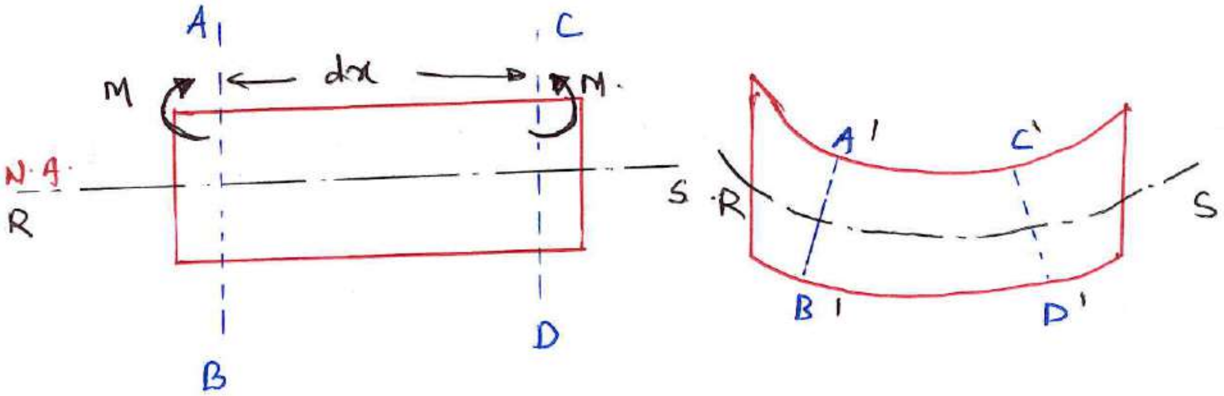
4. Each Layer of beam is free to expand or contract.

5. The value of Young's Modulus is same in the tension and compression.

6. The beam is in equilibrium i.e. there is no Resultant pull or push.

THEORY of SIMPLE BENDING/ PURE BENDING :—

let us consider a simply supported beam subjected to bending moment as shown in fig. (a), there are two section AB and CD are perpendicular to the axis of beam RS, while bending moment whole beam will bend as shown in fig. (b)



consider small length of beam dx, then the curvature of beam is circular, As we can seen from the observation upper layer of beam possess compression and lower layer possess tension while Bending Bending moment (M) applied. As the result of section of beam will be changed as A'B' and C'D'

PURE BENDING:–

When Beam is subjected to external loads, then the shear force offered by material is zero i.e. Bending moment is constant at every c/s of beam is called pure bending.

NEUTRAL AXIS:–

It is ~~Axis~~ an axis at where stress will be zero throughout the section. it means their is no compression nor tension. is called neutral axis.

SECTION MODULUS:–

It is defined as the ratio of MI about Neutral axis to the distance betn extreme layer of the beam from N.A.

- It is denoted by 'Z'

$$\therefore \quad Z = \frac{I}{Y} \qquad \left\{ \begin{array}{l} \text{Where,} \\ I = MI = I_{xx} \\ Y = Y_{max} \end{array} \right.$$

$$= \frac{mm^4}{mm}$$

$$Z = mm^3 \text{ (Unit)}$$

FLEXURAL FORMULA :- [Without derivation]

- It is also known as flexural equation or Bending equation.

As we know that,

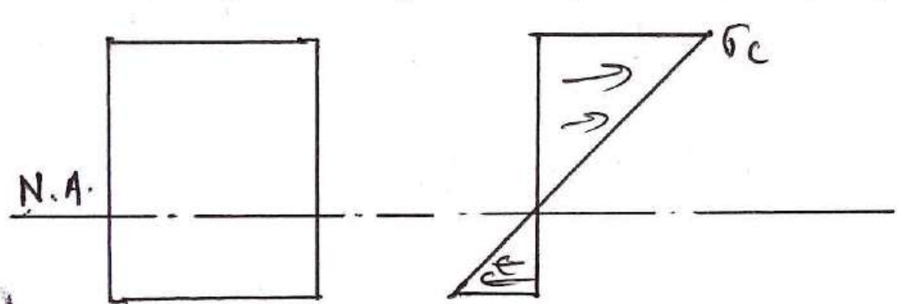
(Imp)
$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

Here,
M = Max. B.M.
$I = I_{xx}$ = M.I
σ = Bending stress.
y = distance from N.A.
R = Radius of curvature.

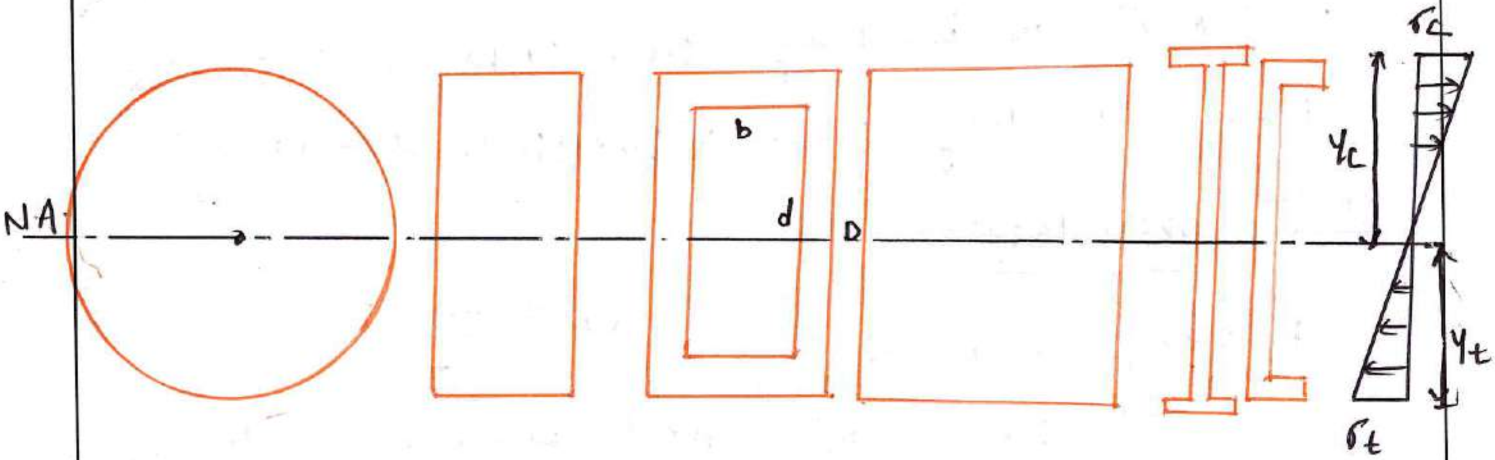
BENDING STRESSES DISTRIBUTION AND IT'S NATURE :-

Consider a simply supported beam, there is no stress at it's N.A. But the compressive stress is developed into upper layer of beam and tensile stress developed below the N.A. i.e. lower layer of beam.

