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SHIRPUR EDUCATION SOCIETY

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The Shirpur Education Society's

R. C. Patel College of Engineering and Polytechnic, Shirpur

Department of Mechanical Engineering

NAME OF COURSE: - Strength of Materials

CODE OF COURSE: - 313308

SEMESTER: - SYME-3K

SUBJECT TEACHER: - Mr. Laxmikant Y.Borse

The Shirpur Education Society's

R. C. Patel College of Engineering and Polytechnic, Shirpur

QUESTION BANK

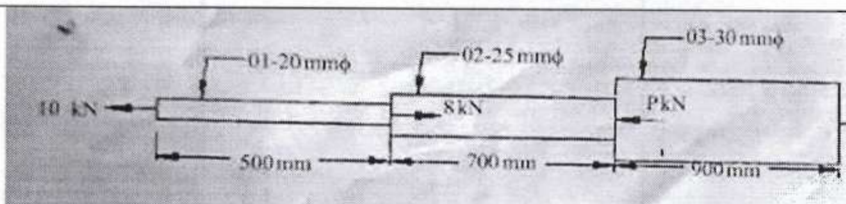
CHAPTER 2.Simple Stresses, Strains & Elastic Constants

Program Name: Mechanical Engineering **Program Code:** ME3K

Name of Subject & Code : Strength of Materials (313308) **Semester :** Third

Date & Time Slot: 20/05/2026

Q. NO.	QUESTION	DETAIL	MAPPING
1	Define Modulus of Elasticity & Modulus of Rigidity.S-26	S-26 Q1B 2M	CO 2.3 R
2	Define Lateral strain & linear Strain.S-26	S-26 Q1C 2M	CO 2.2 R
3	Define the term Rigid body and Plastic Body.S-26	S-26 Q2A 2M	CO 2.1 R
4	Differentiate between single shear and double shear.S-25	S-25 Q1B 2M	CO 2.5U
5	Define shear strain and modulus of elasticity.w-25	W-25 Q1B 2M	CO 1.1 R
6	State Hooke's Law.w-24	W-24 Q2B 2M	CO 1.2 R
7	Define the following terms. : i) Ultimate stress ii) Yield stress iii) Plastic strain iv) Factor of Safety.S-26	S26 Q2C 4M	CO 2.4R,
8	Draw stress-strain diagram with all important points on it for mild steel material subjected to gradually applied axial tensile load.S-25	S25 Q2C 4M	CO 2.4 U
9	For a certain material, modulus of elasticity is 169 MPa. If Poisson's ratio is 0.32 calculate the values of modulus of rigidity and bulk modulus.	W25 Q2B 4M	CO 2.3 A
10	A cube of 50 mm side is subjected to a force of 6 kN (Tensile), 8 kN (compressive) and 4 kN (Tensile) along X, Y, Z respectively. Determine change in volume. Take E = 200 GPa and m as 10/3. S-25	S&W25 Q3B 4M	CO 2.9 A
11	State the relation between E, G, K.W-24	W24 Q3A 4M	CO 2.3 U
12	A cube of 200 mm side is subjected to a compressive force of 3500 kN. on all its faces. The change in volume of the cube is 5000 mm3. Calculate the bulk modulus and modulus of elasticity if Poisson's ratio is 0.28.W-24	W24 Q3B 4M	CO 2.9 A
13	A brass bar having cross sectional area of 1000 mm2 is subjected to axial force as shown in Fig. No. 2 find the net deformation in the bar. Take $E = 1.05 \times 10^5$ N/mm^2.S-25	S25 Q3C 4M	CO 2.8 A
14	Draw stress-strain curve for ductile material and explain the following terms : i) Elastic limit ii) Upper yield point iii) Ultimate load point iv) Breaking load point. W-24&25*	W-24&25* Q3D 4M	CO 2.1 A
15	Determine the magnitude of 'P' for equilibrium and total elongation of the bar Shown in Fig. No.1 take E=210GPa. Also calculate minimum stress induced.S-26	S26 Q3B 4M	CO 1.4 A

	 01-20 mmφ 02-25 mmφ 03-30 mmφ 10 kN 8kN PkN 14 kN 500 mm 700 mm 900 mm Fig. No. 1		

* Simple stresses and strain and Elastic constant.

(1) Cond^n of equilibrium:-

(1) $\Sigma F_x = 0$ ie algebric sum of all force along x-direction must be zero

(2) $\Sigma F_y = 0$ ie —— " —— along y-direction " "

(3) $\Sigma M = 0$ ie algebric sum of moment any point in their plane must be equal to zero.

(2) $$\text{stress} = \frac{\text{Internal Resisting force}}{\text{Cross-sectional area}} = \frac{F \text{ or } P}{A}$$

$$\sigma = \text{sigma} = \frac{P}{A}$$

S.I. unit of stress (σ) is N/m^2 is called pascal & is denoted by (Pa)

$\therefore$ $1 Pa = 1 N/m^2$

|||ly

(1) $1 KPa = 10^3 Pa = 10^3 N/m^2$ ... kilo pascal

(2) $1 MPa = 10^6 Pa = 10^6 N/m^2 = \frac{10^6 N}{10^6 mm^2} = 1 N/mm^2$

(3) $1 GPa = 10^9 Pa = 10^9 N/m^2 = \frac{10^9 N}{10^6 mm^2} = 10^3 N/mm^2$

Note:-

i) K = kilo = 10^3, (ii) M = mega = 10^6, (iii) ~~G = Gg~~

iii) G = Gega = 10^9.

(3) Tensile stress & strain :- (+ve)

$\sigma_t = \frac{P}{A}$ & tensile strain $e_t = \frac{+\delta L}{L}$

(4) Compressive stress & strain :- (-ve)

$\sigma_c = \frac{P}{A}$ & $e_c = \frac{\delta L}{L}$

(5) shear stress (q or τ)

$$q = \frac{\text{Shearing force}}{\text{Shearing Area}} = \frac{P}{A}$$

i) for a single shear stress having circular rivet of diameter (d) is given by, without thickness considering

$$q = \frac{P}{A} = \frac{P}{\pi/4\, d^2}$$

ii) For Rectangular punch. having plate thickness (t)

$$q = \frac{P}{A} = \frac{P}{\text{perimeter of Rectangle} \times \text{thickness of Plate}}$$

$$= \frac{P}{2(a+b)t}$$

IIIy

iii) Double shear stress for circular punch or Rivet. without thickness of plate considering.

$$q = \frac{P}{2A} = \frac{P}{2(\pi/4\, d^2)}$$

iv) For Rectangular punch. having plate thickness (t)

$$q = \frac{P}{2A} = \frac{P}{2 \times 2(a+b) \times t}$$

$$\therefore q = \frac{P}{4(a+b)t}$$

(6) Punching stress:-

single shear stress having diameter of Rivet (d) & plate thickness (t) is given by

$$q \text{ or } \tau = \frac{P}{\text{circumference of circle} \times \text{thickness of Plate}}$$

$$\boxed{q = \frac{P}{(\pi d) \times t}}$$

original

(*) Hook's law or modulus of Elasticity :⇒ or Young's modulus

$$E = \frac{\sigma}{e}$$ ie Young's modulus = $\frac{\text{stress}}{\text{strain}}$.

Hook's law :-

$$\sigma \propto e$$

$$\therefore \sigma = \text{constant} \times e.$$

$$\therefore \frac{\sigma}{e} = \text{constant}$$

S.I. Unit of modulus of Elasticity is N/m^2 or Pascal

* Deformation or (change in length) ie (δL)

$$\delta L = \frac{PL}{AE} \quad \ldots$$

where, P = Applied force
L = original length
A = Cross-sectional area
E = modulus of Elasticity.

* Principle of superposition :⇒

Statement :- "when a number of forces are acting on a body, the resulting strain will be the algebraic sum of strains caused by individual forces"

ie. $\delta L = \delta L_1 + \delta L_2 + \delta L_3 + \cdots$

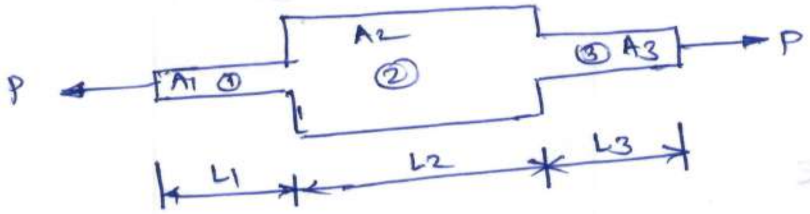
$$\delta L = \frac{P_1 L_1}{AE} + \frac{P_2 L_2}{AE} + \frac{P_3 L_3}{AE} + \cdots = \sum \frac{PL}{AE}$$

Note :-
The net deformation of the body ie $\sum \frac{PL}{AE}$ or (δL) may be "+ve" or "-ve".

1) δL = +ve indicate that the body elongates, i.e. Tensile force.

2) δL = -ve indicate that the body shortens or decreases i.e. compressive force.

* Deformation of a body of stepped cross-section due to axial load :=>



P = axial force on the bar

E_1, E_2, E_3 = modulus of elasticity of sections (1), (2) & (3) resp.

L_1, L_2, L_3 = Lengths of sections (1), (2) & (3) resp.

A_1, A_2, A_3 = cross-sectional areas of sections (1), (2) & (3) resp.

∴ change in length of section L_1

$$\delta L_1 = \frac{PL_1}{A_1E_1}$$

III^ly

$$\delta L_2 = \frac{PL_2}{A_2E_2} \quad \& \quad \delta L_3 = \frac{PL_3}{A_3E_3}$$

∴ Total change in length, $\delta L = \delta L_1 + \delta L_2 + \delta L_3$

$$\therefore \delta L = P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2} + \frac{L_3}{A_3E_3}\right]$$

Note:- If the bar is of same material throughout,

$$E_1 = E_2 = E_3,$$

then, $$\delta L = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

* Examples Based on stress-strain curve.

① ~~Nomin~~ Nominal Breaking stress = $\dfrac{\text{Load at breaking point}}{\text{original Area.}}$

② Actual Breaking stress = $\dfrac{\text{Load at Breaking point}}{\text{Reduced cross-section area at fracture.}}$

③ Yield stress = $\dfrac{\text{Yield Load}}{\text{original cross-section area.}}$

④ Ultimate stress = $\dfrac{\text{Maximum load}}{\text{original cross-sectional area.}}$

(5) Working stress = $\dfrac{\text{Actual load}}{\text{original cross-sectional area.}}$

(6) Factor of Safety = $\dfrac{\text{Ultimate Stress}}{\text{Working stress.}}$

(7) % reduction in Area = $\left(\dfrac{\text{original area - Final area}}{\text{original area}}\right) \times 100$

(8) % elongation = $\left(\dfrac{\text{Final length - Initial length}}{\text{Initial length}}\right) \times 100$

(*) Temperature stresses and strains in uniform Bar.

(1) Free to expand or contraction ie change in length of Bar because of temp. difference. is given by.

$\delta L = \alpha t L$

where,

δL = change in length in (mm) or Temp. deformation
α = coefficient of linear expansion in ($/ ^\circ C$).
t = temp. differnce betn two points in ($^\circ C$)
L = original length of the bar. in (mm)

(2) Temp. strain (e) = $\dfrac{\delta L}{L} = \dfrac{\alpha t L}{L} = \alpha t$.

$\therefore \boxed{e = \alpha t}$

(3) Temp. stress according to Hook's law. is given by.

$E = \dfrac{\text{stress}}{\text{strain}} = \dfrac{\sigma}{e}$

$\therefore \sigma = Ee = E\alpha t$.

$\boxed{\text{Temp stress } (\sigma) = E\alpha t}$

Unit No :- 03
mechanical properties & Elastic constant of metals.

(1) Linear strain or Longitudinal strain

$$e = \frac{\delta L}{L}$$

(2) Lateral strain = $\frac{\text{change in lateral dimension}}{\text{original lateral dimension}}$

i) For a rectangular bar :-

Lateral strain = $\frac{\delta b}{b}$ or $\frac{\delta t}{t}$

where,

δb = change in width, b = original width

δt = change in thickness, t = original thickness.

ii) For circular bar :-

Lateral strain = $\frac{\delta d}{d}$

where,

δd = change in diameter, d = original diameter.

(3) Poisson's Ratio := (μ or $\frac{1}{m}$)

$$\mu \text{ or } \frac{1}{m} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{\text{Lateral strain}}{e}$$

$$\therefore \boxed{\text{Lateral strain} = \mu e \text{ or } \frac{1}{m} \times e}$$

(4) Shear modulus or modulus of Rigidity (C, G or N)

modulus of Rigidity $(G) = \frac{\text{shear stress}}{\text{shear strain}} = \frac{q}{\phi}$

(5) Relation of three modli ie (E, G & K)

i) $E = 2G(1+\mu)$

ii) $E = 3K(1-2\mu)$

iii) $E = \frac{9KG}{G+3K}$

(6) Bulk modulus (K)

$$\therefore K = \frac{\text{Direct stress}}{\text{volumetric strain}}$$

$$\boxed{K = \frac{\sigma}{e_v}}$$

(7) **Volumetric strain :- (e_v)**

$$\therefore \boxed{e_v = \frac{\delta V}{V}}$$

e_v = Volumetric strain, δV = change in volume

V = original volume

or,

Volumetric strain is the algebric sum of all axial or linear strains.

