The Shirpur Education Society's

R. C. Patel College of Engineering and Polytechnic, Shirpur

Department of Mechanical Engineering

NAME OF COURSE: - Strength of Materials

CODE OF COURSE: - 3123308

SEMESTER: - SYME-3K

SUBJECT TEACHER: - Mr. Laxmikant Y.Borse

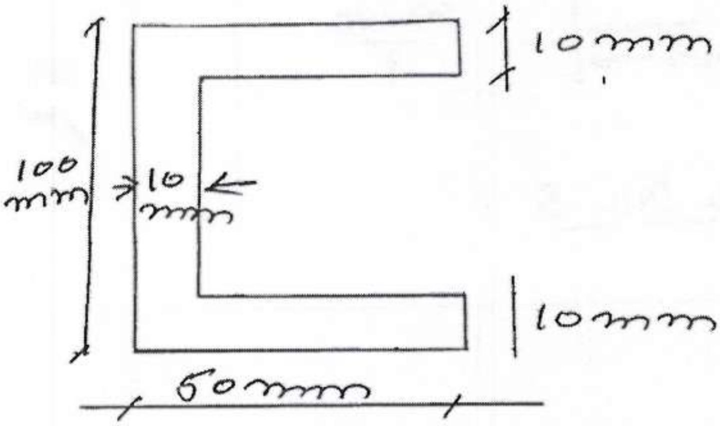
The Shirpur Education Society's

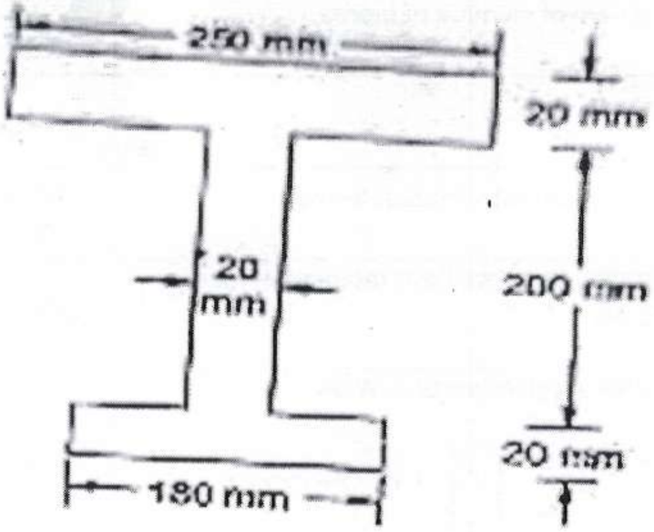
R. C. Patel College of Engineering and Polytechnic, Shirpur

QUESTION BANK

CHAPTER 1. MOMENT OF INERTIA

Program Name: Mechanical Engineering
Program Code: ME3K
Name of Subject & Code : Strength of Materials (313308)
Semester : Third
Date & Time Slot: 20/05/2026

Q. NO.	QUESTION	DETAIL	MAPPING
1	Define radius of gyration.	S-W-25** & W-24 Q1A 2M	
2	Define i) Moment of inertia.	S-25 &W-24 Q1A 2M	CO 1.1 R
3	Define Moment of inertia. State its value for semicircle about its centroid.	S-26 Q1A 2M	
4	State perpendicular axis theorem of moment of Inertia.	w-25 & s-26 Q1A 2M	CO 1.1U
5	State the parallel axis theorem.	S-25 Q1A 2M	CO 1.1 U
6	State the parallel axis theorem with mathematical formula.	W-24&25 Q2A 4M	CO 1.2 U
7	Define 'Polar moment of Inertia'. Calculate Polar moment of Inertia for square lamina of side 40 cm.	W24 Q2B 4M	CO1.2,1.3,R,A
8	Calculate the M.I. for the following given section. W-24  **Fig. No. 1**	W24 Q3B 4M	CO 1.4 A
9	An Angle section 120 mm × 100 mm × 20 mm is placed such as its longer leg is horizontal. Calculate M.I. about centroidal horizontal axis only. (i.e. Ixx only)	W25 Q3A 4M	CO 1.4 A
10	A hollow circular section having 200 mm external diameter and 100 mm internal diameter. Calculate the moment of the section about any of the tangent. Also find polar moment of inertia. W-25	W25 Q3B 4M	CO 1.4 A

11	A hollow square has inner dimensions a × a and outer dimensions 2a × 2a. Find moment of inertia about the outer side.	S25 Q2A 4M	CO 1.4 A
12	A circular disc has diameter of 80 mm. Calculate M.I. about its any one tangent.	S25 Q3A 4M	CO 1.4A
13	Calculate polar M.I. of Semi circle having 60mm diameter. Also calculate minimum radius of gyration. Diameter is parallel to Y-Y axis. W-26	W26 Q3A 4M	CO 1.2 &1.4 A
14	Find M.I of symmetrical I-section having following details : flange s: 100mm × 20 mm, Overall Depth : 280mm and Thickness of web: 10mm W-26	W26 Q3A 4M	CO 1.4 A
15	Determine the MI about X-X and Y-Y axis as shown in fig no.5  Fig. No. 5	W26 Q5C 6M	CO 1.4 A

Name of college :- R.C. Patel college of Engg. & Polytechnic, Shirpur.

Name of Subject I/c :- Mr. L.Y. Borse

Name of subject :- Strength of materials (SOM)

Subject code :- 313308

code & Year :- ME - 3K

Unit → I :- Moment of Inertia

Weightage and Hour's :- 1) Learning Hours :- 10

2) Weightage :- 12 marks.

Question Ask in M.S.B.T.E in question paper :- Q.1. (a) - 2 marks } 04
(b) - 2 marks

Q.2. (a) - 4 marks.

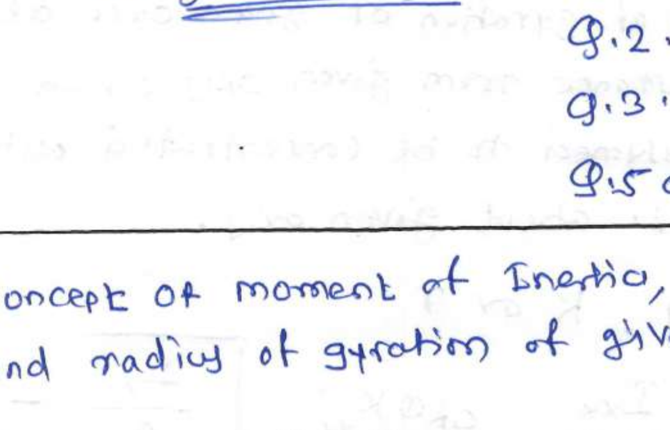
Q.3. (a) - 4 marks

Q.5 OR Q.6. :- (a) or (c) = 6 marks.

1.1 Concept of moment of Inertia, M.I. Plane lamina and radius of gyration of given lamina. [2M]

Def^n. Moment of Inertia :-

moment of inertia of a body about any axis is defined as the sum of second moment of all elementary area about that axis.



where,

X-X ⇒ Co-ordinate ⊥er to Y-axis

&

Y-Y ⇒ co-ordinates ⊥er to X-axis.

a = Cross-sectional area of member.

1) M.I. about Y-Y axis is given by as per definition is -

$$I_{YY} = \Sigma(a_1x_1^2 + a_2x_2^2 + a_3x_3^2)$$

ie $\boxed{I_{YY} = \Sigma a x^2}$ ..... $\{ \because ax^2 = a_1x_1^2 + a_2x_2^2 + a_3x_3^2$

(2) As per defination M.I. about x-x axis is given by

$$I_{xx} = \Sigma a_1 y_1^2 + a_2 y_2^2 + a_3 y_3^2$$

$$\therefore \boxed{I_{xx} = \Sigma a y^2} \cdots\cdots \{\because a y^2 = a_1 y_1^2 + a_2 y_2^2 + a_3 y_3^2$$

S.I. unit is $\Rightarrow$ mm^4, cm^4 or m^4

~~Note~~:

1.1 Radius of Gyration :- [2m]

Defn :- The radius of gyration of given area about any axis that distance from given axis which the entire area is assumed to be concentrated without changing the M.I. about given axis.

It's denoted by, K or r

$$\therefore K^2 = \frac{I_{xx}}{A} \quad \text{OR (1)} \quad K_{xx} = \sqrt{\frac{I_{xx}}{A}} \quad \text{— for x-x axis}$$

$$\text{(2)} \quad K_{yy} = \sqrt{\frac{I_{yy}}{A}} \quad \text{— for y-y-axis.}$$

It's S.I. unit is given by —

mm, cm or m.

1.2. Parallel and Perpendicular axes theorems [2 & 4m] without derivation.

(a) Parallel axes theorem :-

Statement :- "The moment of inertia of a plane section about any axis parallel to the centroidal axis is equal to the moment of inertia of section about the centroidal axis plus the product of the area of the section and square of the distance between two axes.