As we know, that the stress at point is directly proportional to it's distance from the N.A.



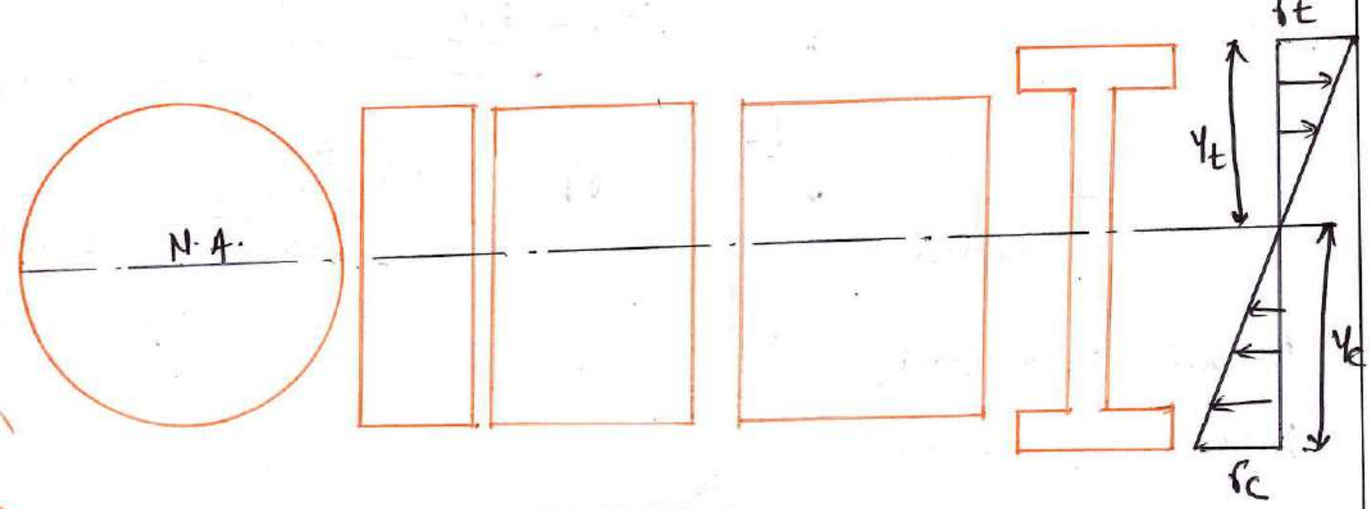
★ BENDING STRESS VARIATION DIAGRAM FOR SSB FOR SYMMETRICAL SECTIONS :—



For symmetrical (SSB) bending stress is equally distributed through out the section.

$$\therefore \boxed{\sigma_c = \sigma_t}$$

★ BENDING STRESS VARIATION DIAGRAM FOR CANTILEVER FOR SYMMETRICAL SECTIONS :—



$$\boxed{\sigma_c = \sigma_t}$$

★ A Rectangular beam section 300mm wide and 500mm deep is simply supported over a span of 4m. It carries a full span UDL of 10kN/m. Find maximum bending stress induced in the section. Draw Bending stress distribution diagram.

Ans:- Given data:-

Rectangular beam

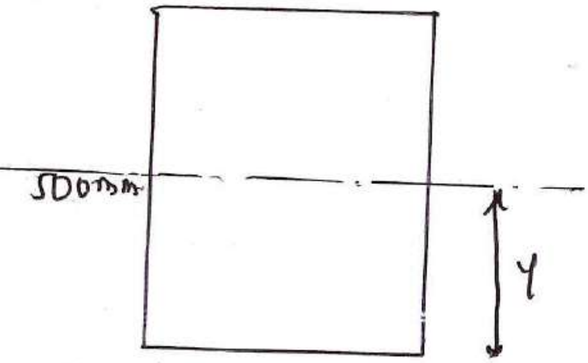
(300 × 500) mm

Span = L = 4m.

UDL = 10kN/m.

To Find :-

σ_b = maximum stress.



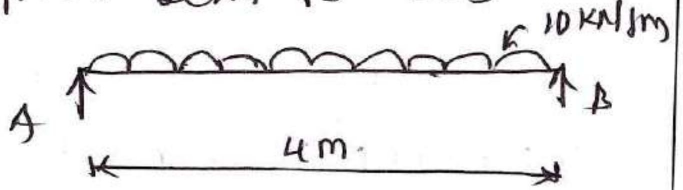
As we know that,

Flexural formula.

$$\frac{M}{I} = \frac{\sigma_b}{Y} \quad —(1)$$

$$\therefore \sigma_b = \frac{MY}{I}$$

given beam is SSB.



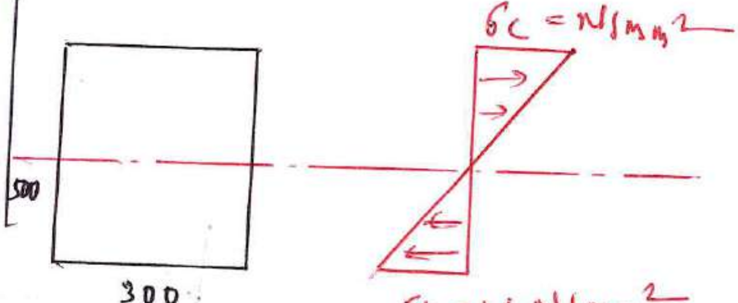
$$\therefore B.M = \frac{wL^2}{8} = \frac{10 \times 4^2}{8}$$

$$M = 20\ kN\text{-}m$$

Here, $Y = \frac{500}{2} = 250$ mm.