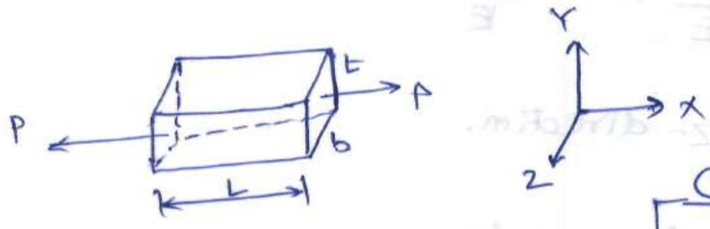
ie $\boxed{e_v = e_x + e_y + e_z}$

where,

e_x = strain in x-direction, & e_z = strain in z-direction

e_y = strain in Y-direction

(8) **Uni-axial Loading**



① strain in x-direction

$$\boxed{e_x = \frac{\sigma_x}{E}}$$

② strain in ~~Y~~ & Z direction

$$\boxed{e_y = e_z = -\mu \cdot \frac{\sigma_x}{E}}$$

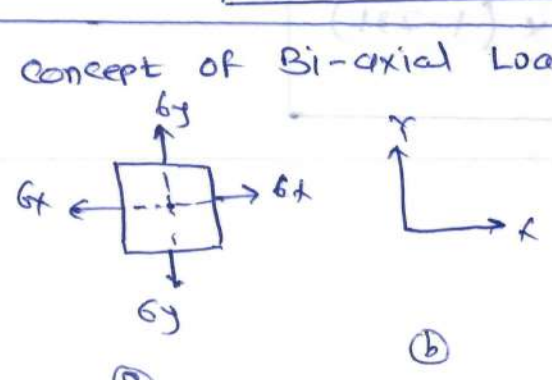
③ Volumetric strain of a rectangular bar,

$$\frac{\delta V}{V} = e_x + e_y + e_z = \frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_x}{E} = \frac{\sigma_x}{E}(1-2\mu)$$

but, $\frac{\sigma_x}{E} = e_x$ = linear strain, e

$$\therefore \boxed{\frac{\delta V}{V} = e(1-2\mu)}$$

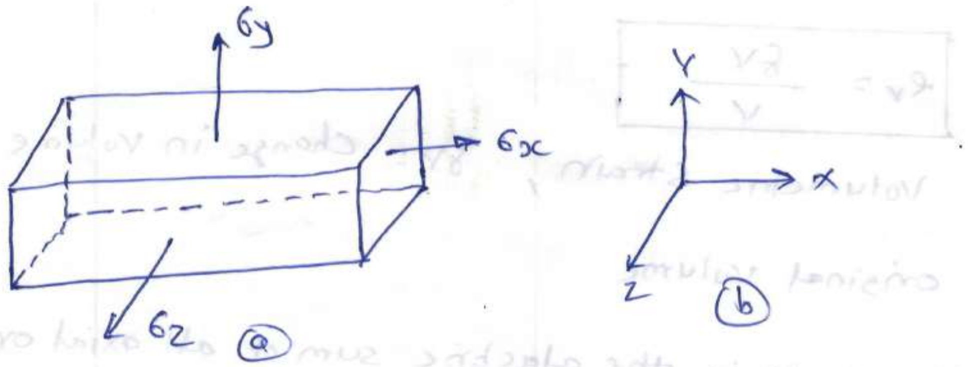
(9) **Concept of Bi-axial Loading.**



$$e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$$e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

(9) Concept of Tri-axial loading :-



(1) Total strain in x-direction.

$$e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E}$$

(2) Total strain in Y-direction:

$$e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E}$$

(3) Total strain in z-direction.

$$e_z = \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

(4) Volumetric strain for Tri-axial loading

$$\boxed{\frac{\delta V}{V} = e_x + e_y + e_z}$$

OR

$$\boxed{\frac{\delta V}{V} = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right) \times (1-2\mu)}$$

Simple stresses, strain & Elastic constant.

2.1 :- Concept of Elastic Body

Introduction :⇒ In any building or machine designing there are many members which are acted upon the different loads.

While designing a member, engineer should know the types of loading, type of material etc. for civil Engineering purpose.

Similarly for designing machine there are various components required which should designed practically to take loads in the consider conditions.

that's why we required some fundamental concepts like stress, strain there types are covered topics.

* Mechanical properties of metal :⇒

It is very important role while designing & manufacturing of components parts of the machine etc.

Mechanical properties

1) Elasticity 2) Plasticity 3) Ductility 4) Brittleness
5) Malleability 6) Fatigue 7) Creep 8) Toughness 9) Hardness.

1) Elasticity :⇒ It is a property of material due to which a material regain it's shape & size on removal of external load is called Elasticity.

e.g. All metals.

2) Plasticity :⇒ It is the property of material due to which material undergoes permanent deformation without failure or rupture on application of load.

(3) strength! - Ability to withstand an applied force without failure.

(4) Ductolity! - Ability to drawn into the thin wires. e.g. Copper, Aluminium, ms, etc.

(5) Malleability! - Ability to be hammered or rolled into thin sheet e.g. (gold, silver).

(6) Hardness! - Resistance to surface identation or scratching

(7) Toughness! - Ability to absorb energy before fracture e.g. shock, accident.

(8) Brittleness! - Tendancy to break or shatter without significant deformation.

(9) Fatigue Strength ! ⇒ Resistance to repeated loading cycle. e.g. machine parts like, axle, shaft springs etc.

(10) Creep Resistance! ⇒ Resistance to slow deformation over a time under constant stress.

* Types of Body :- 3

1) Elastic body 2) Plastic body 3) Rigid body

1) Elastic body :- when the external load is applied on a body or elastic materials it's dimensions may get changed but body regains it's original dimensions after removal of external load called elastic body

e.g. Aluminium, ms, Copper etc.

2) Plastic body ! - when a external load is applied on a body which get permanently, deformed and body will not regain it's original dimensions after removal of external load. called plastic body.

3. Rigid body :-

when a external load is applied to body it's does not undergo any change it's dimensions called Rigid body.

* Deformation :-

when an elastic body is subjected to external force so there will be some changes in size & shape ie change in dimension of the body is called as deformation.

(*) Types of loads :-

1) Direct load or Axial load ⇒ i) Tensile load
ii) compressive load.

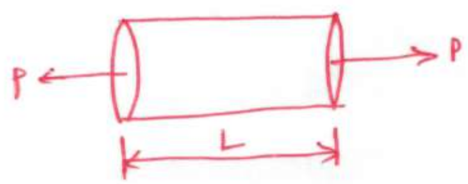
2) Eccentric load

3) Shear load

1) Direct load or Axial Load :-

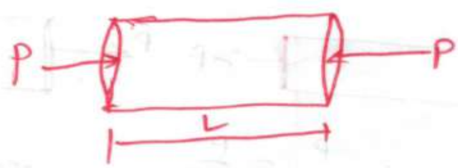
when a load is acting on the body whose line of action coincides with the axis of body is called Direct or Axial load.

i) Direct load may be tensile nature. ie pulling natures



where
P = Tensile load
L = length of the member

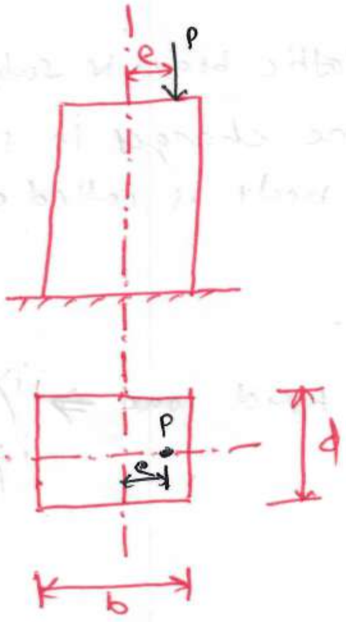
ii) Compressive nature ie pushing nature



P = compressive load
L = Length of the member

2) Eccentric Load :-

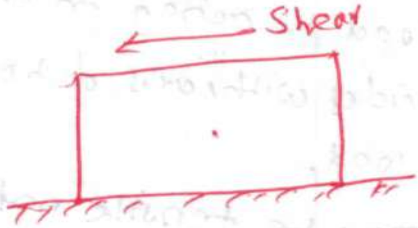
when a load is acting on the body whose line of action will not co-inside with the axis of the body, is called as eccentric load.



Note:- Eccentric load is acting at an eccentricity (e) from the axis of the member. as shown in fig.

3) Shear Load :-

A load which is an act tangential to the surface considered is called as shear load



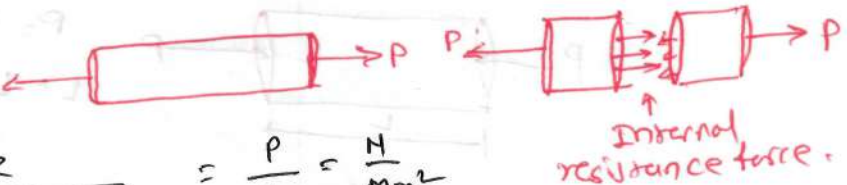
e.g.
1. Cutting on paper by using sizer
2. Cutting cake on nife
3. punching a hole by punching paper m/cs.
4. Nail cutter

Civil structure :-

2.1 :) Defn :-

(*) Stress (σ) :-

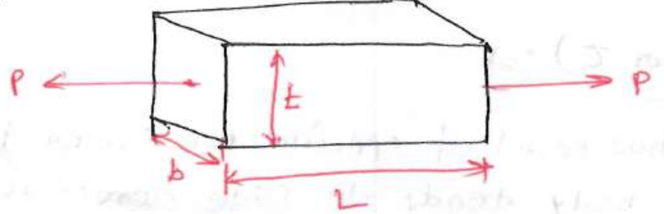
The internal resistance force offered by the body against the deformation per unit cross-sectional area is called stress.



$$\sigma = \frac{\text{Force}}{\text{Cross-sectional area}} = \frac{P}{A} = \frac{N}{mm^2}$$

It's S.I. unit is N/mm^2

i) Rectangular section -



Stress for Rectangular section $(\sigma)_{Rect} = \frac{P}{A} = \frac{P}{b \times t}$

ii) For circular section :- Stress $(\sigma) = \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2}$ -- { $A = \frac{\pi}{4} \times d^2$, d = dia. of circular section)

(*) Types of Stress

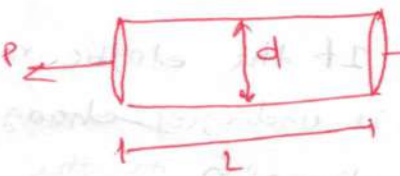
1) Direct / simple or Normal stress.

2) Indirect stress → i) Bending
ii) Torsion

3) Combined stress
ie direct + Bending
or
Normal/direct + Torsion.

(1) **Direct stress :-** The stress which will acts normal (ie ⊥er) to the plane on which the force acts axially is called normal or direct or simple stress.

i) **Tensile stress :-** when two equal & opposite pulls are applied on the body there the body is subjected to tension & due to which stress is developed is called tensile stress

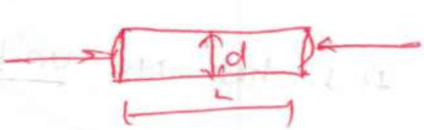


$(\sigma) = \frac{P}{A} = \frac{P}{\frac{\pi}{4} \times d^2}$

ii) **Compressive stress :-**

When two equal & opposite push are applied on the body & nature of stress produced due to push is called of compressive stress.

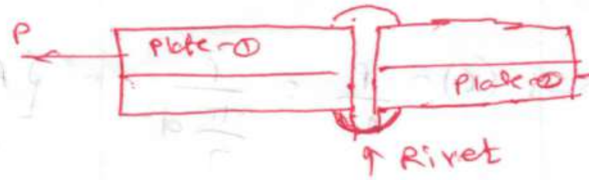
$(\sigma) = \frac{P}{A}$

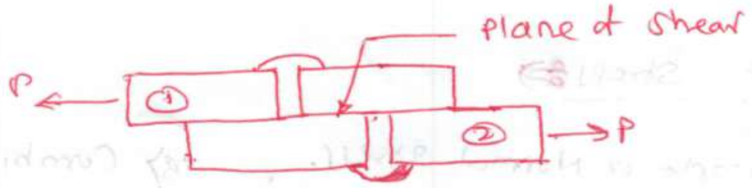


② Indirect stress :-

i) Shear stress (q or τ) :⇒

When two equal & opposite force acts tangentially on the body so body tends to slide across the section due to this stress is developed is called shear stress.





SI unit of shear stress is N/mm²

$$\therefore \text{shear stress } (q \text{ or } \tau) = \frac{\text{Shear force}}{\text{Cross-sectional Area}} = \frac{P}{A} = \frac{P}{\frac{\pi}{4}d^2}$$

where d = diameter of rivet.

② Bending stress :-

When the beam is subjected to external loading applied vertically, the resistance offered by the internal stresses to bending is called as bending stresses.

2.1) DEFN

✱ Strain (e) :- If the elastic material is subjected to an axial load it undergoes change in dimensions. This change in dimension to the original dimension is called as strain.

$$\text{Strain } (e) = \frac{\text{change in dimensions}}{\text{original dimensions}}$$

It's has no unit.

✱ Types of strain ⇒

1) Longitudinal or Linear strain → ① Tensile strain, ② Compressive strain
2) Lateral strain
3) Volumetric strain
4) Shear strain

1) Longitudinal strain or Linear strain:-

An elastic material subjected to load 'p' then there is change in length the direction of applied force. is called Linear / Longitudinal strain.

OR.