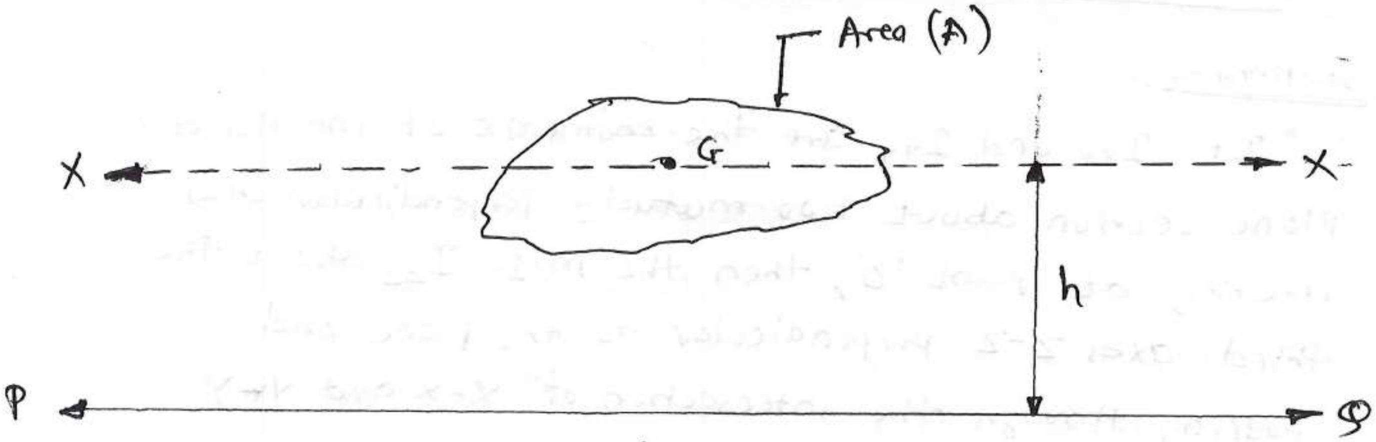


fig. (a)

<u>To find</u> :- moment of inertia about axis 'PQ' is given by according to statement iv.

Applying parallel axis ~~theorem~~ theorem.

$$I_{PQ} = I_{G_{xx}} + A h^2$$

$$\therefore \quad I_{PQ} = I_{xx} + Ah^2 \quad \ldots \left\{ \text{Here } I_{G_{xx}} = I_{xx} \text{ as per given fig.} \right.$$

Similarly we apply parallel axis theorem. for Y-Y axis is given by fig. (b)

According to,

$$I_{RS} = I_{G_{yy}} + A h^2$$

$$\therefore \quad I_{RS} = I_{YY} + Ah^2 \ldots \left\{ \because I_{yy} = I_{G_{yy}} \right.$$

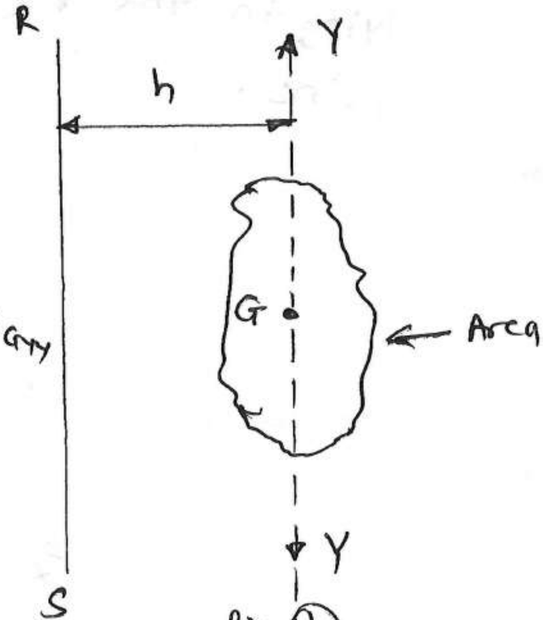


fig. (b)

1.2. Perpendicular axis Theorem :-

Statement :-

"If I_{xx} and I_{yy} are the moments of inertia of a plane section about two mutually perpendicular axes meeting at point 'O', then the M.I. I_{zz} about the third axes Z-Z perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by,

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

OR

" the moment of inertia of a plane lamina about an axis ⊥er it's plane is equal to the sum of the moments of inertia about two mutually ⊥er axes lying in the plane and intersecting at the same point.

Note:. It's also called of polar moment of Inertia.

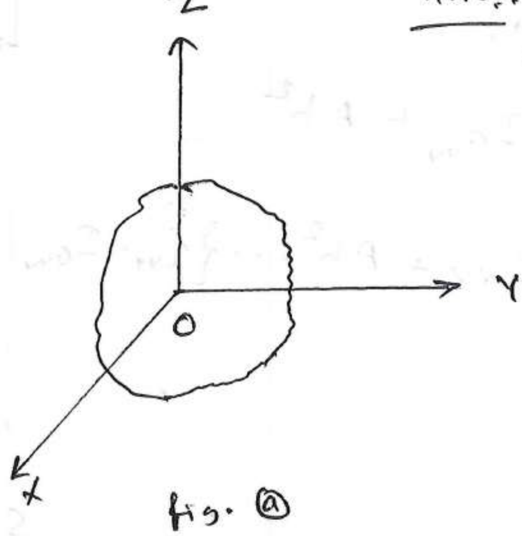


fig. ⓐ

Where,

$I_p = I_{zz} =$ Polar moment of inertia about Z-axis.

$I_{xx} =$ M.I. about X-axis.

$I_{yy} =$ M.I. —"— Y-axis.

Note:

① Polar m.I. for solid circular shaft having diameter (D) is given by.

$$I_p = I_{zz} = \frac{\pi}{32} \times D^4 \quad \ldots\ldots$$

$\because$ $I_p = I_{xx} + I_{yy}$

$= \frac{\pi}{64} D^4 + \frac{\pi}{64} D^4$

$I_p = \frac{2\pi D^4}{64} = \frac{\pi}{32} \times D^4$

② Polar m.I. for Hollow circular shaft having Internal diameter (d) and External diameter (D) is given by-

$$I_p = \frac{\pi}{32} (D^4 - d^4)$$

(*) Formules :- Radius of Gyration

(1) Radius of Gyration for Rectangular section with X-X & Y-Y axis is given by :-

(a) $K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{\left(\frac{bd^3}{12}\right)}{b \times d}} = \frac{d}{2\sqrt{3}}$

(b) $K_{YY} = \sqrt{\frac{I_{yy}}{A}} = \frac{b}{2\sqrt{3}}$

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(2) Hollow Rectangular section.

(a) $K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{\frac{1}{12}(BD^3 - bd^3)}{(BD - bd)}}$

(b) $K_{yy} = \sqrt{\frac{I_{YY}}{A}} = \sqrt{\frac{\frac{1}{12}(DB^3 - db^3)}{(BD - bd)}}$

---

(3) solid circular section: of diameter 'D'

$K_{xx} = K_{yy} = \sqrt{\frac{\frac{\pi}{64} \times D^4}{\pi/4\, D^2}} = \sqrt{\frac{D^2}{16}} = \frac{D}{4}$

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(4) Hollow circular section :- D = External diameter, d = Internal diameter

$K_{xx} = K_{yy} = \frac{\sqrt{D^2 + d^2}}{4}$

(5) For triangular section of base - b & heigh 'h'

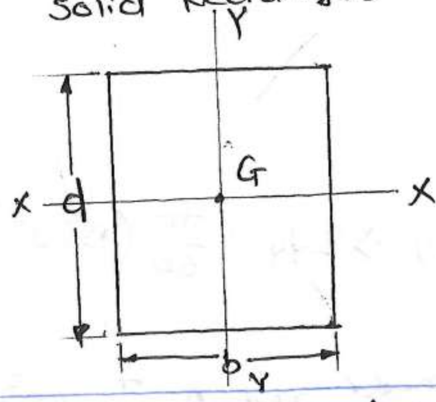
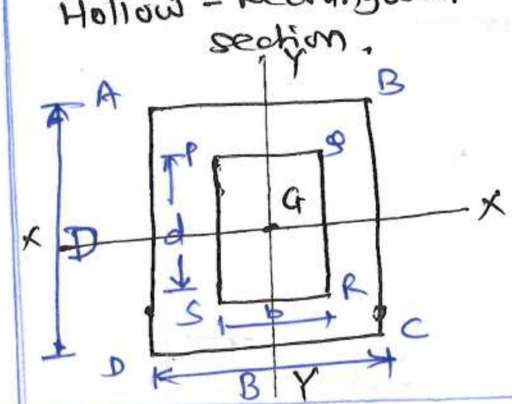
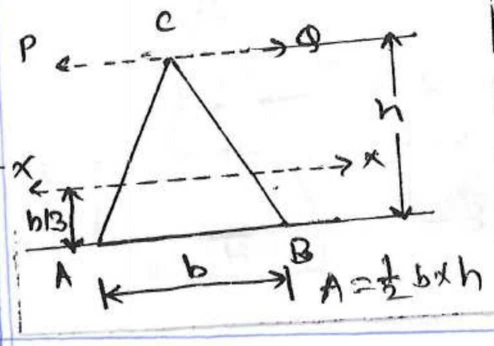
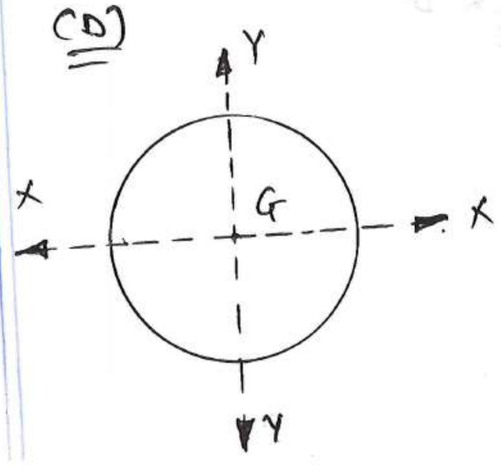
$K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{\frac{1}{36} bh^3}{\frac{1}{2} b \times h}} = \sqrt{\frac{h^2}{18}} = \frac{h}{3\sqrt{2}}$

(6) for semi-circular section of Radius 'R'.