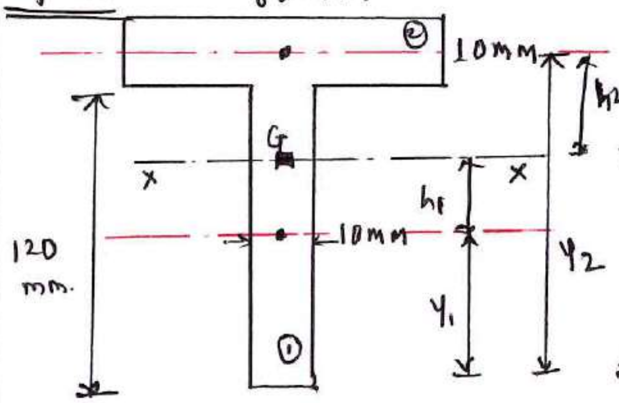
$$\therefore I = \frac{bd^3}{12} = \frac{300 \times 500^3}{12} = 3.125 \times 10^9\ mm^4$$

$$\therefore \sigma_b = \frac{20 \times 10^6 \times 250}{3.125 \times 10^9} = 1.6\ N/mm^2$$



* A cantilever is 2m long and subjected to UDL of 2KN/m, cross section beam is Tee with flange 80mm x 10mm and web of 10mm x 120mm and total ~~depth~~ depth is 130mm. The flange is at top and web is vertical. Calculate maximum tensile stress and compressive stress developed and their position.

Ans:-



$Y_1 = 60mm$, $A_1 = 10 \times 120 = 1200 mm^2$

$Y_2 = 125mm$, $A_2 = 80 \times 10 = 800 mm^2$

$$\bar{Y} = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2} = \frac{(1200 \times 60) + (800 \times 125)}{1200 + 800}$$

$$\boxed{\bar{Y} = 86mm}$$

$I_{xx} = I_{xx_1} + I_{xx_2}$ —— ①

$\therefore I_{xx_1} = \frac{bd^3}{12} + Ah^2$ —— (llel axis thm)

$$= \frac{10 \times 120^3}{12} + 1200 \times 26^2$$

$I_{xx_1} = 2.25 \times 10^6 mm^4$

$$I_{xx_2} = \frac{80 \times 10^3}{12} + [800 \times 39^2]$$

$$= 1.22 \times 10^6 mm^4$$

$eq^n ① \Rightarrow$

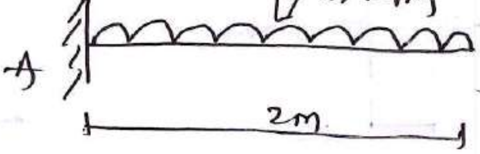
$I_{xx} = [2.25 \times 10^6] + [1.22 \times 10^6]$

$$\boxed{I_{xx} = 3.47 \times 10^6 mm^4}$$

Now,

using flexural formula.

$$\frac{M}{I} = \frac{\sigma b}{Y} = \frac{E}{R}$$



$$\therefore M = \frac{wL^2}{2} = \frac{2 \times 2^2}{2} = 4KN\text{-}m$$

i.e

$$\boxed{M = 4\times10^{6}\ N\text{-}mm}$$

Here,
Tension at top
and comp. at bottom.