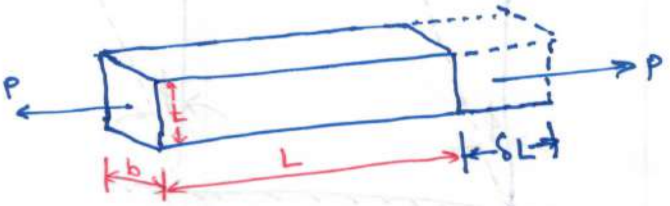
It is the ratio of change in length to it's original length is called as Longitudinal or linear strain.

$$\therefore \text{Longitudinal Strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\delta L}{L}$$

Where, L = original length
δL = Change in length

Note:- i) 'δL' will be increases the nature of applied load is tensile.

ii) δL will be decreses the nature of applied load is compressive.



(2) Lateral strain :-

When an elastic body is subjected to an axial load then there is change in ~~breadth~~ breadth & thickness. ie The change in lateral dimensions to the original dimensions is called as lateral strain.

$$\therefore \text{Lateral strain} = \frac{\text{Change in lateral dimensions}}{\text{original lateral dimensions}}$$

i) For Rectangular bar

$$\text{Lateral strain} = \frac{\delta b}{b} = \frac{\text{change in width or breadth}}{\text{original width or breadth}}$$

$$= \frac{\delta t}{t} = \frac{\text{change in thickness}}{\text{original thickness}}$$

ii) For circular bar, shaft,

Lateral dimension (d) $\therefore$ Lateral strain $= \frac{\delta d}{d} = \frac{\text{change in dia}}{\text{original dia}}$

3) Volumetric strain (e_v) :⇒

An elastic material undergoes some external loads applied on it's faces there will be change in it's volume per original volume is called as Volumetric strain.

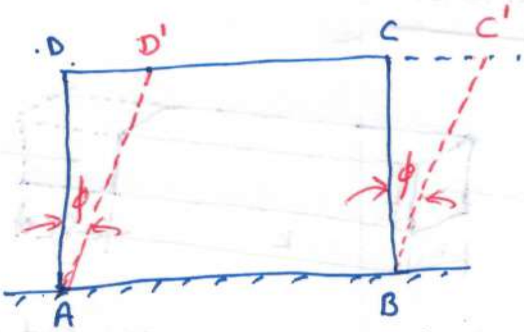
OR

It is the ratio of change in volume to the original volume is called volumetric strain.

$$\therefore \text{Volumetric strain } (e_v) = \frac{\text{change in volume}}{\text{original volume}} = \frac{\delta V}{V}$$

4) Shear strain (ϕ) :⇒

If we apply shearing load (P), a shear strain will be produced which is measured by the angle by which the body shifts.



Consider a rectangular block ABCD, AB is fixed & force 'P' is to be applied. After the application of force 'P' it dissons through an angle 'ϕ' and occupies new position $AD'C'B$.

∴ The shear strain is given by,

$$e_s = \tan\phi = \frac{DD'}{AD} \text{ or } \frac{CC'}{CB} \quad \dots \{\because \phi \text{ is very small}\}$$

2.2 Hooke's Law :⇒ Robert Hooke

It state that, within elastic limit, stress is directly proportional to strain.

mathematically, ∴ stress ∝ strain

$$\sigma \propto e$$

$$\therefore \frac{\sigma}{e} = \text{constant} = E$$

Where, E ⇒ Modulus OR Young's of Elastic modulus.

* Modulus of Elasticity or Young's modulus ("E")

Within elastic limit stress is directly proportional to strain

OR

It is the ratio of stress to the strain is constant. this constant is called as modulus of Elasticity or Young's Modulus.

$$\sigma \propto e$$

$$\therefore \quad E = \frac{\sigma}{e} = \frac{\text{stress}}{\text{strain}}$$

* Modulus of Rigidity (C, G or N)

It is the ratio of shear stress to shear strain is called as modulus of Rigidity.

It is denoted by, C, G or N

S.I. unit is N/mm^2

mathematically,

$$\underset{(G)}{\text{modulus of Rigidity}} = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\tau}{e\phi}$$

* Elastic limit :-

It is the maximum value of stress up to which the material behaves as completely elastic one.

:- Formulae :-

1. $\text{Stress } (\sigma) = \frac{P}{A} = \frac{\text{Force}}{\text{Area}} = \frac{N}{mm^2}$

2. $\text{strain } (e) = \frac{\text{change in dimension}}{\text{original dimension}}$

3. $\text{modulus of Elasticity } 'E' = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{e}$

4. $\text{Deformation } (\delta L) = \frac{PL}{AE} \text{ OR } \frac{\sigma L}{e} \quad \cdots \left\{ \sigma = \frac{P}{A} \right.$

Basic Conversion :

1) $1m = 1\times10^3\ mm$; $1m = 1\times10^2\ cm$

2) $1mm = 1\times10^{-3}\ m$; $1cm = 1\times10^{-2}\ m$

3) $1MPa = 1\ \frac{N}{mm^2}$

4) $1GPa = 1\times10^3\ \frac{N}{mm^2} = 1\ \frac{KN}{mm^2}$

5) $1KN = 1\times10^3\ N$

6) $1N = 1\times10^{-3}\ KN$

7) $1Pa = 1$ Pascal $= 1\ \frac{N}{m^2} = 1\times10^{-6}\ \frac{N}{mm^2}$ .... simplify. $\left\{ \frac{1\ N}{1\times(10^3)^2\ mm^2} = \frac{1}{10^6} = 1\times10^{-6}\ N/mm^2 \right.$

8) $1KPa = 1$ Kilo pascal $= 1\times10^3\ \frac{N}{m^2} = 1\times10^{-3}\ \frac{N}{mm^2}$

9) $1GPa = 1$ Giga pascal $= 1\times10^9\ \frac{N}{m^2} = \cancel{1\times10}\ 1\times10^3\ \frac{N}{mm^2}$

2.3

(*) Volumetric Strain (e_{vol})

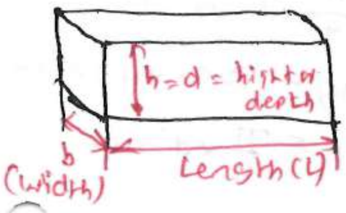
i) strain :- It is the ratio of change in ~~dim~~ dimension to original dimension.

ii) Volumetric strain :- Is the ratio of change in volume to the original volume

$$\therefore \text{ Volumetric strain } (e_{vol}) = \frac{\text{change in volume}}{\text{original volume}} = \frac{\delta V}{V}$$

where

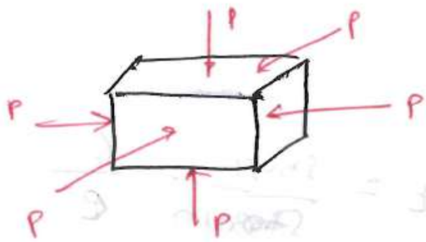
original volume $(V) = L \times b \times h$



(*) Bulk modulus (K) :- It is the ratio of direct stress to volumetric strain.

- It is measure of how resistance to compression that substance is.
- It's unit is N/mm^2

$$\therefore \text{ Bulk modulus } (K) = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{e_{vol}}$$



(*) Poisson's ratio (μ) or ($\frac{1}{m}$) :-

It is the ratio of lateral strain to linear strain is called as Poisson's ration.

$$\therefore \text{ Poisson's ratio } (\mu \text{ or } \tfrac{1}{m}) = \frac{-\text{Lateral strain}}{\text{linear strain}}$$

-ve ~~sign~~ sign indicates that lateral strain & linear strain occurs in opposite nature.

2.9
Volumetric strain due to uniaxial force or Uniaxial stress system

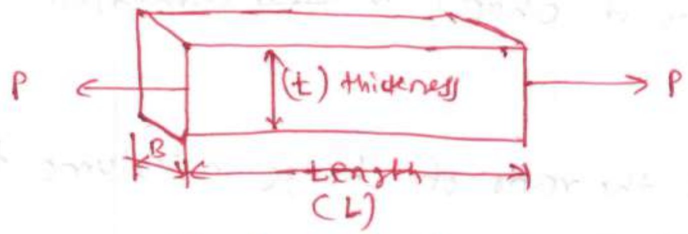


fig. for. (a) Rectangular section

$\therefore$ Area = $b \times t$

Volume = V = Area × Length

$V = b \times t \times L$

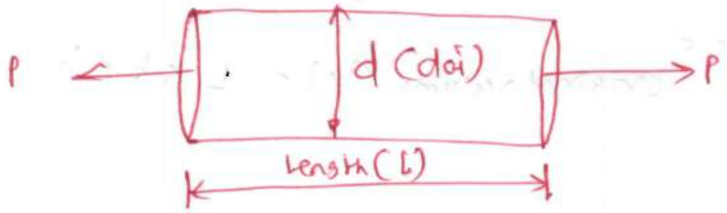


fig. (b) for Circular bar section.

Area $= \frac{\pi}{4} d^2$ or πr^2

Volume (V) = Area × Length

$= \frac{\pi}{4} \times d^2 \times L$

$\therefore$ stress in x-direction in both cases

$$\sigma_x = \frac{P}{A}$$

As there is no load in x & y direction

$$\therefore \sigma_y = 0 \ \& \ \sigma_z = 0 .$$

$\therefore$ strain in x-direction is,

$$E = \frac{\sigma_x}{e_x} \quad \ldots\ldots \left\{ \because E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{e} \right.$$

$$\therefore e_x = \frac{\sigma_x}{E}$$

lly strain in y & z direction must be lateral strain,

$e_y = e_z$ = lateral strain.

$$\therefore \mu = \frac{-e_y}{e_x}$$

$$e_y = -\mu e_x = -\mu \left(\frac{\sigma_x}{E}\right) \ldots\ldots \left\{ \because e_x = \frac{\sigma_x}{E} \right.$$

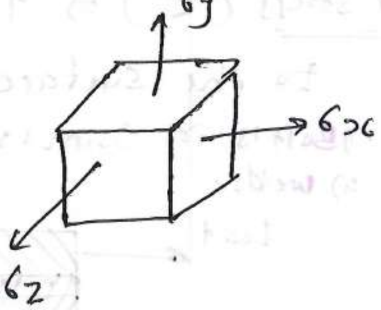
lly

$$e_z = -\mu e_x = -\mu \left(\frac{\sigma_x}{E}\right)$$

Now, volumetric strain $(e_v) = \frac{\delta V}{V}$ = $\frac{\text{change in volume}}{\text{original volume}}$

$$\therefore e_x + e_y + e_z = \frac{\delta V}{V} \longrightarrow (1)$$

2.3 (*) Relation Between modulus of Elasticity 'E' & Bulk Modulus "K". 2.3



⇒ Consider triaxial stress system

∴ Volumetric Strain (e_v)

$$e_v = \frac{\delta V}{V} = \left(\frac{\sigma_x+\sigma_y+\sigma_z}{E}\right) \times (1-2\mu)$$

consider $\sigma_x = \sigma_y = \sigma_z = \sigma$ = Equal stresses acts on a body

$$\therefore \; e_v = \frac{\delta V}{V} = \frac{\sigma+\sigma+\sigma}{E}(1-2\mu)$$

$$e_v = \frac{\delta V}{V} = \frac{3\sigma}{E}(1-2\mu)$$

We Know that,

$$\text{Bulk modulus } (K) = \frac{\text{direct stress } (\sigma)}{\text{volumetric strain } (e_v)}$$

$$\therefore \; K = \frac{\sigma}{e_v}$$

$$K = \frac{\cancel{\sigma}}{\frac{3\cancel{\sigma}}{E}(1-2\mu)} = \frac{E}{3(1-2\mu)}$$

$$\therefore \; \boxed{E = 3K(1-2\mu)}$$

OR

$$\boxed{E = 3K\left(1-2\frac{1}{m}\right)}$$

Where, $\frac{1}{m} = \mu$ = poisson's ratio (unitless quantity)

K = Bulk modulus (N/mm²)

E = — " — Elasticity (N/mm²)

(*) Shear stress :- when a body subjected to

OR,

It is defined as the internal resistance offered by a material to the deformation caused by a tangential (parallel) force. called shear stress.

2.5

OR.

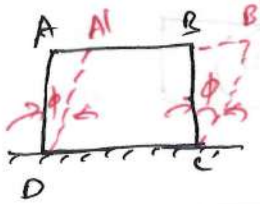
Shear stress (τ) ⇒ The force per unit area acting tangentially to the surface of a material.

e.g. 1) Bolts in joints 2) Scissors [cutting pa: papers] 3) Riveted joints 4) welds. 5) Cutting a cake. etc. 6) Punching machine. (drill the papers)

$\therefore$ Shear stress $(\tau) = \dfrac{\text{Tangential force}}{\text{Resisting cross-section area}} = \dfrac{P}{A}$

Unit is N/mm^2.

(*) **Shear strain (ϕ)** :- Shear strain is the change in angle (in radians) between two lines that were originally at right angles to each other in the material.