$K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{0.11 R^4}{\frac{\pi}{2} R^2}} = \sqrt{\frac{0.22 R^2}{\pi}} = 0.2646 R$

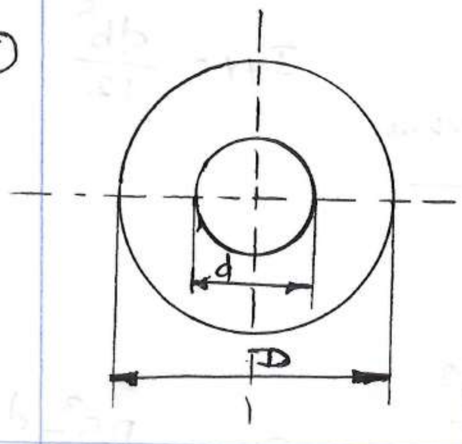
$K_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{R^2}{4}} = \frac{R}{2} = 0.5 R$

13 M.I. about centroidal axes some Basic Figures.

		M.I. Ixx	M.I. YY.
1.	Solid Rectangle 	$I_{xx} = \frac{bd^3}{12}$ where b = width of Rectangle d = depth —"— Area = $A = b \times d$	$I_{YY} = \frac{db^3}{12}$
2.	Hollow - Rectangular section. 	$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$ $= \left(\frac{BD^3 - bd^3}{12}\right)$ where, B = width of outer rectangle D = Depth —"— b = width of inner —"— d = Depth of —"—	$I_{yy} = \frac{DB^3 - db^3}{12}$ Area = $A = [BD - bd]$
3.	M.I. of a triangular section  $A = \frac{1}{2} b \times h$	① $I_{xx} = \frac{bh^3}{36}$ -- for centroidal x-x axis ② $I_{AB} = \frac{bh^3}{12}$ -- for Base of triangle ③ $I_{PQ} = \frac{bh^3}{4}$ -- for Pt. 'C' Apex or Vertex	
4.	M.I. of solid circular section of diameter (D) 	$I_{xx} = \frac{\pi}{64} D^4$ $A = \frac{\pi}{4} D^2$	$I_{yy} = \frac{\pi}{64} D^4$

M.I. of Hollow circular section

(5)



$I_{xx} = \frac{\pi}{64} \times (D^4 - d^4) \Rightarrow I_{yy} = \frac{\pi}{64}(D^4 - d^4)$

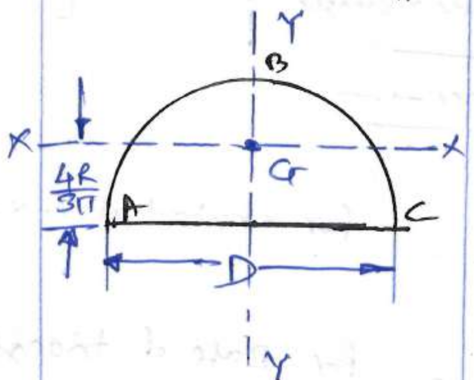
where,

D = External diameter and $A = \frac{\pi}{4}(D^2 - d^2)$

d = Internal "

M.I. of Semi-circular section.

(6)



$I_{xx} = 0.11 R^4$

where

R = Radius of semicircular section.

$I_{yy} = \frac{\pi R^4}{8}$

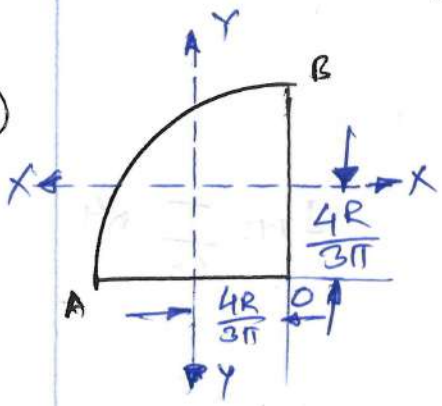
OR

$I_{yy} = \frac{\pi D^4}{128}$

$A = \frac{\pi}{8} \times D^2$

M.I. of Quarter Circular section.

(7)



$I_{xx} = I_{yy} = 0.055 R^4$

$A = \frac{\pi}{16} \times D^2$

* Composite section :- 1.4

1) C.G. of the composite section is calculated by using

$$\bar{X} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{A}$$

$$\bar{Y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{A}$$

$\therefore$ $A = a_1 + a_2 + a_3$ = area of composite section.

(2) $I_{G_1}, I_{G_2}, I_{G_3}$ ---- m.I. of the areas 1, 2, 3 about their own centroidal axes. X-X & Y-Y can be calculated by standard formulae. {i.e. depends upon it's section}

(3) $I_{G_1}, I_{G_2}, I_{G_3}$ ---- m.I. about X-X axis can be calculated by using parallel axis theorem.

$$I_{XX} = I_{XX_1} + I_{XX_2} + I_{XX_3}$$

where,

$$I_{XX_1} = I_{G_1} + a_1h_1^2 \quad \& \quad I_{XX_3} = I_{G_3} + a_3h_3^2$$

$$I_{XX_2} = I_{G_2} + a_2h^2$$

where, h_1 = distance betn G_1 & XX-axis

h_2 = —— " —— G_2 —— " ——

h_3 = —— " —— G_3 —— " ——

(4) similarly,

$$I_{YY} = I_{YY_1} + I_{YY_2} + I_{YY_3}$$

where,

$$I_{YY_1} = I_{G_1} + a_1h_1^2, \quad I_{YY_2} = I_{G_2} + a_2h_2^2 \;\&$$

$$I_{YY_3} = I_{G_3} + a_3h_3^2$$

where, h_1 = distance betn G_1 and Y-Y axis

h_2 = —— " —— G_2 —— " ——

h_3 = —— " —— G_3 —— " ——

1.5. Introduction to M.I. for built-up sections.

Built-up section :-

It is a section formed by combining two or more simple geometrical shapes such as rectangles, triangles circles, channels, angles. etc.

Eg. :- 1) I-section   3) Channel section (C-section)
2) T-"   4) L-section (Angle section)

Built-up section are combination of simple figures. their M.I. found by

a) Dividing the section into simple standard section.
b) finding the centroid (C.G)
c) calculating M.I. of each parts about it's own centroidal axis.
d) Applying parallel axis theorem.
e) Adding all M.I.

∴ Built-up section is given by

$$I = \Sigma \left(I_g + Ad^2 \right)$$

where,
I_g = M.I. of individual part about it's centroidal axis.
A = Area of the part.
d = Distance betn centroidal axis of the part and required axis.

Unit - I

Moment of Inertia

(*) Examples Based circular section.

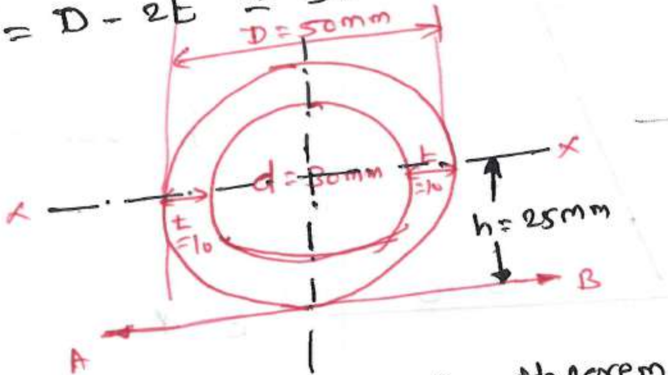
1) A hollow circular section has external diameter 50 mm & wall thickness of 10 mm. calculate moment of inertia about the tangent to external diameter.

⟹ Given :- Hollow circular section.