$\therefore y_c = \bar{y} = 86\ mm.$

$\therefore y_t = 130 - 86$

$y_t = 44\ mm$

$\therefore \sigma_{bt} = \frac{M}{I} \times y_t$

$= \frac{4\times10^{6}}{3.47\times10^{6}} \times 86$

$$\boxed{\sigma_{bt} = 99.135\ N/mm^2}$$

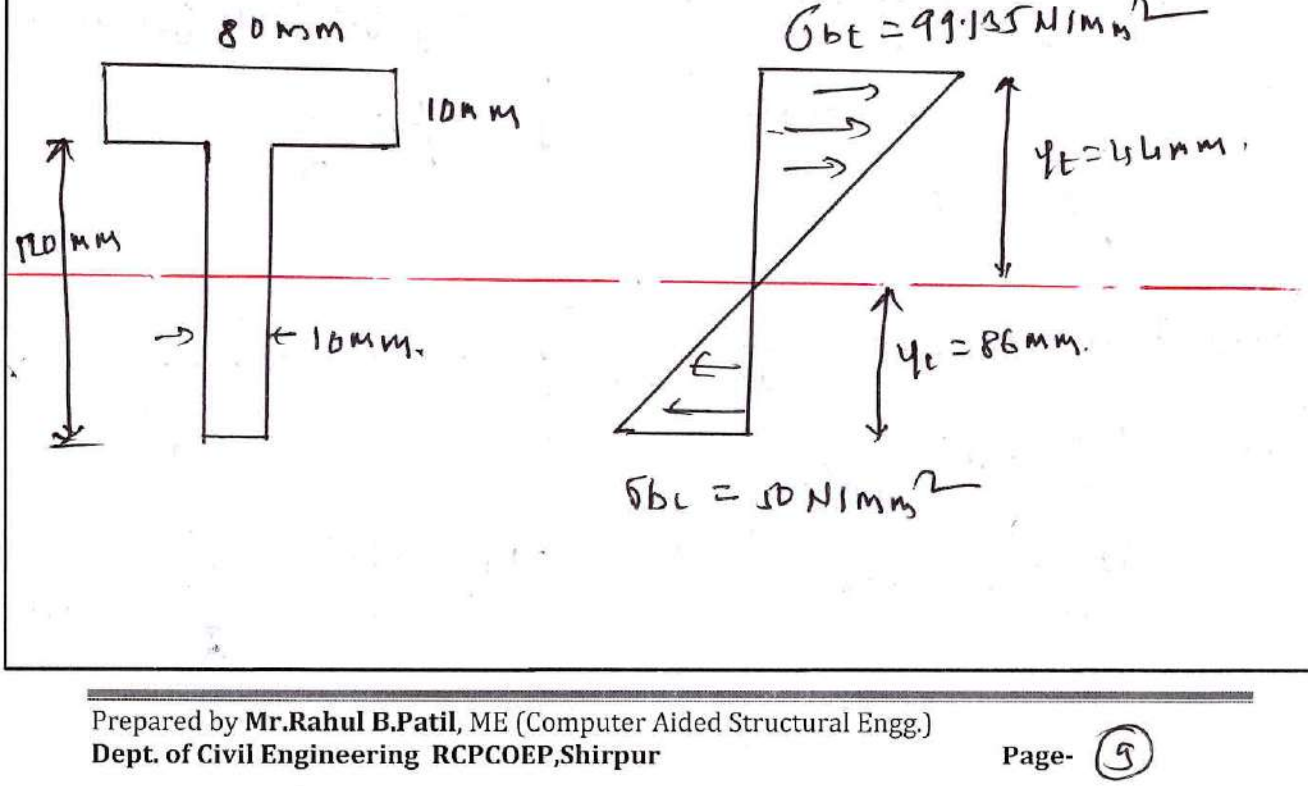
Now,

$\sigma_{bc} = \frac{M}{I} \times y_c$

$= \frac{4\times10^{6}}{3.47\times10^{6}} \times 44$

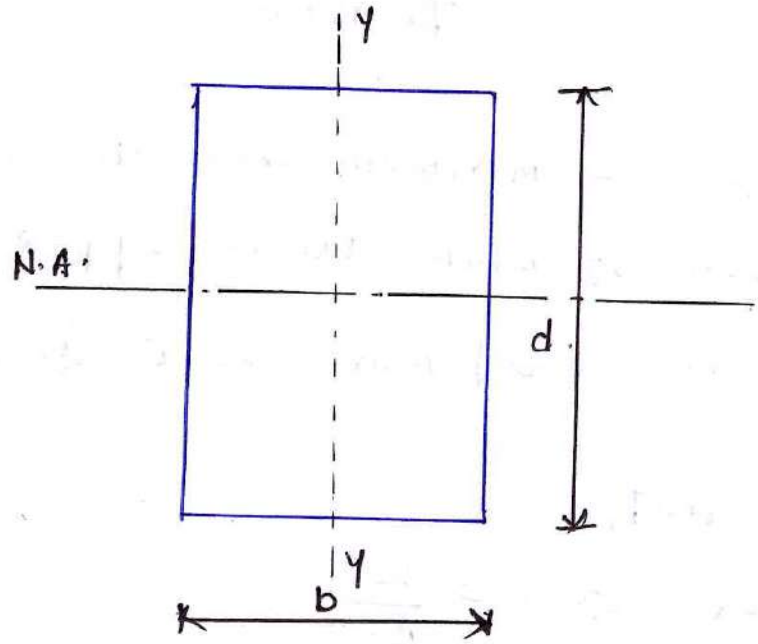
$$\boxed{\sigma_{bc} = 50\ N/mm^2}$$

To Draw stress variation dia for given beam.



★ SHEAR STRESS EQUATION :—

$$\boxed{q = \frac{F A \bar{Y}}{I b}}$$



Rectangular section of Beam.

Here,

q = shear stress (N/mm^2)

b = Width of beam section (mm)

F = Applied shear force

$A\bar{Y}$ = First moment of area.

RELATION BETWEEN AVG. SHEAR STRESS (q_{avg}) and MAXIMUM SHEAR STRESS:—

Avg. shear stress $= \dfrac{\text{shear force}}{\text{c/s area}}$.

Max. shear stress = Maximum shear stress induced at any section of beam due to applied shear force is called max. shear stress.

As we know that,

$$\text{shear stress} = \frac{F}{A}.$$

i) Rectangular section:—

$$\boxed{q_{max} = \frac{3}{2} q_{avg}.}$$

ii) square section:—

$$\boxed{q_{max} = \frac{3}{2} q_{avg}.}$$

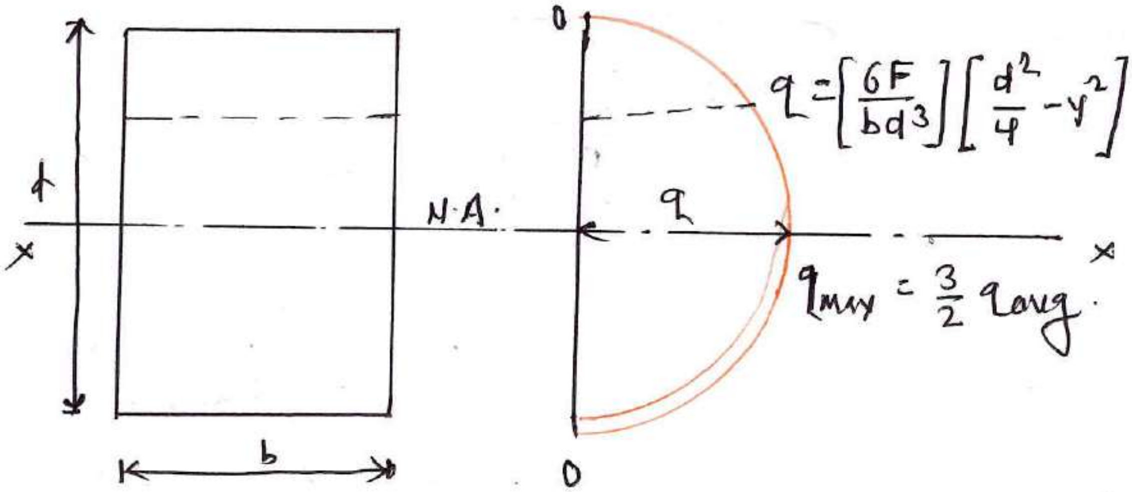
iii) circular section:—

$$\boxed{q_{max} = \frac{4}{3} q_{avg}.}$$

SHEAR STRESS DISTRIBUTION DIAGRAM:-

If we applied the shear force to the beam it possess some stresses across it's section, at different point shear stress is induced, so to show that shear stresses values on graph. or a graphical representation of shear stresses values is called shear stress distribution dia.