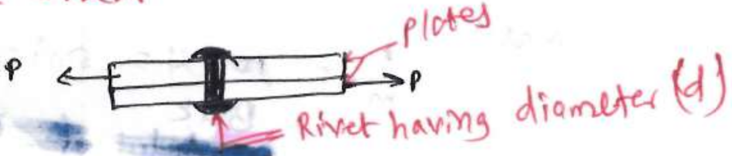
$\Delta DAA'$ or $\Delta CBB'$

$\tan\phi = \dfrac{AA'}{AD}$ or $\dfrac{BB'}{BC}$

$\therefore \phi = \tan^{-1}\left(\dfrac{AA'}{AD}\right)$ ---- { ϕ is very small angle

(*) **Single shear & Double shear**

i) **Single shear :- (τ or q)**



$\therefore$ Shear stress (q or τ) $= \dfrac{\text{Shear force}}{\text{Area.}}$

$= \dfrac{P}{A} = \dfrac{P}{\frac{\pi}{4}d^2}$

where, d = diameter of ~~[illegible]~~ rivet

∴ We know that,

$$\tau_{max} = \frac{P}{A}$$

$$\therefore P = \tau_{max} \times A \quad \text{------} \{ A = \pi d t$$

$$= 560 \times \pi \times 15 \times 30$$

$$\therefore \boxed{P = 791.68 \text{ kN}} \text{ ——— Ans — (1)}$$

Also,

$$\text{Compressive stress } (\sigma_{comp}) = \frac{P}{A} = \frac{791.68 \times 10^3}{\frac{\pi}{4} \times (d)^2}$$

---- { d = dia. of Punch.

$$\therefore \sigma_{comp} = \frac{791.68 \times 10^3}{\frac{\pi}{4} \times (15)^2} = 4.479 \text{ kN/mm}^2$$

$$\boxed{\begin{array}{c} \sigma_{comp} = 4.479 \times 10^3 \text{ N/mm}^2 \\ \text{or} \\ \sigma_{comp} = 4.479 \text{ kN/mm}^2 \end{array}} \text{ ——— Ans — (2)}$$

2.3 (*) Modulus of Rigidity OR Shear modulus (C, G or N)

Defn:- It is the ratio of shear stress to the shear strain called shear modulus or modulus of Rigidity.

- It is denoted by capital letter of C, G, or N.
- S.I. unit is N/mm^2

Mathematically,

$$\text{Modulus of } \cancel{\text{Rigidity}} \text{ Rigidity OR Shear modulus (C, G or N)} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

2.3 (*) Relation betn modulus of Elasticity (E) & modulus of Rigidity (G)

$$E = 2G(1 + \mu)$$

Where,

E = modulus of Elasticity in N/mm^2

G = —"— of Rigidity in —"—

μ = Poisson's ratio

Imp. 2.3

(*) Relationship between modulus of Elasticity (E), modulus of Rigidity (G) & Bulk modulus (K) ie E, G, K

We know that the relationship betn E & K

$$E = 3K(1-2\mu) \quad \text{--- (i)}$$

lly relation betn E & G is,

$$E = 2G(1+\mu) \quad \text{--- (ii)}$$

$$\therefore \frac{E}{2G} = 1+\mu$$

$$\therefore \mu = \frac{E}{2G} - 1$$

from eqn (1) becomes.

$$\therefore E = 3K\left[1 - 2\times\left(\frac{E}{2G} - 1\right)\right]$$

$$= 3K\left[1 - \frac{2E}{2G} + 2\right]$$

$$= 3K\left[1 - \frac{E}{G} + 2\right]$$

$$= 3K\left[3 - \frac{E}{G}\right]$$

$$E = 3K\left[\frac{3G - E}{G}\right]$$

$$\boxed{EG = 9KG - 3KE}$$

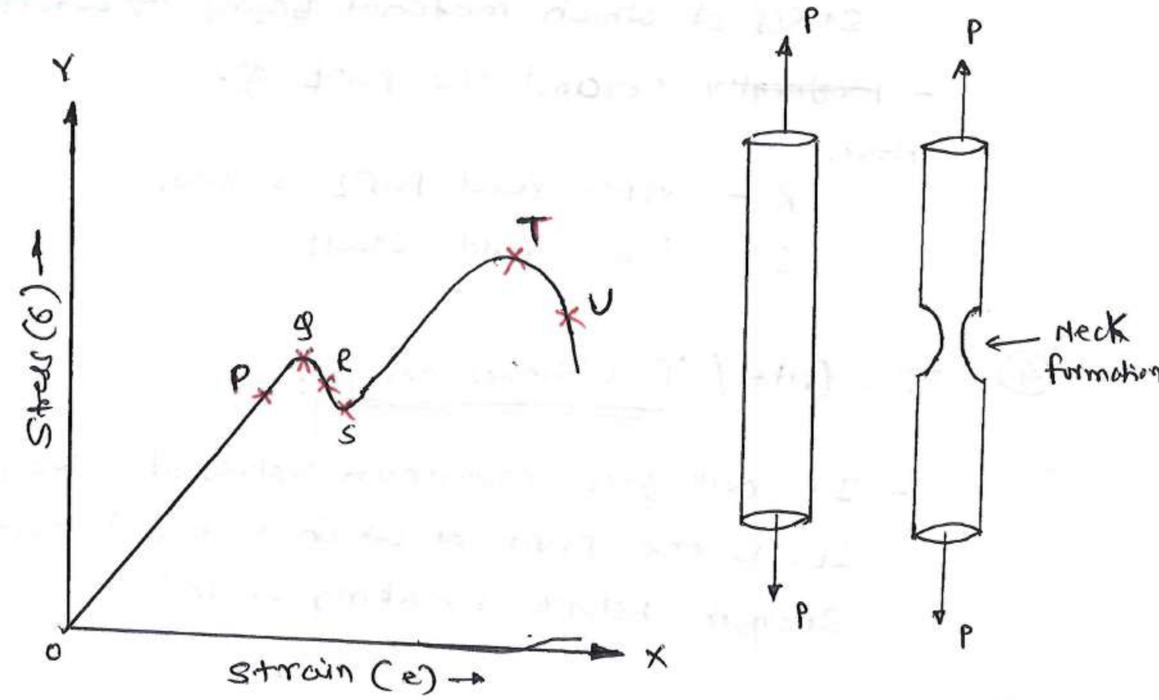
Where,

E = modulus of Elasticity

G = — " — of Rigidity

K = Bulk modulus.

2.4 Stress-strain Curve for mild steel and Tor Steel Bars Under Tensile Loading, and Behaviour of Material with respect to salient points on Graph.



Note:

Location of points.

P - Proportional limits.

Q - Elastic limit

R - Upper Yield point.

S - Lower Yield point

T - Ultimate load Point

U = Breaking point.

Here, OP - limit of Proportionality

PQ - limit of Elasticity.

① P- Proportional limit :-

- If tensile load is applied to mild steel it will have some deformation.
- If the force is in small amount gradually applied then the stress-strain graph will increases in straight line.

② Q - Elastic limit :-

If force increased then material experince elastic deformation upto point. Q.

③ R-S (Yield stress Points)

- The stress after which material deformation occures more quickly with no increase in load.
- Stress at which material begins to deform plastically.
- ~~plasticity~~ beyond the point Q.

Here,
R - Upper Yeild point or stress.
S - lower Yeild stress.

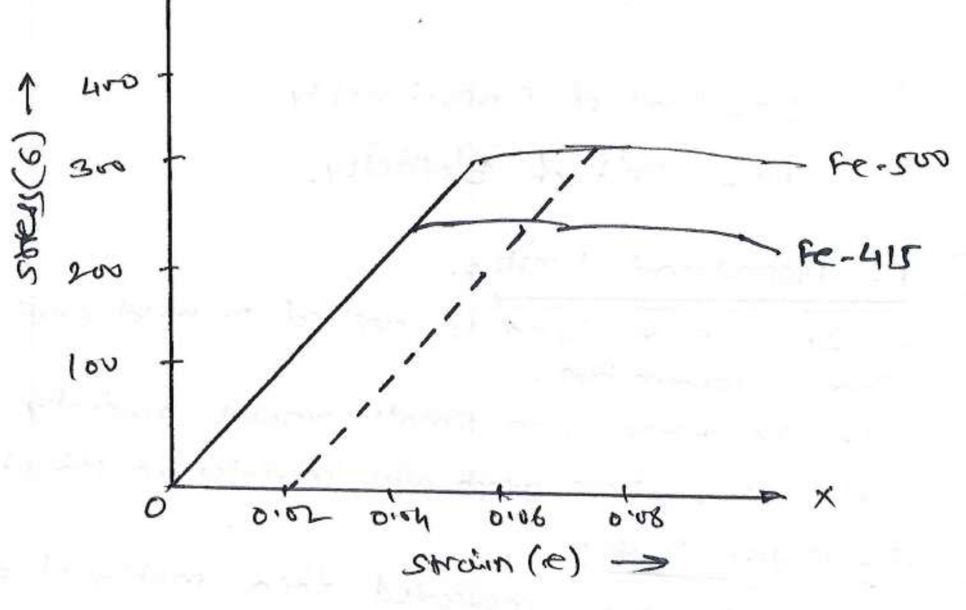
④ ~~T - Ult~~ / T - Ultimate stress :-

- It will give maximum values of stress.
- It is the point at which material gain maximum strength before breaking or failure.

⑤ U - Breaking stress :-

- After ultimate stress point material reach to neck formation at which cross-sectional area will be reduced.

(*) Stress-strain Curve for steel Bar under axial Load



HYSD :- High Yield strength deformed Bars.

- HYSD bar contain high % of carbon as compared to mild steel so the strength of HYSD bar is greater than mild steel.

- Their are two types of HYSD bars.
  1) Hot rolled high ~~Yield~~ Yield strength bars.
  2) cold twisted high yield strength bars. (Tor-steel)

  Fe-500 and Fe-415

Specification :-

- It has 50% more yield stress than mild steel.
- It has corrugations on surface of the bar to increases the bond & prevent sliping.
- This bar does not show any yield point.
- For HYSD bars yield stress point is considered as 0.02 ie 0.2% of proof stress.

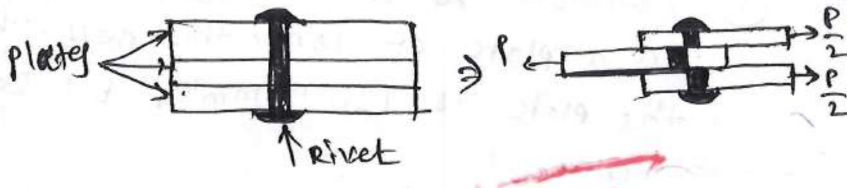
- The point where this lines cuts the stress-strain curve is taken as yield stress.

  ie - 0.2% of proof stress.

Defn :- Proof stress :-

For materials without a clear yield point (like Tor-steel) then the natures of stress is called proof stress.

(*) Double shear (τ or q)



$\therefore$ Double shear stress (q or τ) $= \frac{P}{2A} = \frac{P}{2 \times \frac{\pi}{4} \times d^2}$

where, 'd' = diameter of rivet

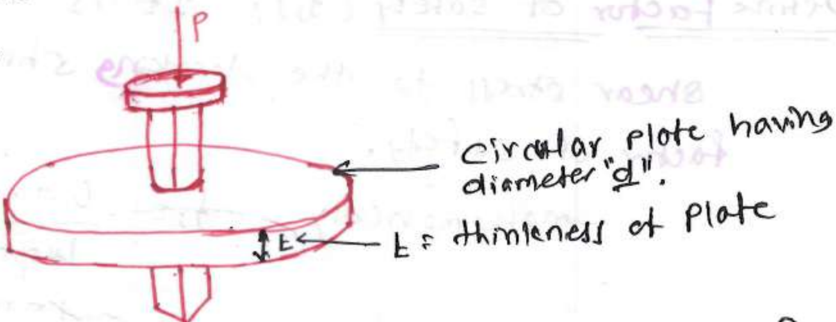
2.5

(*) Punching shear :-

It is defined as when plate faces some punch into a plate, then circumference of the punch faces the shear resistance from that plate is called Punching shear.

OR

It is a type of failure that happens when a heavy load pushes through a slab or footing around a column, causing a break or punch out around the column area, called punching shear.



Where,

P = compressive force across the hole. in Newton (N)

t = thickness of plate in (mm)

F = Force required to punch the hole

q or τ = shear stress (N/mm^2)

A = Area or Shearing Area of plate

= circumference of hole $\times$ thickness of plate.

$\therefore A = \pi d \times t$

(*) Shear stress due to punch (q or τ) :- The shear stress produced due to punching in a plate called as punching shear stress.

$\therefore$ Punching shear stress $= q = \tau = \frac{F}{A} = \frac{\text{shear force}}{\text{shearing Area of plate}}$

$\therefore F = \tau \times A$ or $q \times A = \underline{\underline{\pi d t \times q}}$

2.5 Type - (5) Numerical Based on Punching Shear :-

① Calculate the force required to punch a hole of 14mm diameter in a plate of 18mm thickness. The permissible shear stress for the plate is 130 N/mm², E = 200 GPa & $\frac{1}{m}$ or μ = 0.28.

→ Given

i) $d = 14$ mm

ii) $t = 18$ mm

iii) τ or $q = 130$ N/mm²

iv) E = 200 GPa = 200×10^3 N/mm²

v) μ or $\frac{1}{m} = 0.28$

To find :- ① F or P = ?

Soln :- shear stress τ or $q = \frac{F}{A}$

$\therefore$ $F = q \times A$ .... $\left\{ \begin{array}{l} A = \pi d t \\ = \pi \times 14 \times 18 \\ A = 791.68 \text{ mm}^2 \end{array} \right.$

$F = 130 \times 791.68$

$\boxed{F = 102.91 \text{ KN}}$

* **Define Factor of safety (f_s)** :- It is the ratio of Ultimate shear stress to the working shear stress is called factor of safety.

mathematically, $f_s = \frac{\text{Ultimate shear stress}}{\text{Working shear stress}}$
  (or Permissible shear stress)

② Calculate the force required to punch a hole of 15mm diameter in a metal plate of 30mm thickness. Permissible shear stress in shear is 280 N/mm² & factor of safety is 2. also calculate the compressive stress developed in the punching rod.