$D = 50mm$

$t = 10\ mm$

$\therefore d = D - 2t = 50 - 2 \times 10 = 30\ mm$



$\therefore$ By using parallel axis theorem about tangent line AB.

$$I_{(AB)_{tangent}} = I_{xx} + Ah^2 \quad \text{—— (1)}$$

$$\therefore I_{xx} = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}\left((50)^4 - (30)^4\right)$$

$$I_{xx} = 0.267 \times 10^6\ mm^4$$

$$\therefore A = \frac{\pi}{4} \times (D^2 - d^2) = \frac{\pi}{4}\left((50)^2 - (30)^2\right)$$

$$A = 1256.64\ mm^2$$

$$h = \frac{50}{2} = 25\ mm \text{ from x-x axis}$$

$$\therefore I_{AB} = 0.267 \times 10^6 + 1256.64 \times (25)^2$$

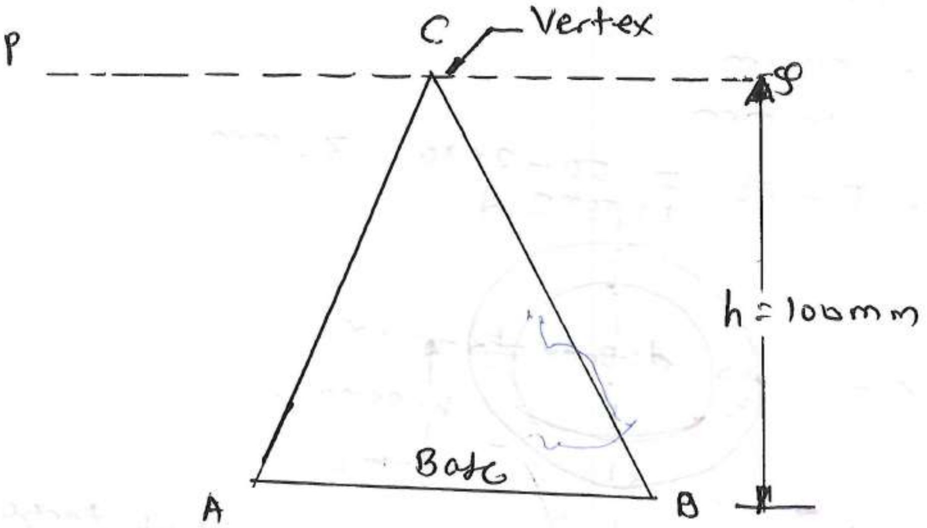
$$\boxed{I_{(AB)_{tangent}} = 1.0524 \times 10^6\ mm^4}$$

M.I. of triangular section

Q.5 (P.B) Given :- Calculate M.I. for a triangle of height 100 mm about an axis passing through vertex and parallel to base, if M.I. about base of same triangle is 10^7 mm^4

1) H = 100 mm
2) I_{Base} = ~~10^7 mm~~ 10^7 mm^4

To find
① I_{vertex} = ?



Step :- I :- To find I_{Base} = ?

$$\therefore I_{Base} \text{ i.e. } I_{AB} = \frac{bh^3}{12}$$

$$\therefore 10^7 = \frac{b \times (100)^3}{12}$$

$$\therefore b = \frac{10^7 \times 12}{(100)^3} = 120 \text{ mm}$$

Step :- II :- Calculate $I_{vertex} = I_{PQ}$ = ?

$$\therefore I_{vertex} = I_{PQ} = \frac{bh^3}{4} = \frac{120 \times (100)^3}{4}$$

$$\boxed{I_{vertex} = 3 \times 10^7 \text{ mm}^4}$$ ← Sol^n

Q.2) Define "Polar moment of Inertia". Calculate polar M.I. for square lamina of side 40 cm. W-24

⇒ Defⁿ: Polar Moment of Inertia:-

(A) The sum of the moments of a plane area about two mutually perpendicular axes (XX & YY) lying in the plane and intersecting at a point.

It's denoted by J or I_p

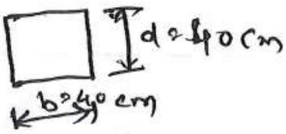
$$\therefore I_p = I_{xx} + I_{yy}$$

Where,

1) $J = I_p =$ Polar M.I.

2) $I_{xx} =$ M.I. about X-axis.

3) $I_{yy} =$ M.I about Y-axis.



(B) Given:- Square lamina $= a = 40cm$

square lamina about centroidal axis

$$\therefore I_x = I_y = \frac{a^4}{12}$$

$$\therefore I_p = I_{xx} + I_{yy} = \frac{a^4}{12} + \frac{a^4}{12}$$

$$= \frac{2a^4}{12} = \frac{a^4}{6} = \frac{(40)^4}{6}$$

$$\therefore I_p = J = \frac{2560000}{6} = 426666.67 \ cm^4$$

$$\therefore \boxed{I_p = J = 4.27 \times 10^5 \ cm^4}$$

Q.4) An angle section 120mm x 100mm x 20mm is placed such as it's longer leg is horizontal. calculate M.I. about centroidal horizontal axis only - W-25

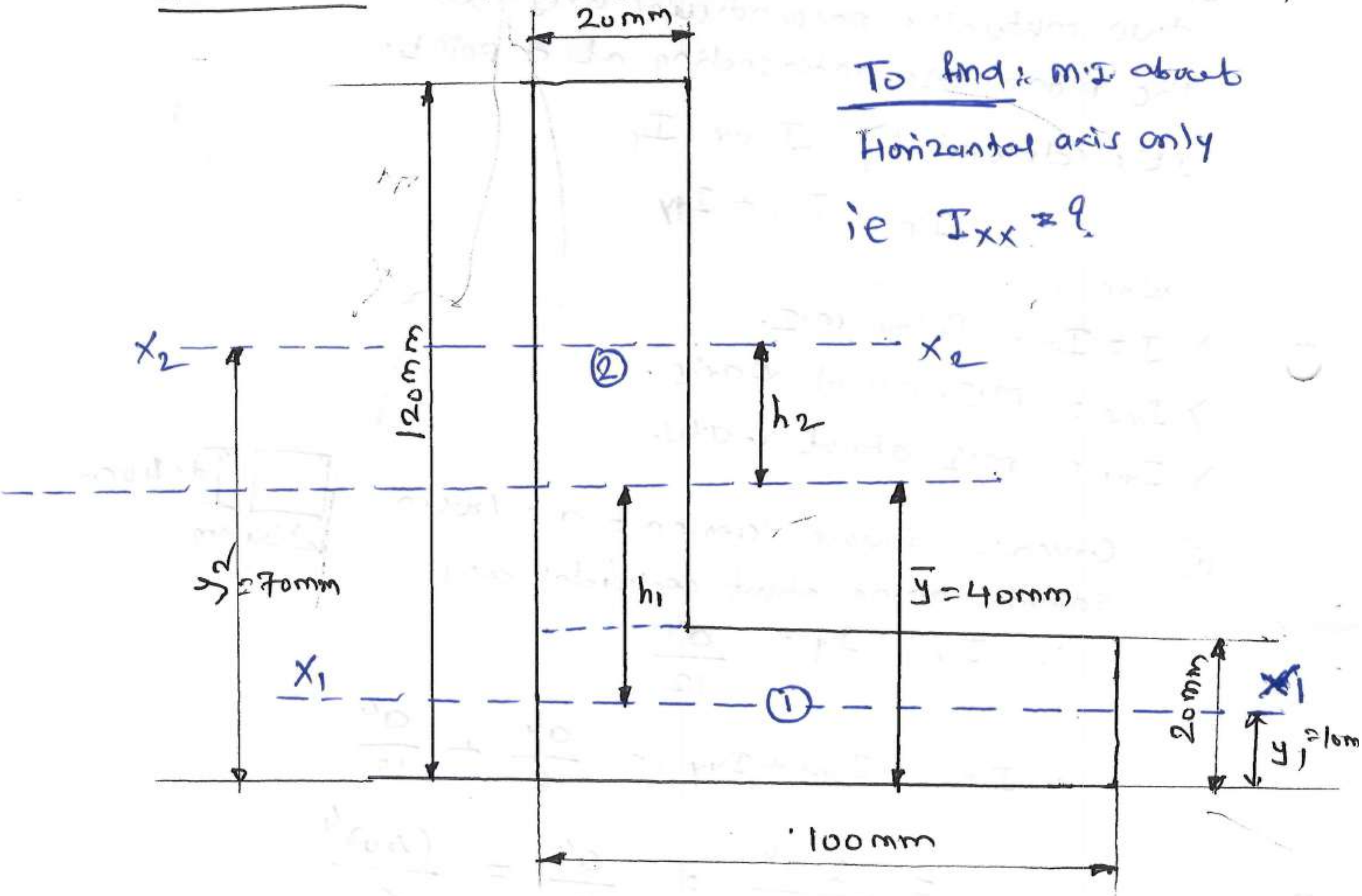
⇒ Soln :. Numerical Based on Angle section.

(L x b x t)

Given Data :- Unequal angle section is 120mm x 100mm x 20mm

To find :- M.I about Horizontal axis only

ie $I_{xx} = ?$



Step :- (I) Split the figure into 2 groups. ① & ② resp.