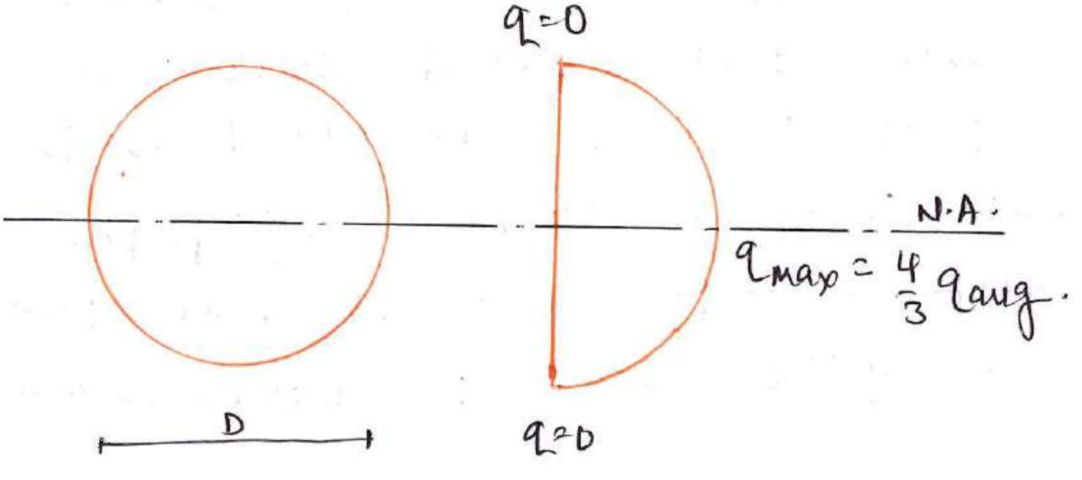
i) Rectangular section:-



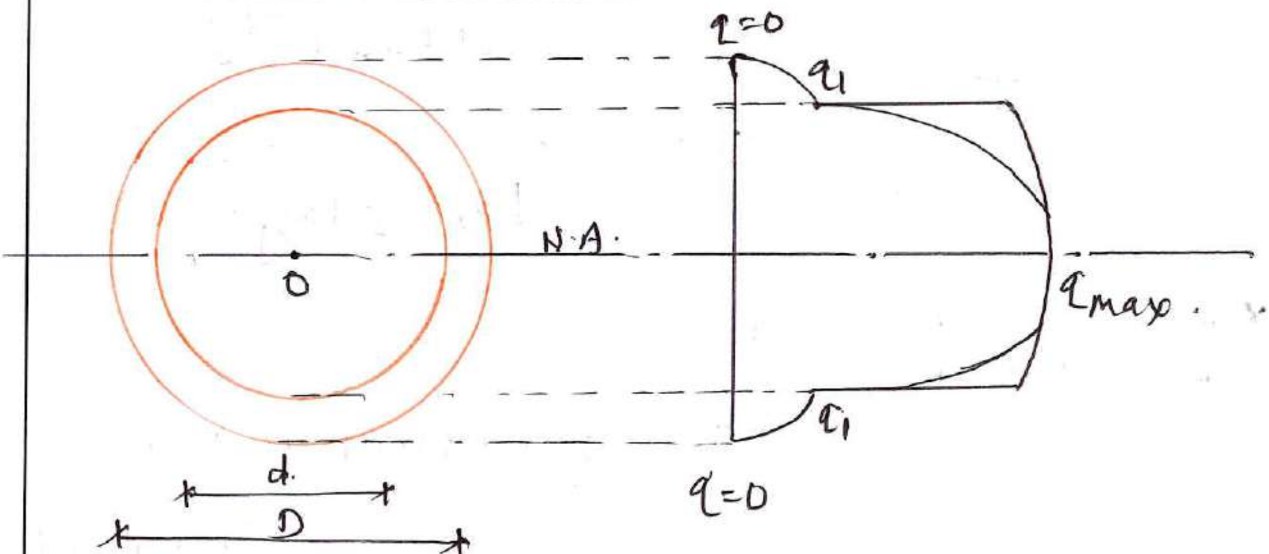
shear stress distribution dia

- From above dia, shear stress is zero at top and Bottom
- Shear stress is maximum at N.A.
- parabolic in shape.

② Circular section :-



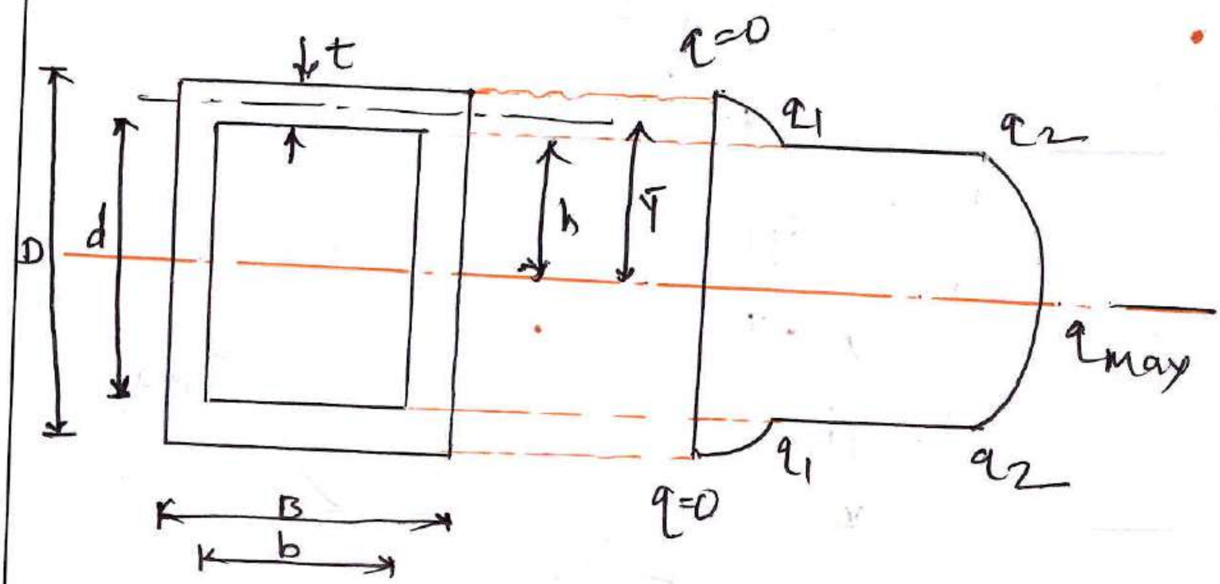
③ Hollow circular section :-



Here, d = Inner dia.