⇒ Given :-

i) $d = 15$ mm

ii) $t = 30$ mm

iii) τ or $q_{permissible} = 280$ N/mm²

iv) $f_s = 2$

To find

1) P = ?

2) $\sigma_{comp} = ?$

Soln :- ⇒ We know that.

Factor of safety (f_s) = $\frac{\text{Ultimate shear stress } (\tau_{max})}{\text{working or permissible shear stress } (\tau_w)}$

$2 = \frac{\tau_{max}}{280}$

$\boxed{\tau_{max} = 2 \times 280 = 560 \text{ N/mm}^2}$

2.1

① A metal rod 12 mm diameter & 2 m long when subjected to tensile force of 50 kN shown an elongation (δL) of 2 mm. calculate the modulus of Elasticity.

⇒ Given :- i) $d = 12\,mm$ 3) $P = 50\,kN = 50 \times 10^3\,N$
2) $L = 2\,m = 2 \times 10^3\,mm$ 4) $\delta L = 2\,mm$

To find :- i) modulus of Elasticity $(E) = ?$

Soln :-

Method - I

i) Stress $(\sigma) = \frac{P}{A} = \frac{50 \times 10^3}{\frac{\pi}{4} \times (d)^2}$

$\therefore A = \frac{\pi}{4} \times (12)^2 = 113.097\,mm^2$

$\sigma = \frac{50 \times 10^3}{113.097} = 442.096\,\frac{N}{mm^2}$

ii) Strain $(e) = \frac{\delta L}{L} = \frac{2}{2 \times 10^3}$

$e = 1 \times 10^{-3}$

We know that

$E = \frac{\sigma}{e} = \frac{442.096}{1 \times 10^{-3}}$

$\therefore \boxed{E = 442.096 \times 10^3\,N/mm^2}$

Method - II

We know that,

$\delta L = \frac{PL}{AE}$

$\therefore E = \frac{PL}{A \times \delta L} = \frac{50 \times 10^3 \times 2 \times 10^3}{113.097 \times 2}$

$\therefore \boxed{E = 442.098 \times 10^3\,\frac{N}{mm^2}}$

② A wooden tie 8 m, 100 mm wide & 140 mm thick is subjected to an axial pull of 60 kN & elongation is 3 mm. calculate modulus of Elasticity of the material.

Given :- i) $L = 8\,m = 8 \times 10^3\,mm$; 2) $b = 100\,mm$ 3) $t = 140\,mm$
4) $P_{tension} = 60\,kN = 60 \times 10^3\,N$
5) $\delta L = 3\,mm$

To find :- E = modulus of Elasticity = ?.

Sol^n :- $\delta L = \frac{PL}{AE}$ — ① $\Rightarrow E = \frac{PL}{A \delta L}$

$\therefore A_{rectangle} = b \times d = 100 \times 140 = 14000 \ mm^2$

from eqn ① becomes.

$\therefore E = \frac{60 \times 10^3 \times 8 \times 10^3}{14000 \times 3} = 11.4285 \times 10^3 \frac{N}{mm^2}$

$$\boxed{E = 11.428 \times 10^3 \frac{N}{mm^2}}$$

3) A steel rod 32 mm in diameter and 2 m long is subjected to an axial pull of 60 KN. Find
i) stress ii) strain iii) Elongation

Take E = 200 GPa

Given :- 1) $d = 32 mm$ 2) $L = 2 m = 2 \times 10^3 mm$

3) $P = 60 KN \Rightarrow 60 \times 10^3 N$

4) $E = 200 GPa = 200 \times 10^3 N/mm^2$

Sol^n :- i) stress $(\sigma) = \frac{P}{A}$ .... $\left\{ A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (32)^2 = 804.2496 \ mm^2 \right.$

$\therefore \sigma = \frac{60 \times 10^3}{804.2496} = 74.603 \frac{N}{mm^2}$ — ① Ans

ii) $e = \frac{\delta L}{L}$ — ①

we know that

$E = \frac{\sigma}{e} \Rightarrow e = \frac{\sigma}{E} = \frac{74.603}{200 \times 10^3} = 0.373 \times 10^{-3}$

$\therefore \boxed{e = 0.373 \times 10^{-3}}$ ← ② Ans

iii) from eqn ① becomes. $e = \frac{\delta L}{L} \therefore \delta L = e \times L = 0.373 \times 10^{-3} \times 2 \times 10^3$

$\boxed{\delta L = 0.74603}$ — ③ Ans

(4) A load of 6 KN is to be raised with the help of steel cable. Find the minimum diameter of steel cable if stress is not exceed 110 N/mm².

Given :- 1) $P = 6\ kN = 6 \times 10^3\ N$

2) $\sigma = 110\ N/mm^2$

To find :- ① diameter (d) = ?

Soln :- We know that,

$$\text{stress } (\sigma) = \frac{P}{A}$$

$$\therefore A = \frac{P}{\sigma} = \frac{6 \times 10^3}{110} = \underline{\underline{54.54}}\ mm^2$$

$$\therefore A = \frac{\pi}{4} \times d^2$$

$$\frac{54.54 \times 4}{\pi} = d^2$$

$$d^2 = 69.4424$$

$$\therefore \boxed{d = 8.332\ mm} \longrightarrow \text{Ans}$$

Practice for Student as an Homework or Assignment

Topic - I

① A bar of cross-sectional are 150 mm² is axially pulled by a force 'P' kN. If the maximum stress induced in the bar is 25 MPa, determine 'P'. If the elongation of 1.2 mm is observed over a gauge length 3 m. determine Young's modulus.

② A mild steel flat 95 mm wide & 11 mm thick & 4 m long carries an axial load of 20 kN. Find stress, strain & elongation. $E = 2.1 \times 10^5$ MPa

2.6

(3) Determine the tensile force on a steel bar of circular cross-section 35 mm diameter, if strain equal to 0.95×10^{-3}. Consider E for steel = 205 GPa.

* Deformation of Body OR Bars of stepped cross-section subjected to Axial Load

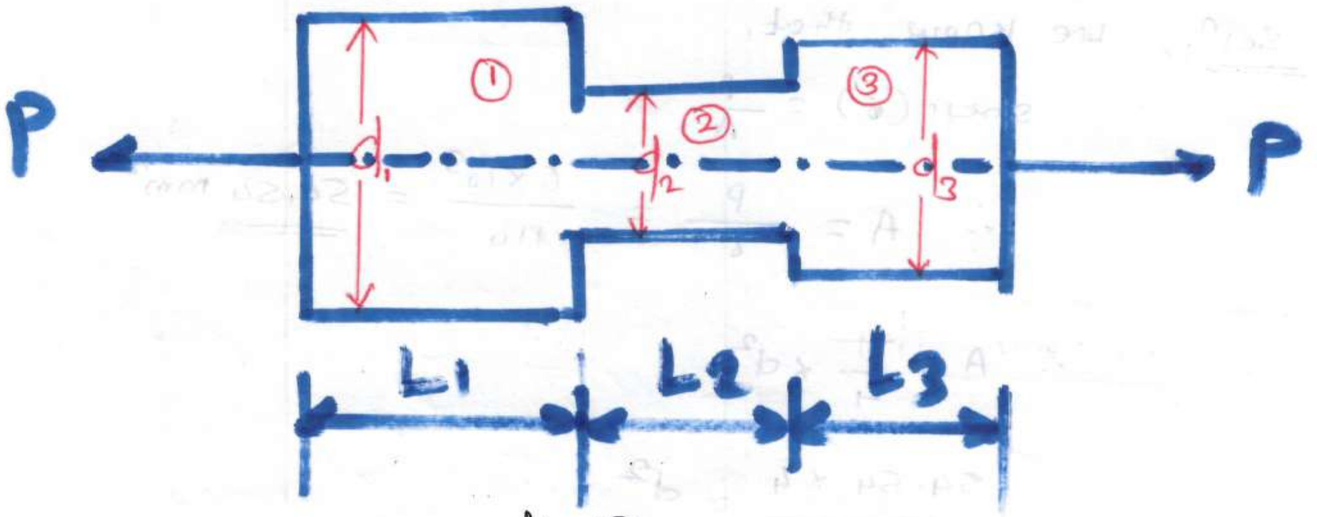


fig. (a)

fig. (a) shows a metal bar of varying section 1, 2 & 3 subjected to pull P.

let A_1, A_2, A_3 = Area of section 1, 2, 3

L_1, L_2, L_3 = Length of part 1, 2, & 3

∴ Total elongation $(\delta L)_{Total} = \delta L_1 + \delta L_2 + \delta L_3$

where, $\delta L_1, \delta L_2$ & δL_3 = change in length of part 1, 2 & 3 resp.

$$\therefore (\delta L)_{Total} = \left(\frac{PL}{AE}\right)_1 + \left(\frac{PL}{AE}\right)_2 + \left(\frac{PL}{AE}\right)_3$$

$$\therefore = P\left[\left(\frac{L}{AE}\right)_1 + \left(\frac{L}{AE}\right)_2 + \left(\frac{L}{AE}\right)_3\right] \cdots \{ \because \text{P is same for all part, 1, 2, 3}$$

* Concept of maximum & minimum stress induced in the section, using fig. (A).

fig (A) indicates that the stepped section subjected to axial Load 'P'.

∴ maximum stress induced in section (2) because it has minimum diameter ∴ ie minimum area ∴ $(\sigma_2)_{max} = \frac{P}{A}$

Note! Stress is always maximum where the cross-section Area minimum.

2.6 Numerical type-II Bar subjected to stepped cross-section Area

① A mild steel stepped bar 5m length is 20mm in diameter of 2m length and 15 mm diameter for remaining length. It is subjected to tensile load of 65 kN. Calculate the change in length if its modulus of Elasticity is 200 kN/mm².

⇒ Given:- Total length $(L) = 5m$

section (I) i) $L_1 = 2m = 2\times10^3$ mm
$d_1 = 20mm$

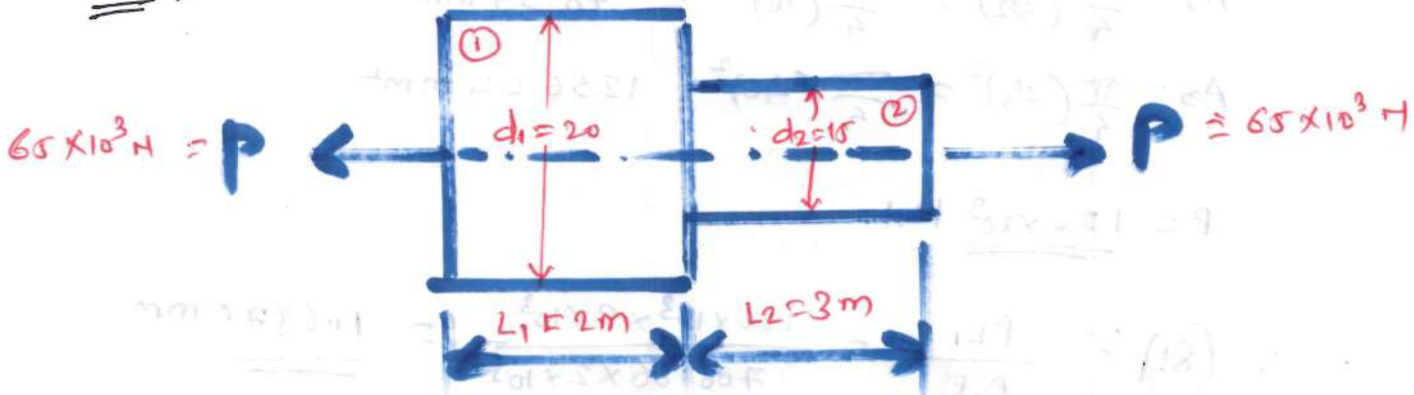
section (II) i) $L_2 = 3m = 3\times10^3$ mm
ii) $d_2 = 15mm$

$$P_{tensile} = 65\,kN = 65\times10^3\,N$$

$$E = 200\times10^3\,\frac{N}{mm^2}$$

To find :- ① change in length ie $(\delta L)_{Total} = ?$

Soln :-



$$\therefore (\delta L)_{Total} = \delta L_1 + \delta L_2 \quad ----- ①$$

$$\therefore \delta L_1 = \frac{P_1 L_1}{A_1 E} = \frac{65\times10^3}{\frac{\pi}{4}(20)^2\times200\times10^3} = \underline{0.2069\,mm}$$

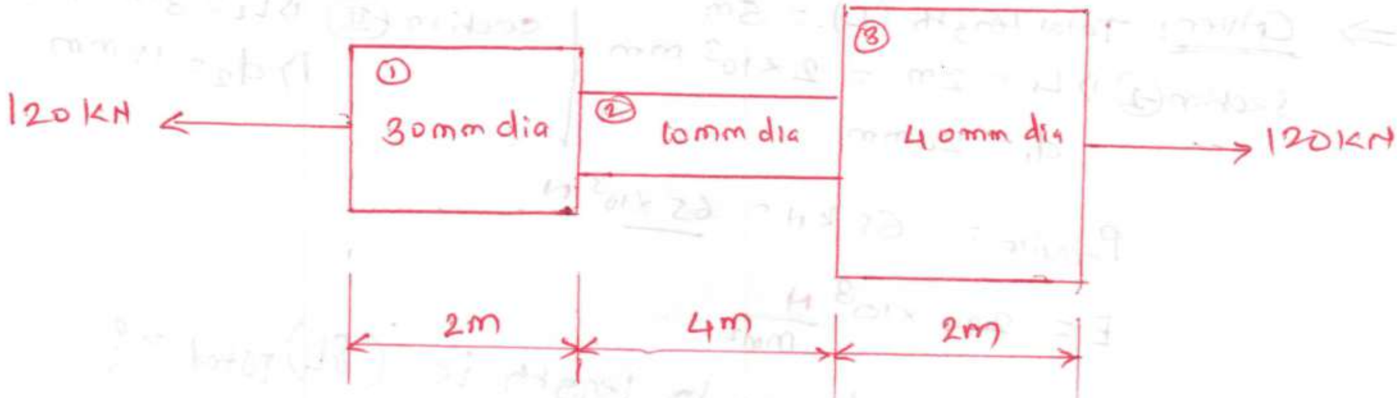
$$\delta L_2 = \frac{P L_2}{A_2 E} = \frac{65\times10^3}{\frac{\pi}{4}(15)^2\times200\times10^3} = \underline{0.3678\,mm}$$

from eqn ① becomes,

$$\therefore (\delta L)_{Total} = 0.2069 + 0.3678 = 0.5747\,mm$$

$$\boxed{(\delta L)_T = 0.5747\,mm}$$

② A bar having cross-section as given in fig. ⓐ subjected to a tensile load 120 KN. Calculate the change in length of each part along with the total change in length, if $E = 2\times10^5\ N/mm^2$.