∴ Part ①, $b_1 = 100mm$ & $d_1 = 20mm$

part ②, $b_2 = 20mm$ & $d_2 = 100mm$

Here Fig. is un-symmetrical about the both axes X-X & Y-Y, we have to use parallel axis therom to find I_{xx} & I_{yy}.

∵ But we required here only $I_{xx} = ?$

$$\therefore I_{xx} = I_{x_1} + I_{x_2}$$

=

Q.4. Ans ⇒ Angle section

$$\therefore I_{xx} = \left(I_{xx_1} + A_1 h_1^2\right) + \left(I_{xx_2} + A_2 h_2^2\right)$$

$$\therefore \quad I_{xx_1} = \frac{b_1 d_1^3}{12} = \frac{100 \times (20)^3}{12}$$

$$= 0.0667 \times 10^6 \text{ mm}^4$$

$$I_{xx_2} = \frac{b_2 d_2^3}{12} = \frac{20 \times (100)^3}{12}$$

$$= 1.667 \times 10^6 \text{ mm}^4$$

Now,

$$A_1 = b_1 \times d_1 = 100 \times 20 = 2000 \text{ mm}^2$$

$$A_2 = b_2 \times d_2 = 20 \times 100 = 2000 \text{ mm}^2$$

To find h_1 & h_2 = ?

Also we find y_1 and y_2 and $\bar{y}$

$\therefore$ for section ① $y_1 = \frac{20}{2} = 10\text{mm}$

for section ② $y_2 = 20 + \frac{100}{2} = 70 \text{ mm}$

$$\therefore \bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(2000 \times 10) + (2000 \times 70)}{2000 + 2000}$$

$$\bar{y} = \underline{\underline{40 \text{ mm}}}$$

$\therefore$ h_1 = Distance betn axes XX and $X_1 - X_1$

$\therefore h_1 = \bar{y} - y_1 = 40 - 10 = 30\text{mm}$

$h_2 = y_2 - \bar{y} = 70 - 40 = 30 \text{mm}$

Q.4. Ans Angle section.

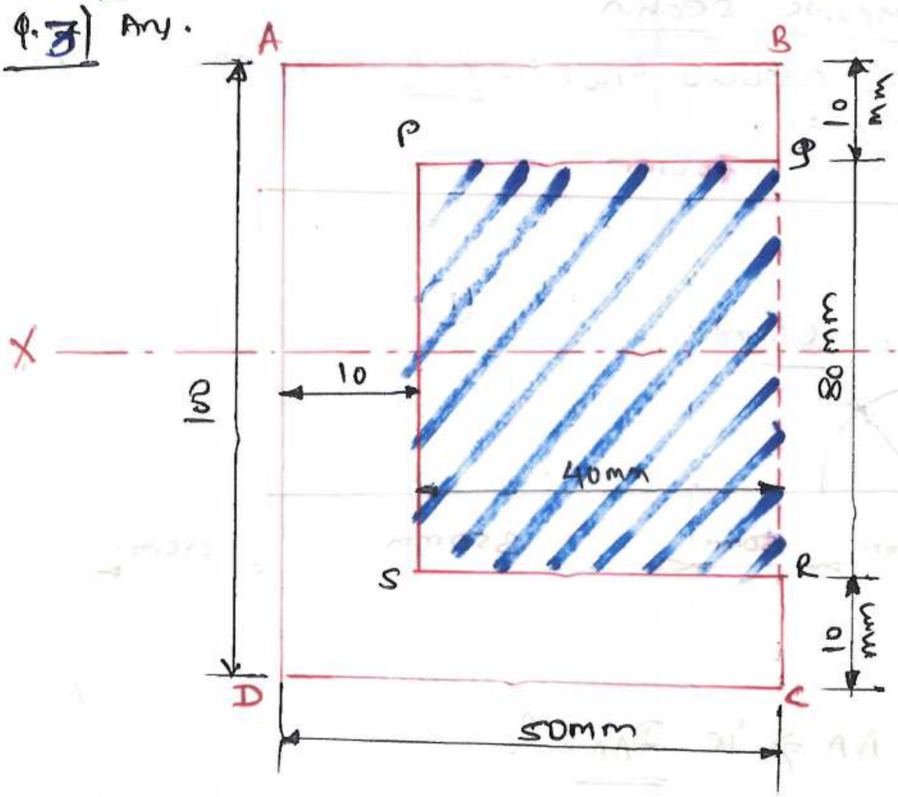
$$\therefore \quad I_{xx} = \left(I_{xx_1} + A_1 h_1^2\right) + \left(I_{xx_2} + A_2 h_2^2\right)$$

$$= \left[0.0667\times10^6 + 1000\times(30)^2\right] + \left[1.667\times10^6 + 1000\times(30)^2\right]$$

$$I_{xx} = \left[0.9667\times10^6 + 2.567\times10^6\right]$$

$$\boxed{I_{xx} = 3.5337\times10^6\ mm^4}$$

Q.37 Calculate the M.I. for the following given section. W-24
Q.7] Any.



Given :- C-section

We find $I_{xx} = ?$

Consider a rectangle "ABCD" having

width (B) = 50mm

Depth (D) = 100mm

& for rectangle "PQRS" having

width (b) = 40mm

depth (d) = 80mm

As the figure is symmetrical about (x-x-axis) there is
∴ ~~Ixx~~ no need to use parallel axis theorem to find $I_{xx} = ?$

$$\therefore I_{xx} = \left[\frac{BD^3}{12}\right]_{ABCD} - \left[\frac{bd^3}{12}\right]_{PQRS}$$

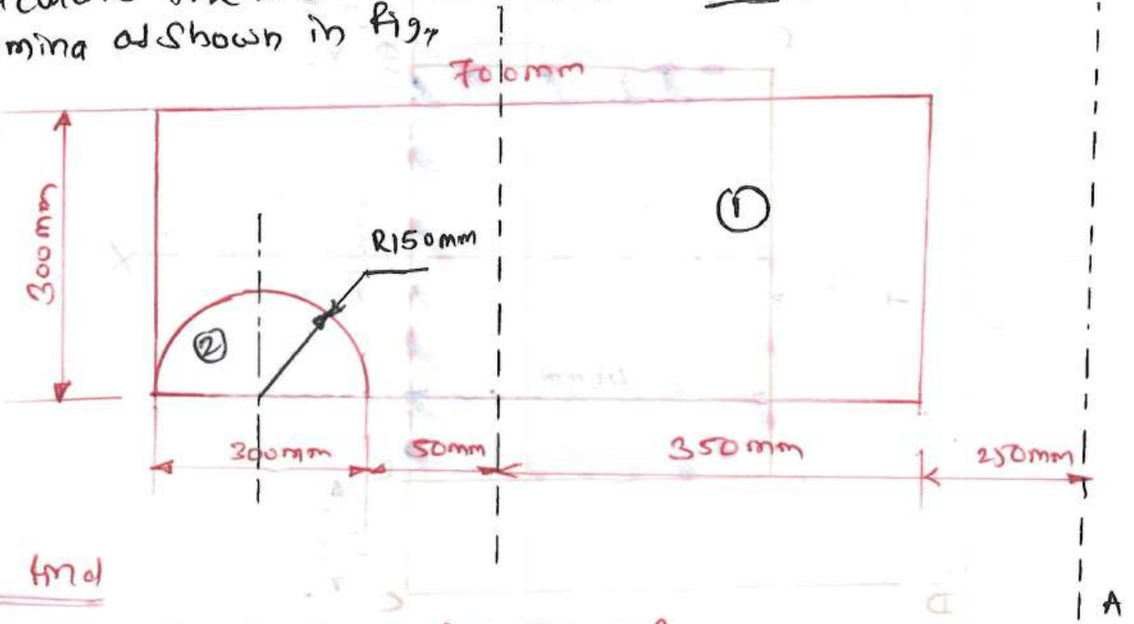
$$= \left[\frac{50 \times (100)^3}{12}\right]_{ABCD} - \left[\frac{40 \times (80)^3}{12}\right]_{PQRS}$$

$$= 4.167 \times 10^6 - 1.706 \times 10^6$$

$$\therefore \boxed{I_{xx} = 2.460 \times 10^6 \text{ mm}^4}$$

Q.6. Sol^n M.I. of composite section

Que :- Calculate the M.I. about the Axis A-A, for the lamina as shown in fig.,



To find

M.I. About AA ⇒ ie $I_{AA} = ?$

Sol^n :- Split the fig. into 2 parts. ① & ②

$$\therefore \quad I_{AA} = I_{11} - I_{22} \quad (I_{Rectangle} - I_{semicircle}) \quad -①$$

$$= \left[I_{yy} + Ah^2\right]_{11} - \left[I_{yy} + Ah^2\right]_{22}$$

$$= \cancel{\left[\frac{hb^3}{12} + (b \times h)\right]}$$

$$= \left[\frac{hb^3}{12} + (b \times b) \times (250+350)^2\right]$$

$$+ \left[\frac{\pi (d)^4}{8} + \frac{\pi R^2}{2} \times (250+350+200)^2\right]$$