D = outer dia.

④ Hollow Rectangular section :-



Here, $q=0$

and q_1 = shear stress at junction of web and flange

$$\therefore q_1 = \frac{FA\bar{Y}}{IB} \qquad \left\{ \bar{Y} = h + \frac{t}{2} \right.$$

Shear stress is Increased upto q_2

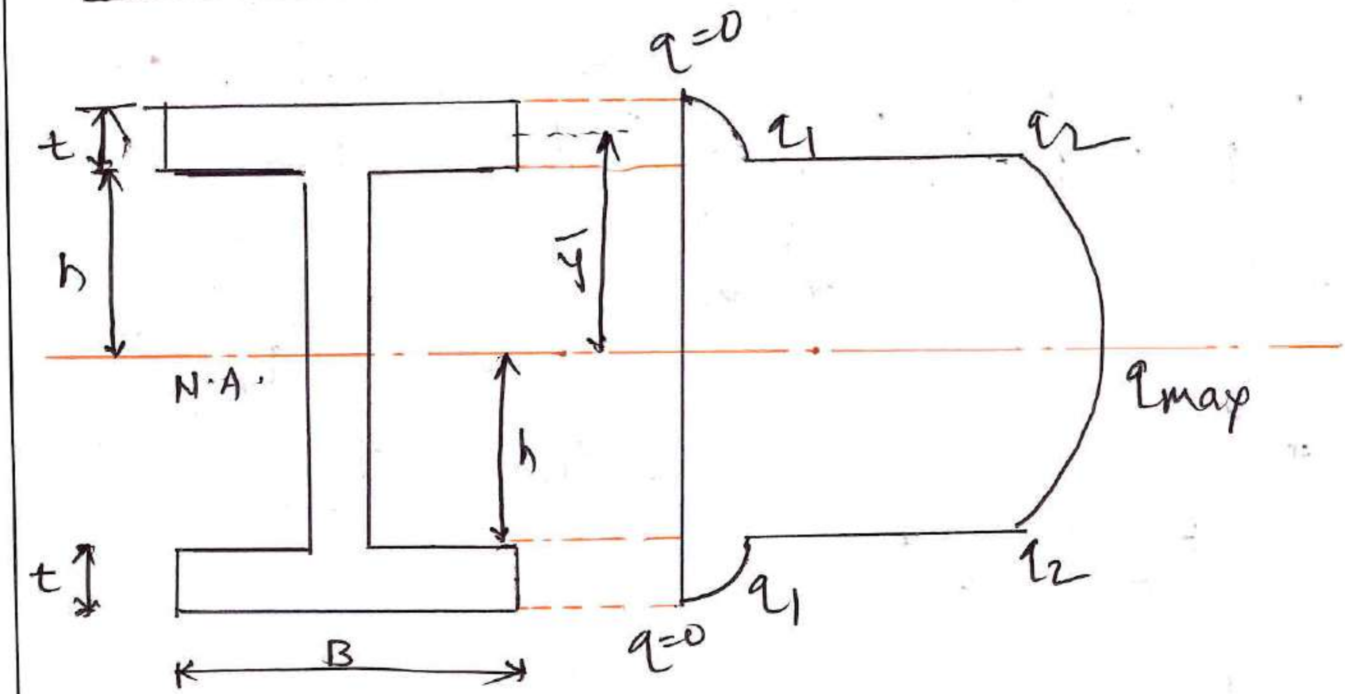
$$\therefore q_2 = \frac{B}{2b} \times q_1$$

$$\boxed{\therefore q_{max} = q_2 + q_{add}} \qquad \left\{ q_{add} = \frac{FA\bar{Y}}{Ib} \right.$$

$$q_{add} = \frac{FA\bar{y}}{Ib}$$

$$\left\{ \begin{array}{l} A = 2[b \times h] \\ \bar{y} = h/2 \text{ — from N.A.} \end{array} \right.$$

(c) I-section:—



- shear stress at top and bottom of flanges should be zero.

$$\therefore q_1 = \frac{FA\bar{y}}{IB}$$

Here, $\bar{y} = h + \frac{t}{2}$

$A = B \times t$

and shear stress $q_2 = \frac{B}{b} \times q_1$

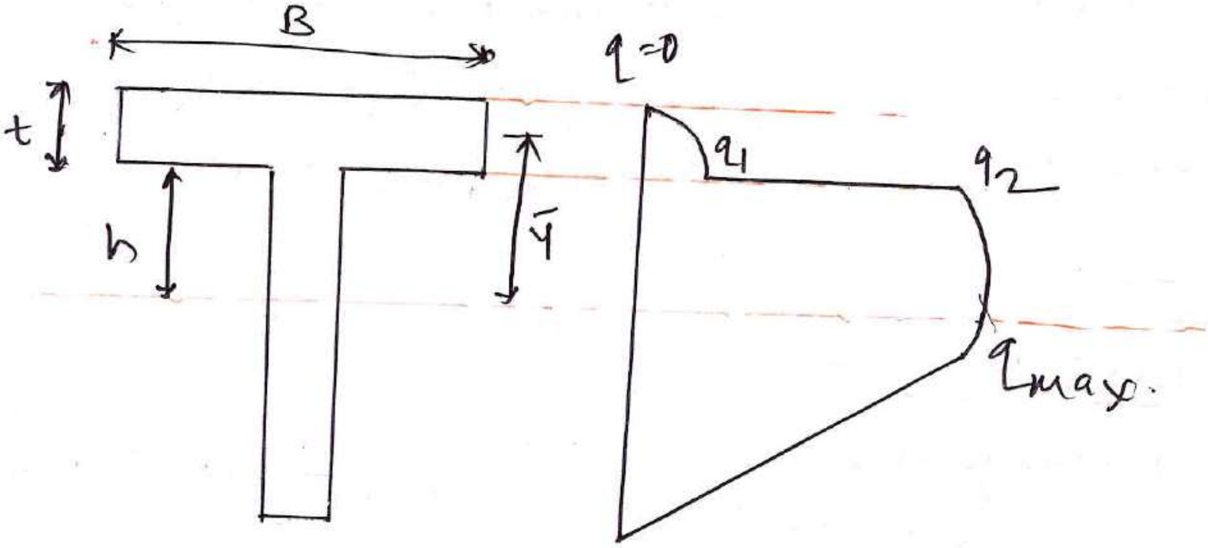
Now, maximum shear stress at N·A·

$$q_{max} = q_2 + q_{add}.$$

$\left\{ q_{add} = \frac{FA\bar{y}}{Ib} \right.$

$A = b \times h, \ \bar{y} = h/2$

T - section :-



WINTER-25

(1) Sketch shear stress distribution dia for Rectangular section beam of (600×200) mm subjected to shear force of 20 kN.