Soln.

$$A_1 = \frac{\pi}{4}(d_1)^2 = \frac{\pi}{4}(30)^2 = 706.86\ mm^2$$

$$A_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4}(10)^2 = 78.54\ mm^2$$

$$A_3 = \frac{\pi}{4}(d_3)^2 = \frac{\pi}{4}(40)^2 = 1256.64\ mm^2$$

$$P = 120\times10^3\ KN$$

$$\therefore (\delta L)_1 = \frac{PL_1}{A_1E} = \frac{120\times10^3\times2\times10^3}{706.86\times2\times10^5} = 1.6976\ mm$$

$$(\delta L)_2 = \frac{PL_2}{A_2E} = \frac{120\times10^3\times4\times10^3}{78.54\times2\times10^5} = 30.55\ mm$$

$$(\delta L_3) = \frac{PL_3}{A_3E} = \frac{120\times10^3\times2\times10^3}{1256.64\times2\times10^5} = 0.9549\ mm$$

$$\therefore (\delta L)_{total} = \delta L_1 + \delta L_2 + \delta L_3$$

$$= 1.6976 + 30.55 + 0.9549$$

$$\boxed{(\delta L)_{total} = 33.2025\ mm}$$

2.6

Q.10) A steel bar $800\,mm^2$ cross-sectional area is subjected to axial forces as shown in fig. No (2), find total change in length of bar. If $E = 2 \times 10^5\ N/mm^2$.

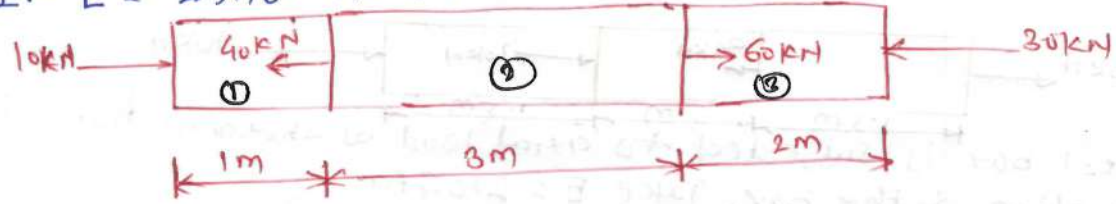


fig. No (2)

Given :- 1) $A = 800\,mm^2$

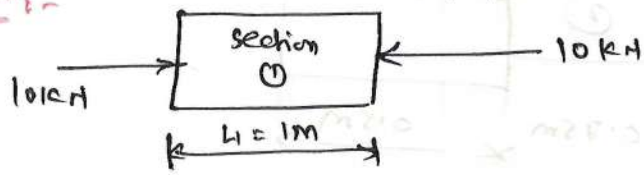
2) $E = 2 \times 10^5\ N/mm^2$

To find

i) change in length $(\delta L)_{total} = ?$

Soln :-

$P = -40 + 60 - 30 = -10\,kN$ (Compressive)



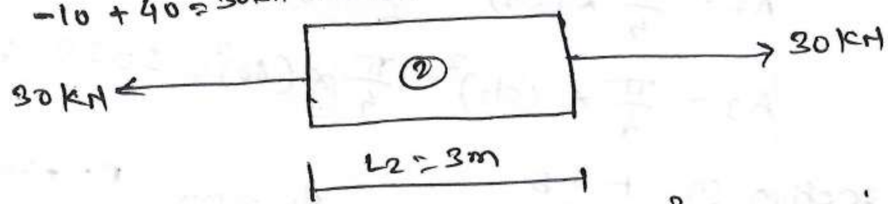
(1.) $\delta L_1 = \dfrac{P_1 L_1}{AE} = \dfrac{10 \times 10^3 \times 1 \times 10^3}{800 \times 2 \times 10^5} = 0.0625\ mm$

$\therefore \boxed{\delta L_1 = 0.0625\ mm}$ ⟵ Compressive ~~stress~~ (decreases) length.

Section (2) FBD

$P = 60 - 30 = 30\,kN$ (Tensile)

$P = -10 + 40 = 30\,kN$ (Tensile)

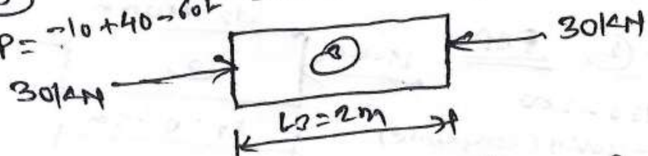


$\therefore \delta L_2 = \dfrac{P_2 L_2}{AE} = \dfrac{30 \times 10^3 \times 3 \times 10^3}{800 \times 2 \times 10^5} = 0.5625\ mm$

$\boxed{\delta L_2 = 0.5625\ mm}$ —— Tensile. (Increases) length

Section (3) FBD

$P = -10 + 40 - 60 = -30\,kN$ (Compressive)



$\therefore \delta L_3 = \dfrac{P_3 L_3}{AE} = \dfrac{30 \times 10^3 \times 2 \times 10^3}{800 \times 2 \times 10^5} = 0.375\ mm$

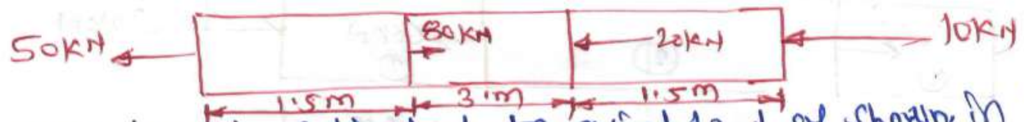
$\boxed{\delta L_3 = 0.375\ mm}$ —— Compressive (decreases length)

$\therefore (\delta L)_{Total} = -\delta L_1 + \delta L_2 - \delta L_3 = -0.0625 + 0.5625 - 0.375$

$\boxed{(\delta L)_{Total} = 0.125\ mm}$ —— Final Ans

2.6 H.W

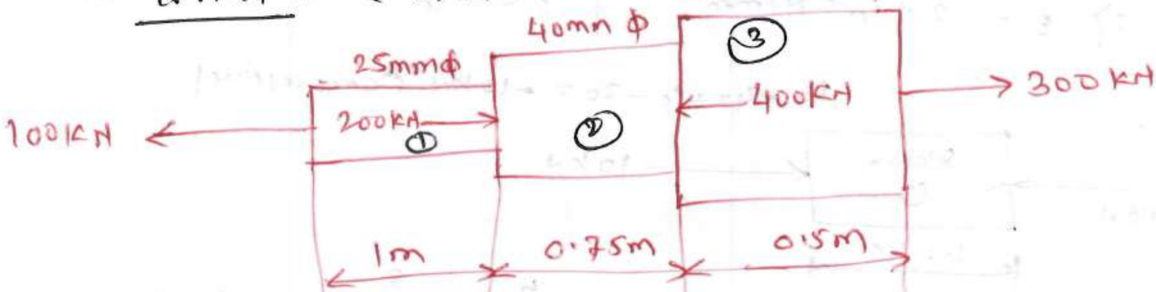
Q.1) Ans :- A steel bar 800 mm^2 cross-sectional area is subjected to axial forces as shown in fig. Find change in length of bar if $E = 2 \times 10^5 \frac{N}{mm^2}$



Q.) A steel bar is subjected to axial load as shown in fig. calculate deformation of the bar. Take E = 210 GPa.

Q.1) Given :- i) $E = 210 \text{ GPa} = 210 \times 10^3 \frac{N}{mm^2}$

To find :- $(\delta L)_{total} = ?$



Soln :- i) $d_1 = 25$ mm ; $L_1 = 1 m = 1 \times 10^3$ mm

ii) $d_2 = 40$ mm ; $L_2 = 0.75 m = 0.75 \times 10^3$ mm

iii) $d_3 = 60$ mm ; $L_3 = 0.5 m = 0.5 \times 10^3$ mm

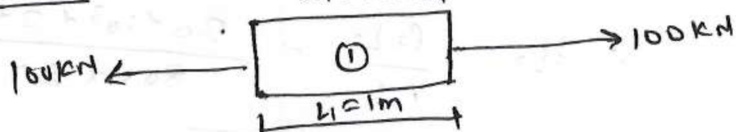
$$\therefore A_1 = \frac{\pi}{4} \times (d_1)^2 = \frac{\pi}{4} \times (25)^2 = 490.875 \text{ mm}^2$$

$$A_2 = \frac{\pi}{4} \times (d_2)^2 = \frac{\pi}{4} \times (40)^2 = 1256.64 \text{ mm}^2$$

$$A_3 = \frac{\pi}{4} \times (d_3)^2 = \frac{\pi}{4} \times (60)^2 = 2827.44 \text{ mm}^2$$

* section ① F.B.D

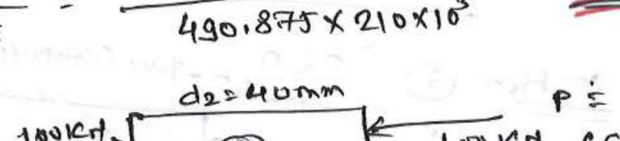
$P = +200 - 400 + 300 = 100$ KN (tensile)



$$\delta L_1 = \frac{P_1 L_1}{A_1 E} = \frac{100 \times 10^3 \times 1 \times 10^3}{490.875 \times 210 \times 10^3} = 0.970 \text{ mm}$$ (Tensile Increase in length)

* section ② FBD

$P = -400 + 300 = 100$ KN (compressive)



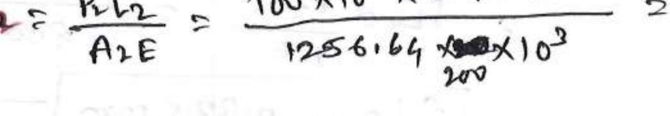
$P = 100 - 200$
$= -100$ KN (compressive)

$$\therefore \delta L_2 = \frac{P_2 L_2}{A_2 E} = \frac{100 \times 10^3 \times 0.75 \times 10^3}{1256.64 \times \cancel{210} \times 10^3} = 0.2984 \text{ mm}$$ (Compressive decrease)

200

* section ③ FBD

$P = 100 - 200 + 400 =$
$= 300$ KN (Tensile)



$$\therefore \delta L_3 = \frac{P_3 L_3}{A_3 E} = \frac{300 \times 10^3 \times 0.5 \times 10^3}{2827.44 \times 200 \times 10^3}$$

$\delta L_3 = 0.2652$ mm (Increase length)

$\therefore (\delta L)_{total} = 0.970 - 0.2984 + 0.2652$

$\delta L_T = 0.9368$ mm — Final Ans

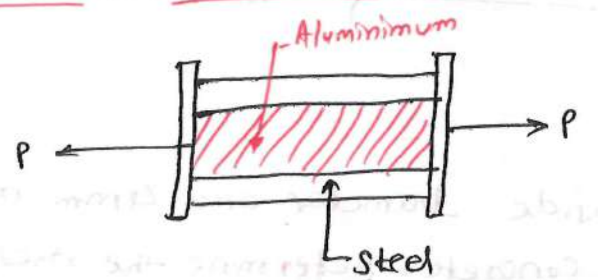
(*) Composite section :

A structural member composed of two or more dissimilar materials joined together to act as a unit is called composite section. e.g. concrete & steel, Cu & Aluminimum etc.

(*) Modular ratio : (m)

It is the ratio of Young's modulus of elasticity of two different materials $m = \frac{E_1}{E_2}$

(*) Stresses in Composite Section :



Composite section made up of Steel & Aluminium.