$\therefore$ (h) Rectangle from AA = 250+350 = 600 mm

(h) Semicircle = 250+350+200 = 800 mm

$$= \left[\frac{300 \times (700)^3}{12} + (300 \times 700) \times (600)^2\right] + \left[\right.$$

$$+ \left[\frac{\pi (150)^4}{8} + \frac{\pi (150)^2}{2} \times (800)^2\right]$$

$$= (8.575 \times 10^9 + 7.56 \times 10^{10}) - (198.8 \times 10^6 + 2.26 \times 10^{10})$$

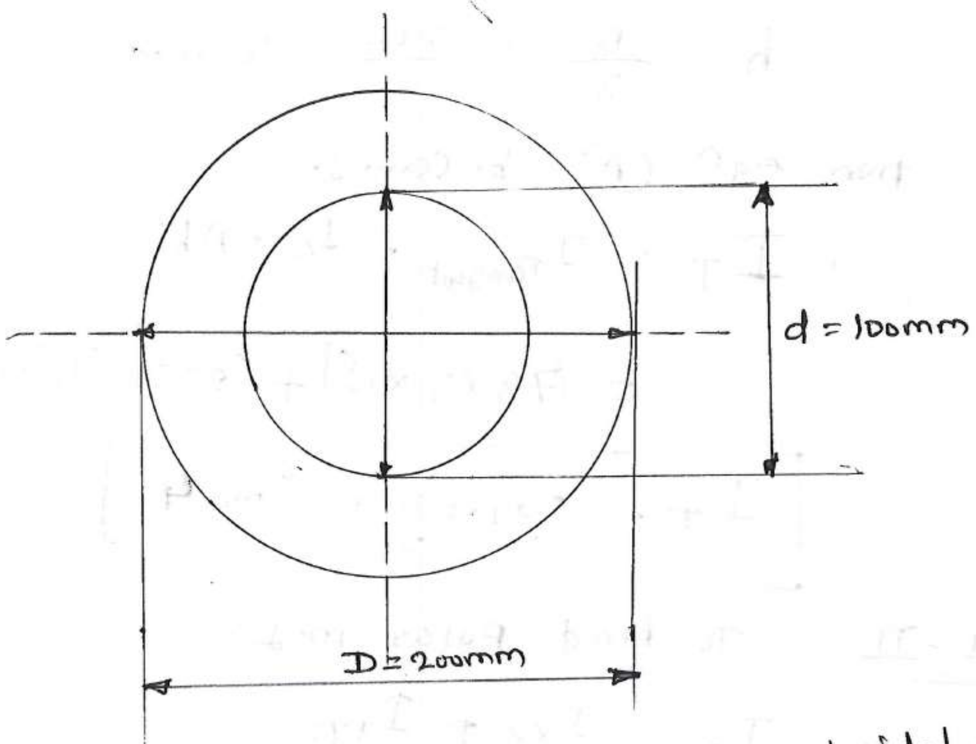
$$= 8.417 \times 10^{10} - 2.28 \times 10^{10}$$

$$\boxed{I_{AA} = 6.1375 \times 10^{10} \text{ mm}^4}$$ —— final Ans

Q. (5) A hollow circular section having 200mm external diameter and 100 mm internal diameter. calculate the M. moment of the section about any of the tangent. Also find polar moment of Inertia. W-25

⇒ Soln :- Given data :- $D = 200 mm$ - External dia.

$d = 100 mm$ - Internal dia.



M.I. of hollow circular section about centroidal axis is -

$$I_G = \frac{\pi}{64}\left(D^4 - d^4\right)$$

$$= \frac{\pi}{64}\left(200^4 - 100^4\right)$$

$$I_G = 73.63 \times 10^6 \ mm^4 \text{ —— } I^{st}$$

To find :-

moment of inertia about any tangent.

Using parallel axis therom

$$I_{Tangent} = I_G + Ah^2 \text{ —— } (A)$$

Q.(5) Answer

Area of Hollow section is given by -

$$\therefore A = \frac{\pi}{4}(D^2 - d^2)$$

$$= \frac{\pi}{4}(200^2 - 100^2)$$

$$= 23565 \text{ mm}^2$$

Distance from centroid from tangent is -

$$h = \frac{D}{2} = \frac{200}{2} = 100 \text{ mm}$$

from eqn (A) becomes.

$$\therefore I_T = I_{Tangent} = I_G + Ah^2$$

$$= (73.63 \times 10^6) + (23565)(100)^2$$

$$\boxed{I_T = 309.28 \times 10^6 \text{ mm}^4}$$

Part - II :- To find polar M.I.

$$\therefore I_p = I_{xx} + I_{yy}$$

for circular section $I_{xx} = I_{yy} = I_G$

$$\therefore I_p = J = I_{xx} + I_{yy} = 2I_G$$

$$\therefore I_p = J = 2 \times I_G = (2 \times 73.63 \times 10^6)$$

$$\boxed{J = I_p = 147.26 \times 10^6 \text{ mm}^4}$$

Summary

∴ (1) Answer :- i) M.I. about any tangent ⇒ 309.28×10^6 mm^4

ii) Polar M.I ⇒ 147.26×10^6 mm^4

Q.6. A hollow square section had inner dimensions $(a \times a)$ and outer dimensions are $(2a \times 2a)$, find M.I. about outer side.

⇒ Given :-

① Innear square section $= (a \times a)$

② outer —||— $= (2a \times 2a)$

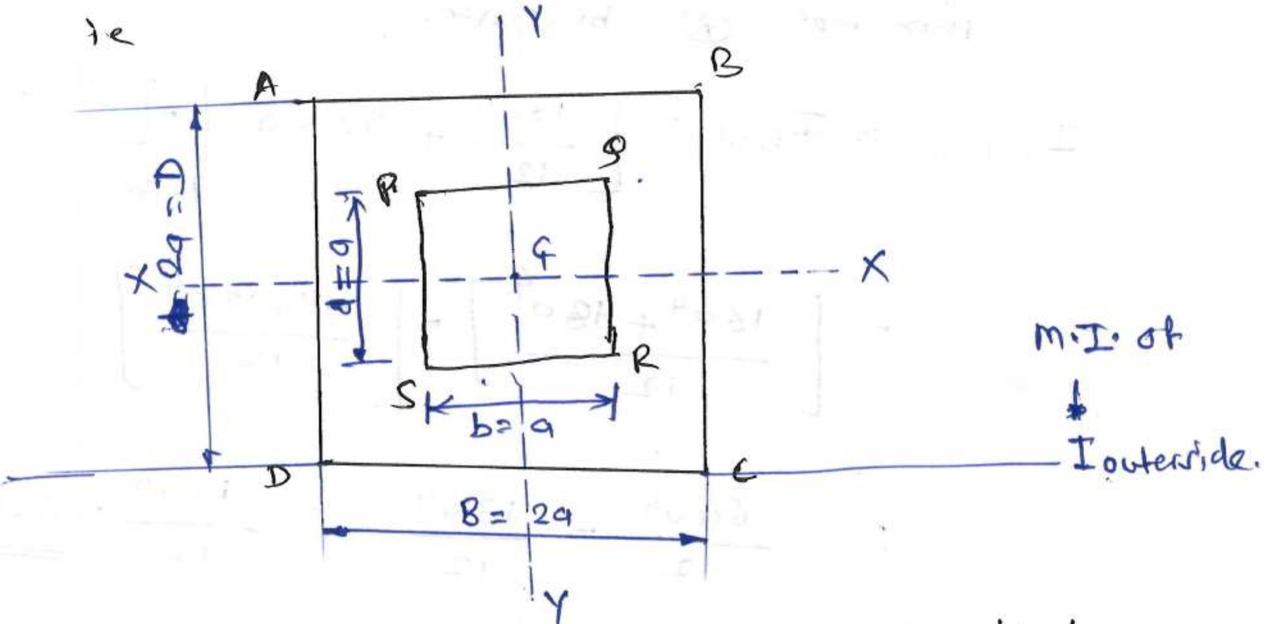
To find :-

① M.I. about outer side ie $I_{outerside} = ?$

Soln :-

$$I_{outerside} = \underset{\substack{\text{square} \\ (ABCD)}}{I_{outerside}} - \underset{\text{square (PQRS)}}{I_{innear\ side}}$$

ie



Now applying Parallel axis therom to find M.I. of outer side of square section

$$\therefore \underset{\substack{\text{Consider here} \\ \text{Base DC}}}{(M.I.)_{outerside}} = \underset{\substack{\text{ie outside} \\ \text{square.}}}{\left[I_{G_1} + A_1 y_1^2\right]_{ABCD}} - \underset{\substack{\text{Inner side} \\ \text{square.}}}{\left[I_{G_2} + A_2 y_2^2\right]_{PQRS}}$$

——— (A)

① Outer side Square dimension and calculation. ie 

$A = 2a \times 2a = 4a^2$

$I_{G_1} = \frac{BD^3}{12} = \frac{2a \times (2a)^3}{12} = \frac{16a^4}{12}$

$y_1 = \frac{2a}{2} = a$

② Inner side squre dimension & calculation :- ie, 

$A = a \times a = a^2$

$I_{G_2} = \frac{bd^3}{12} = \frac{(a)^4}{12}$

$y_2 = \frac{2a}{2} = a$

from eqn Ⓐ becomes.