Ans:-

Given data:-

A Rectangular section = (600×200) mm.

shear force(F) = 20 kN = 20×10^3 N.

Sol^n -

For rectangular section shear stress at TOP and Bottom is zero.

i.e $q = 0$ N/mm^2

As we know that,

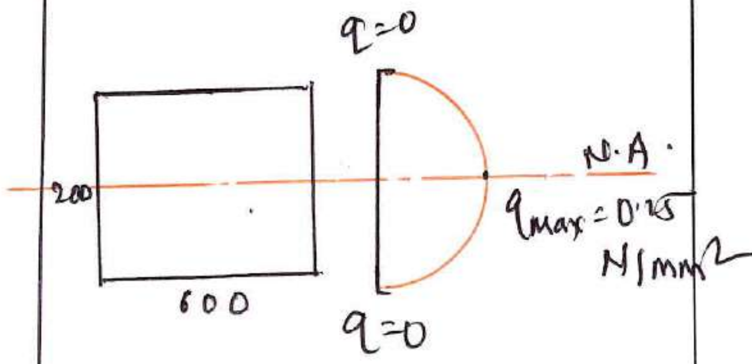
$$\boxed{q_{max} = \frac{3}{2} q_{avg}}$$

$$\therefore q_{avg} = F/A = \frac{20 \times 10^3}{(600 \times 200)}$$

$q_{avg} = 0.167$ N/mm^2

$$\therefore q_{max} = \frac{3}{2} \times 0.167$$

$$\boxed{q_{max} = 0.25 \text{ N/mm}^2}$$



Shear stress distribution dia.

WINTER -24

(2) Draw shear stress distribution along 4s of circular section beam for 300mm diameter carrying 400KN shear force also determine the ratio of maximum shear stress to avg. shear stress.

Ans:—

Given data! —

Beam is circular having dia = 300mm.

Shear force (F) = 400KN = 400×10^3 N.

Soln:-

As We know that, For circular section Relation betn q_{avg} and q_{max}.

i.e $q_{max} = \frac{4}{3} q_{avg}$.

$\therefore q_{avg} = \frac{F}{A}$.

Here, $A = \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times 300^2$

$= 70.68 \times 10^3 \ mm^2$

$\therefore q_{avg} = \frac{400 \times 10^3}{70.68 \times 10^3} = 5.65$

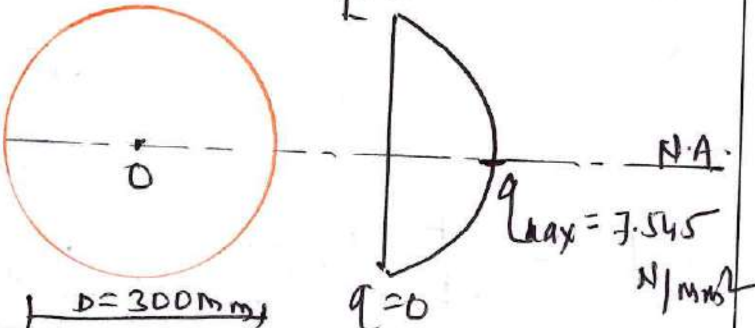
$\boxed{q_{avg} = 5.65 \ N/mm^2}$

i.e. $q_{max} = 4/3 \ q_{avg}$.

$= 4/3 \times 5.65$

$\boxed{q_{max} = 7.545 \ N/mm^2}$

$\therefore$ ratio of q_{max} to $q_{avg} = 1.33$



WINTER-24 (Imp)

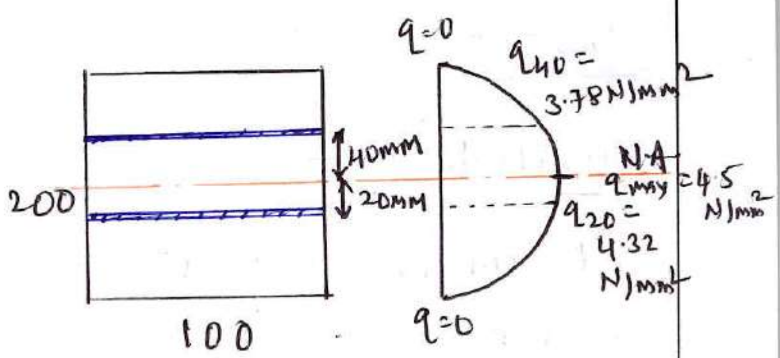
③ A beam section 100mm x 200mm is subjected to a shear force of 60KN, Determine the shear stresses induced on the layer at 40mm above N.A. and 20mm below N.A.

Ans:-

given data:-

Beam section = (100 x 200) mm

Shear force = 60KN = 60×10^3 N.