Let

P = Axial load applied

P_s = Load taken by steel

P_{Al} = — " — by Aluminimum

$\therefore \quad P = P_s + P_{Al}$

We know that

$$\sigma = \frac{P}{A} \quad \therefore P = \sigma A$$

$$P = \sigma_s A_s + \sigma_{Al} \cdot A_{Al}$$

where, σ_s = stress in steel & A_s = Area in steel rod,

σ_{AL} = stress in Aluminimum & A_{Al} = — " — Aluminimum rod

As both ends of rods are fixed elongation of two rods will be same

$$\therefore \delta L_{Al} = \delta L_s$$

$$\left(\frac{PL}{AE}\right)_{Al} = \left(\frac{PL}{AE}\right)_s \quad \because \left\{ \delta L = \frac{PL}{AE} \right.$$

$$\left(\frac{P}{A}\right)_{AL} \cdot \left[\frac{L_{Al}}{E_{Al}}\right] = \left(\frac{P}{A}\right)\left(\frac{L_s}{E_s}\right)$$

$$\therefore \sigma_{Al} \cdot \frac{L_{Al}}{E_{Al}} = \sigma_s \cdot \frac{L_s}{E_s} \quad \text{—— (1)}$$

As length of Aluminium rod & steel rod are same

$$\therefore L_{Al} = L_s$$

from eqn (1) becomes.

$$\sigma_{Al} \times \frac{1}{E_{Al}} = \sigma_s \times \frac{1}{E_s}$$

$$\therefore \sigma_{Al} = \frac{E_{Al}}{E_s} \times \sigma_s$$

$$\therefore \boxed{\sigma_{Al} = m\, \sigma_s} \quad \text{———} \left\{ \because m = \text{modular ratio} = \frac{E_{Al}}{E_s} \right.$$

Examples :

(1) A steel tube 40 mm inside diameter and 4 mm metal thickness is filled with concrete. Determine the stress in each material due to an axial thrust of 100 kN.
Take $E_s = 2.1 \times 10^5$ N/mm² & $E_c = 0.14 \times 10^5$ N/mm².

→ Given

(1) $d = 40$ mm (2) $t = 4$ mm

(3) External dia $D = d + 2t = 40 + 2\times4 = 48$ mm

(4) Axial thrust $P = 100$ kN $= 100 \times 10^3$ N

(5) $E_s = 2.1 \times 10^5$ N/mm²

(6) $E_c = 0.14 \times 10^5$ N/mm².

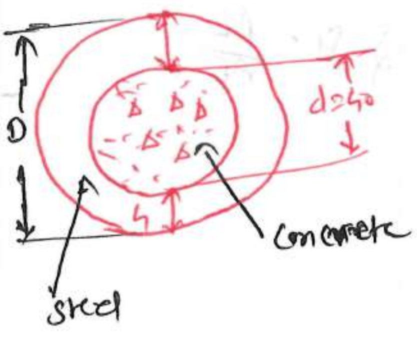
To find :- (1) σ_s = stress in steel = ?

(2) σ_c = stress in concrete = ?

— Soln :-

$$P = P_s + P_c$$

$$P = \sigma_s A_s + \sigma_c A_c \quad \text{—— (1)} \quad \left\{ \because \sigma = \frac{P}{A} \quad \therefore P = \sigma A \right.$$



$\therefore$ Area of steel (A_s)

$$A_s = \frac{\pi}{4}\left(D^2 - d^2\right) = \frac{\pi}{4}\left(48^2 - 40^2\right) = 552.92 \text{ mm}^2$$

Area of concrete, $A_c = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (40)^2$

$= 1256.64 \ mm^2$

We know that,

Extension or Elongation of steel & concrete it will be same

$$\therefore (\delta L)_s = (\delta L)_c$$

we can say $\Rightarrow e_s = e_c$

$$\therefore E = \frac{\sigma}{e} \quad \therefore e = \frac{\sigma}{E}$$

$$\therefore \frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\therefore \sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{2.1 \times 10^5}{0.4 \times 10^5} \times \sigma_c$$

$$\boxed{\sigma_s = 15 \ \sigma_c} \longrightarrow (2)$$

from eqn (1) becomes.

$$P = \sigma_s A_s + \sigma_c A_c$$

$$P = 15 \sigma_c \times A_s + \sigma_c A_c$$

$$100 \times 10^3 = (15 \times 552.92) \sigma_c + 1256.64 \ \sigma_c$$

$$= 9550.44 \ \sigma_c$$

$$\therefore \sigma_c = \frac{100 \times 10^3}{9550.44}$$

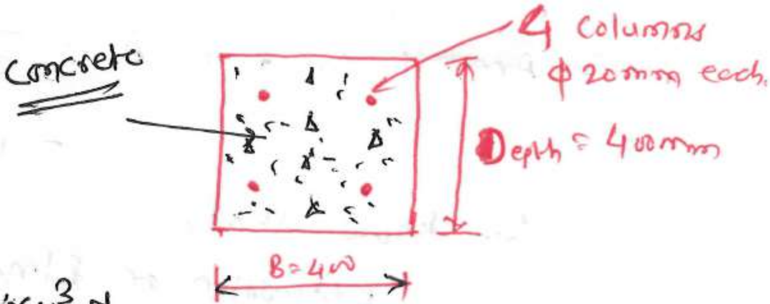
$$\boxed{\sigma_c = 10.47 \ N/mm^2}$$

$$\& \ \sigma_s = 15 \times \sigma_c = 15 \times 10.47$$

$$\boxed{\sigma_s = 157.06 \ N/mm^2}$$

(2) A RCC column 400mm × 400mm is reinforced with 4 bars of 20mm diameter. Determine the stresses induced in steel & concrete. if it is subjected to an axial load of 500 kN & modular ratio is 13

⇒ Given data



RCC column is 400 × 400 mm

i.e. $D = B = 400$ mm

Number of steel bars = 4

Axial load $(P) = 500\ kN = 500 \times 10^3\ N$

modular ratio$(m) = 13$

To find: (1) $\sigma_s = ?$

(2) $\sigma_c = ?$

Sol^n:
$$P = P_s + P_c$$
$$= \sigma_s A_s + \sigma_c A_c \quad \text{—— (1)} \quad \left[\sigma = \frac{P}{A} \therefore P = \sigma A\right.$$

$\therefore$ A_s = Area of steel $= 4 \times \frac{\pi}{4}(20)^2 = \underline{1256.64\ mm^2}$

Gross Area $A_g = A_s + A_c$

$$400 \times 400 = 1256.64 + A_c$$
$$\therefore A_c = 160000 - 1256.64 = \underline{158.74 \times 10^3\ mm^2}$$

We know that,
$$m = \frac{E_s}{E_c}$$
$$m = \frac{\cancel{\sigma_s / e_s}}{\cancel{\sigma_c / e_c}} \quad \frac{\sigma_s / e_s}{\sigma_c / e_c}$$
$$13 = \frac{\sigma_s}{\sigma_c} \quad \cdots \quad \left\{ \because e_s = e_c \therefore \text{strain are same} \right.$$
$$\therefore \boxed{\sigma_s = 13\,\sigma_c}$$

OR In

$\sigma_s = m\,\sigma_c$

$\therefore \underline{\sigma_s = 13\ \sigma_c} \cdots \{\because m = 13$ given

from eqn (1) becomes,
$$P = \sigma_s A_s + \sigma_c A_c = 13\,\sigma_c \times A_s + \sigma_c A_c$$
$$P = (13 \times 1256.64)\,\sigma_c + (158.74 \times 10^3)\,\sigma_c$$
$$P = 175.076 \times 10^3\ \sigma_c$$
$$\therefore \sigma_c = \frac{500 \times 10^3}{175.076 \times 10^3} = \boxed{2.856\ N/mm^2 = \sigma_c}$$
$$\& \boxed{\sigma_s = 13 \times 2.856 = 37.126\ N/mm^2}$$

(3) A compound tube consist of steel & brass, steel tube having 140 mm internal diameter & 160 mm external diameter, an outer side 160 mm internal diameter & 180 mm external diameter. Two tubes having same length of 140 mm. The compound tube carries an axial load of 900 kN. Calculate the stresses in the tube materials & load carrying capacity of each material. Also calculate the amount the tubes get shortens. Take $E_s = 2\times10^5$ N/mm² & $E_{br} = 1\times10^5$ N/mm².

⇒ Soln:- Given

(A) Steel material.

i) $d_{is} = 140$ mm

ii) $d_{os} = 160$ mm

iii) $E_s = 2\times10^5$ N/mm²

(B) Brass material

i) $d_{iBr} = 160$ mm

ii) $d_{oBr} = 180$ mm

iii) $E_{br} = 1\times10^5$ N/mm²

(C) $L_s = L_{Br} = 140$ mm

(D) $P = 900$ kN $= 900\times10^3$ N

To find :- (A) (1) $\sigma_s = ?$ (2) $\sigma_{Br} = ?$ | (B) (1) $P_s = ?$ (2) $P_{br} = ?$ | (C) $(\delta L)_s = (\delta L)_{Br} = ?$

$$\therefore \quad P = P_s + P_{br} \Rightarrow \sigma_s A_s + \sigma_{br} A_{br} \quad \text{—(1)}$$

$$\therefore \quad A_s = \frac{\pi}{4}\left(d_{os}^2 - d_{is}^2\right) = \frac{\pi}{4}\left[(160)^2 - (140)^2\right] = 4712.39 \text{ mm}^2$$

$$A_{Br} = \frac{\pi}{4}\left(d_{obr}^2 - d_{ibr}^2\right) = \frac{\pi}{4}\left[(180)^2 - (160)^2\right] = 5340.71 \text{mm}^2$$

We know that $(e_s = e_{br})$

$$\therefore \quad \sigma_s = m\,\sigma_{Br} \quad \cdots \left\{ m = \frac{E_s}{E_{br}} \right.$$

$$\sigma_s = \frac{E_s}{E_{br}} \times \sigma_{Br} = \left(\frac{2\times10^5}{1\times10^5}\right) \times \sigma_{Br}$$

$$\therefore \quad \sigma_s = 2\,\sigma_{br} \quad \text{—(2)}$$

from eqn (1) becomes.

$$\therefore \quad P = (2\times\sigma_{br}) \times A_s + \sigma_{br} A_{br}$$

$$900\times10^3 = (2\times4712.39)\,\sigma_{br} + (5340.71)\,\sigma_{br}$$

$$900\times10^3 = 14765.49\;\sigma_{br}.$$

$$\therefore \sigma_{Br} = \frac{900 \times 10^3}{14765.49} = \underline{\underline{60.95 \frac{N}{mm^2}}}$$

from eq^n ② becomes-

$$\therefore \sigma_s = 2 \times \sigma_{Br} = 2 \times 60.95$$

$$\boxed{\sigma_s = 121.90 \ N/mm^2} \text{ — Part ① finish}$$

<u>Part - II</u>

i) $$\sigma_s = \frac{P_s}{A_s}$$

$$\therefore P_s = \sigma_s \times A_s = 121.90 \times 4712.39$$

$$\boxed{P_s = 574.46 \ KN}$$

ii) $$\sigma_{Br} = \frac{P_{Br}}{A_{Br}}$$

$$\therefore P_{Br} = \sigma_{Br} \times A_{Br} = 60.95 \times 5340.71$$

$$\boxed{P_{Br} = 325.516 \ KN}$$

<u>Part → III</u>

$e_s = e_{Br} \Longrightarrow$ So change in length will be same bcz length of shaft for both material same.

$$(\delta l)_{Br} = (\delta L)_s = \frac{P_s L}{A_s E_s} =$$

$$= \frac{574.46 \times 10^3 \times 140}{4712.39 \times 2 \times 10^5} = \underline{\underline{0.0853 \ mm}}$$

$$\boxed{(\delta L)_s = (\delta l)_{Br} = 0.0853 \ mm}$$

$$\therefore \frac{\delta V}{V} = \frac{\sigma_x}{E} - \mu\left(\frac{\sigma_x}{E}\right) - \mu\left(\frac{\sigma_x}{E}\right)$$

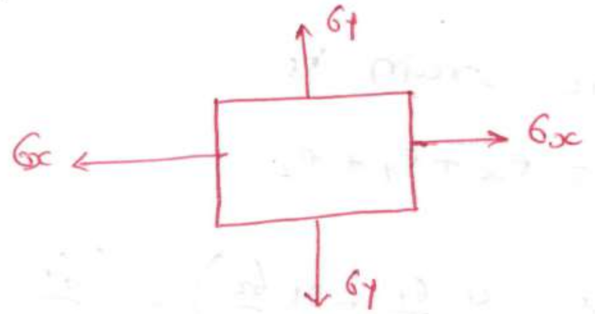
$$= \frac{\sigma_x}{E}(1-\mu-\mu) = \frac{\sigma_x}{E}(1-2\mu)$$

$$\therefore \boxed{\delta V = V.\left[e_x(1-2\mu)\right]} \quad \left\{\because e_x = \frac{\sigma_x}{E}\right.$$

2.8

(*) Bi-axial stress system:-

When two forces are applied on two mutually perpendicular planes then the stresses are developed in the system is called bi-axial stress system.