$I_{outerside}$ or $I_{Base} = \left[\frac{16a^4}{12} + 4a^2 \times a^2\right]_{ABCD} - \left[\frac{a^4}{12} + a^2 \times a^2\right]_{PQRS}$

$= \left[\frac{16a^4 + 48a^4}{12}\right] - \left[\frac{a^4 + 12a^4}{12}\right]$

$= \frac{64a^4}{12} - \frac{13a^4}{12} = \frac{51a^4}{12} = 4.25a^4$

∴ I_{CD} = M.I. outerside or M.I. of Base is given by

$$\boxed{I_{Base} = I_{CD} = I_{outer\,side\,of\,squre} = 4.25\,a^4}$$

Q.7) A circular disc has diameter of 80 mm. Calculate M.I about it's any one tangent. S-25

⇒ Soln:- Given ① $D = 80mm$ ∴ $r = \frac{D}{2} = \frac{80}{2} = 40mm$

To find:- $I_{tangent} = ?$

Apply the parallel axis theorm of a circular disc about tangent

∴ $I_{tangent} = I_G + Ar^2$

$$= \frac{\pi d^4}{64} + \frac{\pi}{4} d^2 \times r^2$$

$$= \frac{\pi \times (80)^4}{64} + \frac{\pi}{4} (80)^2 \times (40)^2$$

$$\boxed{I_{tangent} = 10.05 \times 10^6 \ mm^4}$$

Q.8) Calculate polar M.I. semi circle having 60mm diameter Also calculate minimum radius of gyration. diameter is parallel to Y-Y axis. W-26

⇒ Soln:- Given

① $d = 60mm$

② $r = \frac{d}{2} = \frac{60}{2} = 30mm$

To find:- ① K_{min} = minm radius of Gyration = ?

② $I_p = ?$ ~~also~~

Cond^n:- Diameter is parallel to Y-Y axis are.

∴ $I_{xx} = \frac{\pi d^4}{128} = \frac{\pi (60)^4}{128} = 318086.26 \ mm^4$

$I_{yy} = \frac{\pi d^4}{128} - A\bar{x}^2$ ... { Here parallel axis theorm apply becz diar is parallel to Y-axis.

OR

~~$I_{yy} = I_G - A\bar{x}^2$~~

Q-8. Ans $A = \frac{\pi}{2} \times r^2$ and $\bar{x} = \frac{4r}{3\pi}$

$$\therefore I_{yy} = 318086.26 - \left(\frac{\pi \times \cancel{r}^2}{2}\right) \times \frac{4R}{3\pi}$$

$$= 318086.26 - \left[\left[\frac{\cancel{\pi} \times (30)^2}{2}\right] \times \left(\frac{4 \times 30}{3\cancel{\pi}}\right)^2\right]$$

$$= \cancel{318086.26 - \frac{(30)^3 \times 4}{6}}$$

$$= \cancel{318086.26 - 18000}$$

$$= 318086.26 - \left[(1413.12) \times (12.73)^2\right]$$

$$I_{yy} = 88882.7 \text{ mm}^4$$

Now.

① Polar M.I. $\Rightarrow I_p = J = I_{xx} + I_{yy}$

$$I_p = J = 318086.26 + 88882.7$$

$$\boxed{I_p = J = 406968.96 \text{ mm}^4}$$ — ① Ans

② Minimum radius of Gyration :-

$$K_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{I_{yy}}{A}}$$

$$= \sqrt{\frac{88882.7}{1413.12}}$$

$$\boxed{K_{min} = 7.93 \text{ mm}}$$ — ② Ans

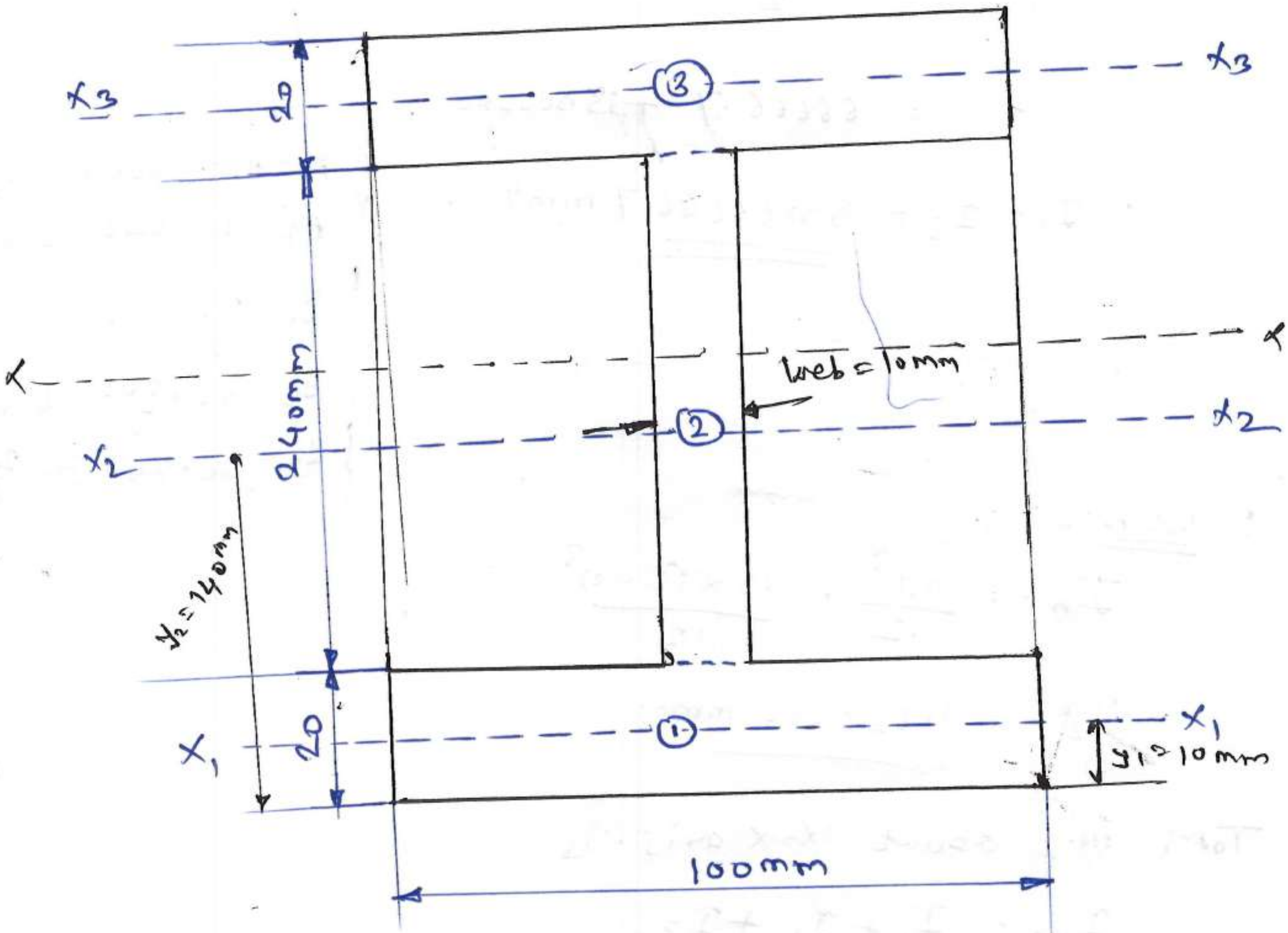
Q-9) Find M.I. of symmetrical I-section having following details. — W-26

(1) Flanges :- 100mm × 20mm

(2) Overall depth : 280 mm

(3) thickness of web :- 10 mm

⟹ Soln :- (1) Draw the images I-section 4 diveded into (3) parts shown in below.



Step :- (1) find centroid (Netural axis)

since it's symmetric about x-x-axis

$$\therefore \quad \bar{Y} = \frac{280}{2} = 140mm$$

(2) M.I. about x-x axis :-

Using Parallel axis therom.