TO Find:-

i) shear stress @ 40mm above N.A.

ii) shear stress @ 20mm below N.A.

soln:-

we know that.

shear stress at top and Bottom = 0 N/mm²

i) Shear stress at 20mm above N.A.

$$\therefore q_{40} = \frac{6F}{bd^3}\left[\frac{d^2}{4} - y^2\right]$$

$$\therefore q_{40} = \frac{6 \times 60 \times 10^3}{100 \times 200^3}\left[\frac{200^2}{4} - 40^2\right]$$

$$\boxed{q_{40} = 3.78\ N/mm^2}$$

above N.A.

ii) shear stress at 20mm below N.A.

$$q_{20} = \frac{6 \times 60 \times 10^3}{100 \times 200^3}\left[\frac{200^2}{4} - 20^2\right]$$

$$\boxed{q_{20} = 4.32\ N/mm^2}$$

below N.A.

q_{max} = q at N.A. { y = 0

$$q_{max} = \frac{6 \times 60 \times 10^3}{100 \times 200^3}\left[\frac{200^2}{4}\right]$$

$$\boxed{q_{max} = 4.5\ N/mm^2}.$$

SUMMER-26

(4) A Hollow Rectangular section (200 x 400) mm externally with uniform thickness of 40 mm carries a shearing force of 100 kN at section, construct the shear stress distribution dia. giving all important values, Also calculate the ratio of max to avg. shear stress. 6M

Ans :-

Given data :-

Hollow Rectangular section (200x400) mm.

thickness 40 mm.

Shear force = 100 kN
$= 100 \times 10^3$ N

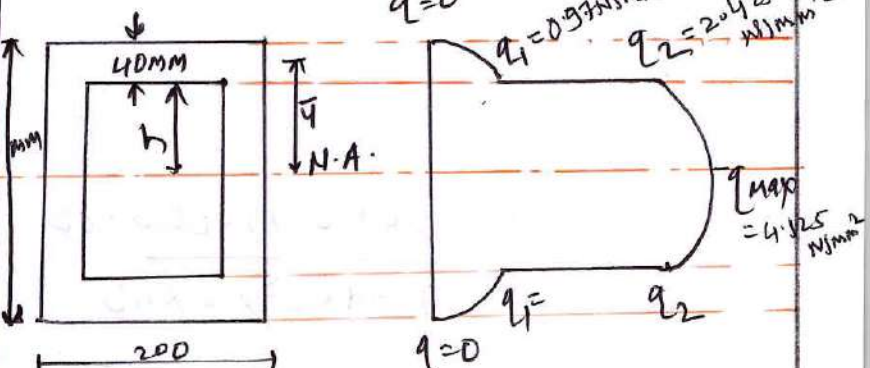
Soln

For hollow Rectangular section shear stress is zero @ top and bottom also, shear stress at junction of flange and web

$$q_1 = \frac{FA\bar{y}}{IB}$$

$\therefore A = 200 \times 40 = 8000\ mm^2$

$\bar{y} = 200 - 40/2 = 180\ mm$



$$\therefore I_{xx} = \frac{BD^3 - bd^3}{12} = \frac{(200 \times 400^3)}{12} - \frac{(120 \times 320^3)}{12}$$

$$= 1.067 \times 10^9 - 3.27 \times 10^8$$

$$I = 7.39 \times 10^8\ mm^4$$

$$q_1 = \frac{100 \times 10^3 \times 8000 \times 180}{7.39 \times 10^8 \times 200}$$

$$= 0.97\ N/mm^2$$

Now, to find q_2

$$\therefore q_2 = \frac{B}{2b} \times q_1 = \frac{200}{2 \times 40} \times 0.97$$

$$q_2 = 2.425\ N/mm^2$$

Now, to find q_{max}

$\therefore q_{max} = q_2 + q_{add}$ — (1)

$$q_{add} = \frac{FA\bar{y}}{IB} = \frac{FA\bar{y}}{I(2b)}$$

Here,
$A = 2 \times h \times t$
$= 2 \times 160 \times 40$
$= 12800 \text{ mm}^2$
$\bar{y} = h/2$
$= \frac{160}{2} = 80 \text{ mm}$

$$= \frac{100 \times 10^3 \times 12800 \times 80}{7.39 \times 10^8 \times 2 \times 40}$$

$q_{add} = 1.73 \text{ N/mm}^2$

$\therefore q_{max} = 2.425 + 1.73$

$$\boxed{q_{max} = 4.157 \text{ N/mm}^2}$$

$\therefore$ Ratio of q_{max} to q_{avg}.

But $q_{avg} = \frac{F}{A} =$

$A = BD - bd$
$= 41600 \text{ mm}^2$

$$= \frac{100 \times 10^3}{41600} = 2.403 \text{ N/mm}^2$$

$$\therefore \frac{q_{max}}{q_{avg}} = \frac{4.157}{2.403} = \underline{\underline{1.73}}$$

Unit - IV

Bending and Shear Stresses in beams

Question Bank

1. State any four assumptions in theory of pure bending
2. Define section modulus and neutral axis.
3. Define flexural rigidity.
4. Define pure bending.
5. State shear stress equation and meaning of each term used in it.
6. State shear stress equation and meaning of each term used in it.
7. Give the relation between average and maximum shear stress for rectangular and circular cross-section.
8. Draw shear stress and bending stress distribution diagram for hollow rectangular beam section
9. Sketch the shear distribution diagram for a rectangular beam of 600 × 200 mm (deep) subjected to a shear force of 20 kN.
10. A rectangular beam section 300 mm wide and 500 mm deep is simply supported over a span of 4 m. It carries a full span uniformly distributed load of 10 kN/m. Find the maximum bending stress induced in the section. Draw the bending stress distribution diagram.
11. A cantilever beam of rectangular metal cross section is 4m in length carries an UDL of 5 kN/m. If permissible bending stress in the material is 5 N/mm2, determine the size of the section. Assume depth to width ratio = 2
12. Draw shear stress distribution along cross section of circular beam for 300 mm diameter carrying 400 kN shear force. Also determine the ratio of maximum shear stress to average stress
13. A beam section 100 mm × 200 mm is subjected to a shear force of 60 kN. Determine the shear stresses induced on a layer at 40 mm above N.A. and 20 mm below the N.A
14. A hollow rectangular section, 200 mm x 400 mm externally with uniform thickness of 40 mm caries a shearing force of 100 kN at section. Consttuct the shear stress distribution diagram giving all important values. Also calculate the ratio of max. to average shear stress.
15. A cantilever beam of rectangular section support udl of 5 kN/m. The span of the beam is 3 m. If the maximum bending stress is 100 N/mm2 and depth of the beam is 1.5 times the width, Determine the size of beam.