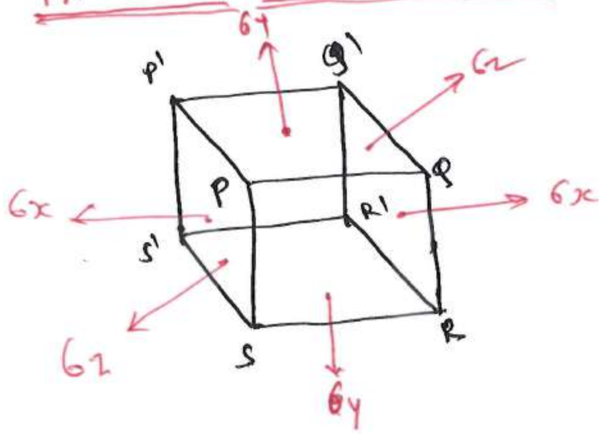
∴ Strain are produced in x & y direction

$$\therefore e_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} \quad \text{..... strain in x-direction}$$

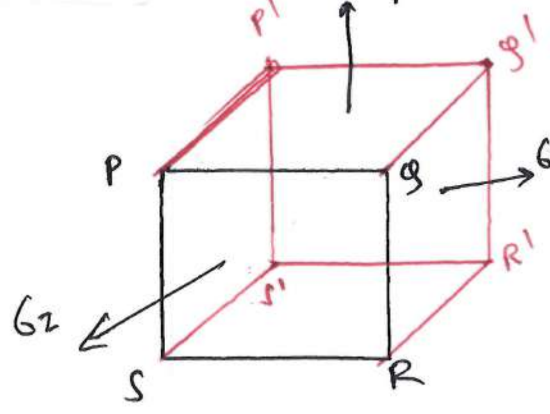
$$e_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} \quad \text{..... strain in y-direction}$$

Note:- σ_x & σ_y are tensile stresses take as +ve.

(*) Tri-axial stress system:-



OR



Let, σ_x, σ_y & σ_z are tensile stresses acting in x, y & z direction respectively as shown in fig. ⓐ

∴ let, e_x = strain in x-direction

$$e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E}$$

lly

$$e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_z}{E}$$

lly

$$e_z = \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

∴ Volumetric strain is

$$\frac{\delta V}{V} = e_v = e_x + e_y + e_z$$

$$\frac{\delta V}{V} = \left(\frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E}\right) + \left(\frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}\right) + \left(\frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}\right)$$

$$\frac{\delta V}{V} = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right) \times (1 - 2\mu)$$

$$\therefore \boxed{\delta V = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right) \cdot (1 - 2\mu) \times V}$$

1) A mild steel bar is subjected to a load of 80 kN. The diameter of the bar is 16 mm & it's length is 320 mm. Calculate elongation if modulus of elasticity is 196 kN/mm². Calculate change in diameter if poisson's ratio is $\frac{1}{m} = 0.28$.

⇒ Given

i) $P = 80\,kN = 80 \times 10^3\,N$

ii) $d = 16\,mm$

iii) $L = 320\,mm$

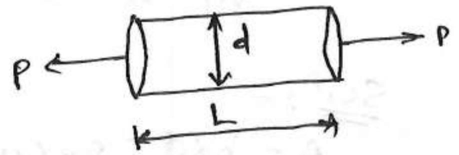
iv) $E = 196\,kN/mm^2 = 196 \times 10^3\,\frac{N}{mm^2}$

v) $\mu = \frac{1}{m} = 0.28$

To find

① $\delta L = ?$

② $\delta d = ?$



Soln:

i) $\delta L = \frac{PL}{AE}$ — ①

$\therefore A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (16)^2 = \underline{201.06\,mm^2}$

from eqn ① becomes.

$$\delta L = \frac{80 \times 10^3 \times 320}{201.06 \times 196 \times 10^3} = \underline{0.6496\,mm}$$ — ① Ans

2) We know that,

$$\mu = \frac{e_{lateral}}{e_{linear}}$$

$\therefore e_{lateral} = \mu \times e_{linear} = 0.28 \times \frac{\delta L}{L}$

$$e_{lat} = \frac{0.28 \times 0.6496}{300} = \underline{6.063 \times 10^{-4}}$$

$$\therefore e_{lat} = \frac{\delta d}{d}$$

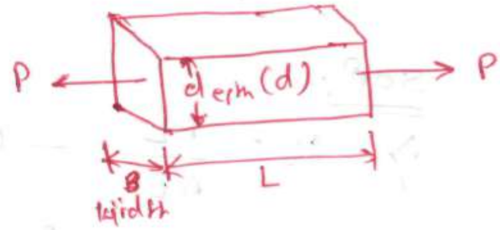
$\therefore \delta d = d \times e_{lateral} = 16 \times 6.063 \times 10^{-4}$

$$\boxed{\delta d = 9.70 \times 10^{-3}\,mm}$$ — ② Ans

(2) A bar of cross-section dimensions 20 mm × 40 mm & length 500 mm is subjected to axial tensile force of 50 KN. The change in length is 0.20 mm. Determine change in width, change in depth, change in volume & volumetric strain if $\mu = 0.30$

⇒ **Given**
i) $b = 20\,mm$
ii) $d = 40\,mm$
iii) $L = 500\,mm$
iv) $P = 50\,KN = 50 \times 10^3\,N$
v) $\delta L = 0.20\,mm$
vi) $\mu = 0.30$

To find
(1) $\delta b = ?$
(2) $\delta d = ?$
(3) $\delta v = ?$
(4) $e_v = ?$



Soln:-

$A = b \times d = 20 \times 40 = 800\,mm^2$

$e_{linear} = \frac{\delta L}{L} = \frac{0.20}{500}$

$$\boxed{e_{linear} = 4 \times 10^{-4}}$$

we know that,

poisson's ratio $(\mu = \frac{1}{m}) = \frac{e_{later}}{e_{linear}} = \mu$

$\therefore \quad 0.30 = \frac{e_{later}}{4 \times 10^{-4}}$

$\therefore \; e_{lateral} = 0.30 \times 4 \times 10^{-4}$

$$\boxed{e_{lat} = 1.2 \times 10^{-4}}$$

$\therefore \; e_{lateral} = \frac{\delta b}{b}$

$\therefore \; \delta b = b \times e_{later} = 20 \times 1.2 \times 10^{-4}$

$$\boxed{\delta b = 2.4 \times 10^{-3}\,mm}$$ —— Ans - (I)

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$e_{lateral} = \frac{\delta d}{d}$

$\therefore \; \delta d = d \times e_{lat} = 40 \times 1.2 \times 10^{-4}$

$$\boxed{\delta d = 4.8 \times 10^{-3}\,mm}$$ —— Ans - (II)

(3) volumetric strain $(e_v) = \frac{\delta v}{V} = e_x(1-2\mu)$

$\therefore e_v = 4\times10^{-4}(1-2\times0.30)$

$\boxed{e_v = 1.6\times10^{-4}}$ —— Ans (4)

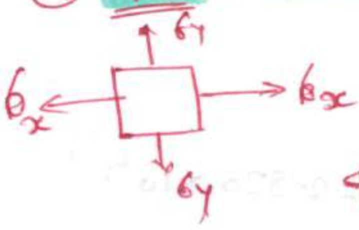
$\therefore e_v = \frac{\delta V}{V}$

$\therefore \delta V = V \times e_v$ ... { V = Area x Length $= 20\times40\times500 = 400\times10^3\ mm^3$

$\therefore \delta V = 400\times10^3\times1.6\times10^{-4} = 64\ mm^3$

$\boxed{\delta V = 64\ mm^3}$ —— Ans – (3)

(4) ~~4.2~~ 2.8 Numerical Based on Bi-axial Stresses System



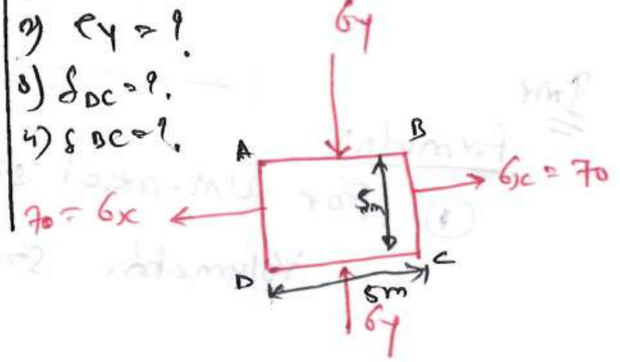
(1) In a bi-axial stress system, the stresses along the two perpendicular directions are 70 N/mm² (tensile) & 40 N/mm² (compressive) & poisson's ratio = 0.28 Calculate the strain along these two direction. Take E = 2.1×10⁵ N/mm² also find change in length in both direction if section is square of 5m.

⇒ Given:

1) $\sigma_x = 70\ \frac{N}{mm^2}$ (Tensile)
2) $\sigma_y = 40\ \frac{N}{mm^2}$ (compressive)
3) $\mu = 0.28$
4) $E = 2.1\times10^5\ \frac{N}{mm^2}$
5) $L_x = L_y = 5m = 5\times10^3\ m$

To find

1) $e_x = ?$
2) $e_y = ?$
3) $\delta_{DC} = ?$
4) $\delta_{BC} = ?$



Soln:

$$e_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E}$$

$$= \frac{70}{2.1\times10^5} - \frac{0.28\times(-40)}{2.1\times10^5} = \frac{1}{2.1\times10^5}\left(70 - 0.28\times(-40)\right)$$

$$= \frac{70-0.28\times(-40)}{2.1\times10^5} = \boxed{3.867\times10^{-4} = e_x}$$ —— Ans (1)

Now,

$$e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} = \frac{1}{E}\left(\sigma_y - \mu \sigma_x\right)$$

$$e_y = \frac{1}{2.1 \times 10^5}\left[(-40) - 0.28 \times 70\right]$$

$$e_y = \frac{-40 - 0.28 \times 70}{2.1 \times 10^5} = -2.838 \times 10^{-4}$$ — ② Ans

Now,

$$e_x = \frac{\delta L_{BC}}{L_{BC}}$$

$$\therefore \delta L_{BC} = e_x \times L_{BC} = 3.867 \times 10^{-4} \times 5 \times 10^3$$

$$\boxed{\delta L_{BC} = 1.9335 \text{ mm}}$$ — Ans ③

lly

$$e_y = \frac{\delta L_{BC}}{L_{BC}}$$

$$\therefore \delta L_{BC} = e_y \times L_{BC} = 5 \times 10^3 \times (-2.838 \times 10^{-4})$$

$$\boxed{\delta L_{BC} = -1.419 \text{ mm}}$$ — Ans ④

{ ∵ -ve sign indicates decreases in length.

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Formulas.

① For Uni-axial stress system

Volumetric strain $(e_v) = \frac{\delta V}{V} = e_x(1 - 2\mu)$

② For Bi-axial stress system

Volumetric strain in x direction is

$$e_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} \Rightarrow \frac{1}{E}(\sigma_x - \mu \sigma_y)$$

lly

$$e_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E} \rightarrow \frac{1}{E}(\sigma_y - \mu \sigma_x)$$

Type 4.3 ~~2.9~~ Numerical Based on Triaxial Stress System

2.9

formula :-

i) $e_x = \frac{1}{E}\left(\sigma_x - \mu\sigma_y - \mu\sigma_z\right)$

ii) $e_y = \frac{1}{E}\left(\sigma_y - \mu\sigma_x - \mu\sigma_z\right)$

iii) $e_z = \frac{1}{E}\left(\sigma_z - \mu\sigma_x - \mu\sigma_y\right)$

iv) Volumetric Strain $(e_v) = e_x + e_y + e_z = \frac{\delta V}{V}$

$\therefore (e_v) = \frac{\delta V}{V} = \left(\frac{\sigma_x + \sigma_y + \sigma_z}{E}\right) \times (1 - 2\mu)$

(1) A metal bar 200 mm long, 40 mm × 30 mm in cross-section is subjected to stress of 110 MPa along the length & 50 MPa on another two faces. All stresses are tensile. Calculate strain along the three direction & also the volumetric strain. Assume E = 120 MPa & μ = 0.30, also find change in volume.

⇒ Given :-

i) L = 200 mm

ii) b = 40 mm

iii) t = 30 mm

iv) A = b × t = 40 × 30 = 1200 mm²

v) σ_x = 110 MPa = 110 N/mm²

vi) $\sigma_y = \sigma_z$ = 50 MPa = 50 N/mm²

vii) E = 120 MPa = 120 N/mm²

viii) μ = 0.30

To find

$\left.\begin{array}{l} (1)\ e_x = ? \\ (2)\ e_y = ? \\ (3)\ e_z = ? \end{array}\right\}$ — I

(4) $e_v = ?$ — II

Soln :- We know that,

$e_x = \frac{(\sigma_x - \mu\sigma_y - \mu\sigma_z)}{E} = \frac{110 - 0.30 \times 50 - 0.30 \times 50}{120}$

$\boxed{e_x = 0.667 \text{ mm}}$

$||^{ly}$ $e_y = \frac{\sigma_y - \mu\sigma_x - \mu\sigma_z}{E} = \frac{50 - 0.30 \times 110 - 0.30 \times 50}{120}$

$\boxed{e_y = 0.0167 \text{ mm}}$

$$e_z = \frac{\sigma_z - \mu\sigma_x - \mu\sigma_y}{E} = \frac{50 - 0.30\times 80 - 110\times 0.30}{120}$$

$$\boxed{e_z = 0.0167\ mm}$$ —— Ans (I)

We know that

$$e_v = \frac{\delta v}{V} = e_x + e_y + e_z$$

$$\boxed{e_v = 0.6993}$$ —— (II) — Ans

I^{ly}

$$\text{Volume} = b \times t \times L = 40\times 30\times 200 = 240000\ mm^3$$

$$\therefore e_v = \frac{\delta v}{V}$$

$$\therefore \delta v = e_v \times V = 0.6993 \times 240000$$

$$\boxed{\delta v = 167.832\times 10^3\ mm^3}$$ —— (III) Ans