Q.9. Ans

For Top & Bottom flanges:-

$$I_{xx} = \frac{bd^3}{12} + Ah^2 \quad \cdots \left\{ \begin{array}{l} h_1 = \bar{y} - y_1 = 140 - 10 = 130mm \\ h_3 = \bar{y} - y_3 = \cancel{240-10}\ 270 - 140 = 130mm \end{array} \right.$$

$$\therefore \quad I_1 = I_3 = \frac{100 \times (20)^3}{12} + (100 \times 20)(130)^2$$

$$= 66666.7 + 33800000$$

$$\therefore I_1 = I_3 = \underline{\underline{33866666.7\ mm^4}} \quad \cdots$$

$A_1 = A_3 = 100 \times 20 = 2000mm^2$

$A_2 = 10 \times 240 = 2400mm^2$

$y_1 = \frac{20}{2} = 10mm$

$y_2 = 20 + \frac{240}{2} = 140mm$

$y_3 = 20 + 240 + \frac{20}{2} = 270mm$

$\therefore$ Web:-

$$I_2 = \frac{bd^3}{12} = \frac{10 \times (240)^3}{12}$$

$$I_2 = \underline{\underline{11520000\ mm^4}}$$

Total M.I. about X-X axis. is

$$I_{xx} = I_1 + I_2 + I_3$$

$$= 33866666.7 + 11520000 + 33866666.7$$

$$\boxed{I_{xx} = 79.25\ mm^4}$$

Step:- 2 M.I. about Y-Y axis

$$I_{yy} = \frac{db^3}{12}$$

Flanges:- $I_{yy_1} = I_{yy_3} = \frac{db^3}{12} = \frac{20 \times (100)^3}{12} = 1666666.7\ mm^4$

Web :- $I_{yy_3} = \frac{db^3}{12} = \frac{240 \times (10)^3}{12} = 20000\ mm^4$

P.T.O.

Total M.I. about Y-Y-axis is

$$I_{yy} = I_1 + I_2 + I_3$$

$$= 1666666.7 + 20000 + 1666666.7$$

$$\boxed{I_{yy} = 3.35 \times 10^6 \ mm^4}$$

6-Marks Questions.

Un-Symmetrical Sections :-

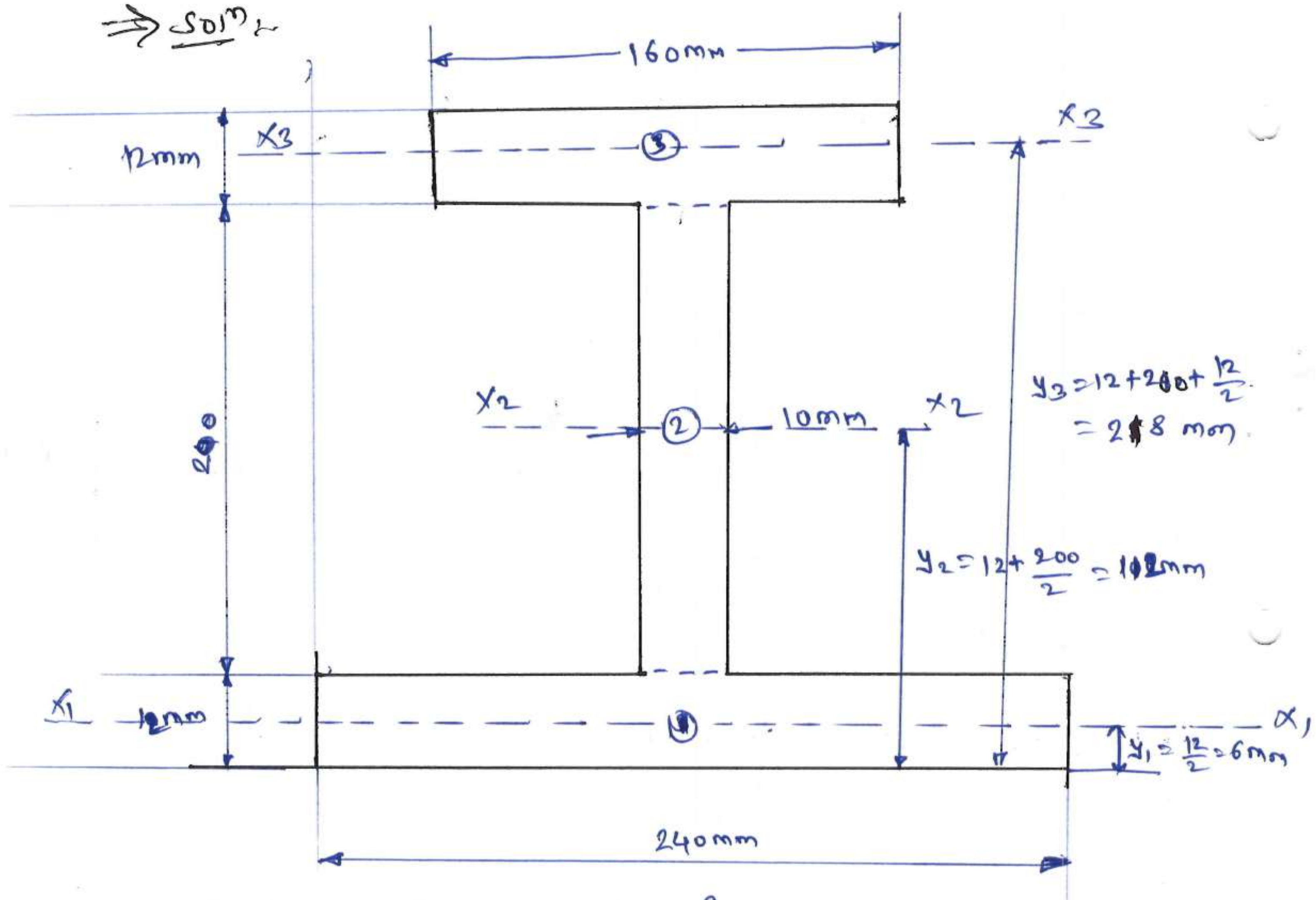
Q.10) Determine the M.I. of unsymmetrical section - I having following details. S.25

① Top Flange :- 160mm × 12 mm

② Bottom Flange :- 240mm × 12 mm

③ Web :- 200 mm × 10 mm.

⇒ Soln :-



To find :- I_{xx} and I_{yy} = ?

⇒ Soln :- $A_1 = \overset{b_1 \times d_1}{240 \times 12} = b_1 d_1 = 2880\ mm^2$ { ∵ $b_1 = 240$mm & $d_1 = 12$mm

$A_2 = 200 \times 10 = 2000 mm^2$ ... { $b_2 = 10$mm, & $d_2 = 200$mm

$A_3 = 160 \times 12 = 1920 mm^2$ ... { $b_3 = 160$mm & $d_3 = 12$mm

Q.10 Ans

Fig. is un-symmetrical about x-x-axis

∴ We use Parallel axis theorem,

$$\therefore I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3} \quad \text{— (A)}$$

$$I_{xx} = \left(I_{G_1} + A_1 h_1^2\right) + \left(I_{G_2} + A_2 h_2^2\right) + \left(I_{G_3} + A_3 h_3^2\right) \quad \text{— (1)}$$

Now,

$$I_{G_1} = \frac{b_1 d_1^3}{12} = \frac{240 \times (12)^3}{12} = 34.56 \times 10^3 \text{ mm}^4$$

$$I_{G_2} = \frac{b_2 d_2^3}{12} = \frac{10 \times (200)^3}{12} = 6666.67 \times 10^3 \text{ mm}^4$$

$$I_{G_3} = \frac{b_3 d_3^3}{12} = \frac{160 \times (12)^3}{12} = 23.04 \times 10^3 \text{ mm}^4$$

Now we have to find y_1, y_2 and y_3, $\bar{y}$

for section-(1) : $y_1 = \frac{12}{2} = 6$ mm

— " — (2) $y_2 = 12 + \frac{200}{2} = 112$ mm

— " — (3) $y_3 = 12 + 200 + \frac{12}{2} = 218$ mm

$$\therefore \bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(2880 \times 6) + (2000 \times 112) + (1920 \times 218)}{2880 + 2000 + 1920}$$

$$\boxed{\bar{y} = 97.035 \text{ mm}} \simeq \underline{97.04 \text{ mm}}$$

Now,

$h_1 = \bar{y} - y_1 = 97.04 - 6 = 91.04$ mm

$h_2 = y_2 - \bar{y} = 112 - 97.04 = 14.96$ mm

$h_3 = y_3 - \bar{y} = 218 - 97.04 = 120.96$ mm

Q.10 Ans

Put all values in eqn ① becomes.

$$I_{xx} = \left[(34.56 \times 10^3 + 2880 \times (91.04)^2 \right]_1 + \left[(6666.67 \times 10^3 + 2000 \times (14.96)^2 \right]_2$$

$$+ \left[(23.04 \times 10^3 + 1920 \times (120.96)^2 \right]_3$$

$$I_{xx} = (23.904 \times 10^6)_1 + (7.114 \times 10^6)_2 + (28.115 \times 10^6)$$

$$\boxed{I_{xx} = 59.133 \times 10^6 \text{ mm}^4}$$

Step-③ To find I_{yy} = ?

As the fig. is symmetrical about y-y axis so no need to use parallel axis theorem.

$$\therefore I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}$$

$$= \left[\frac{d_1 b_1^3}{12} \right] + \left[\frac{d_2 b_2^3}{12} \right] + \left[\frac{d_3 b_3^3}{12} \right]$$

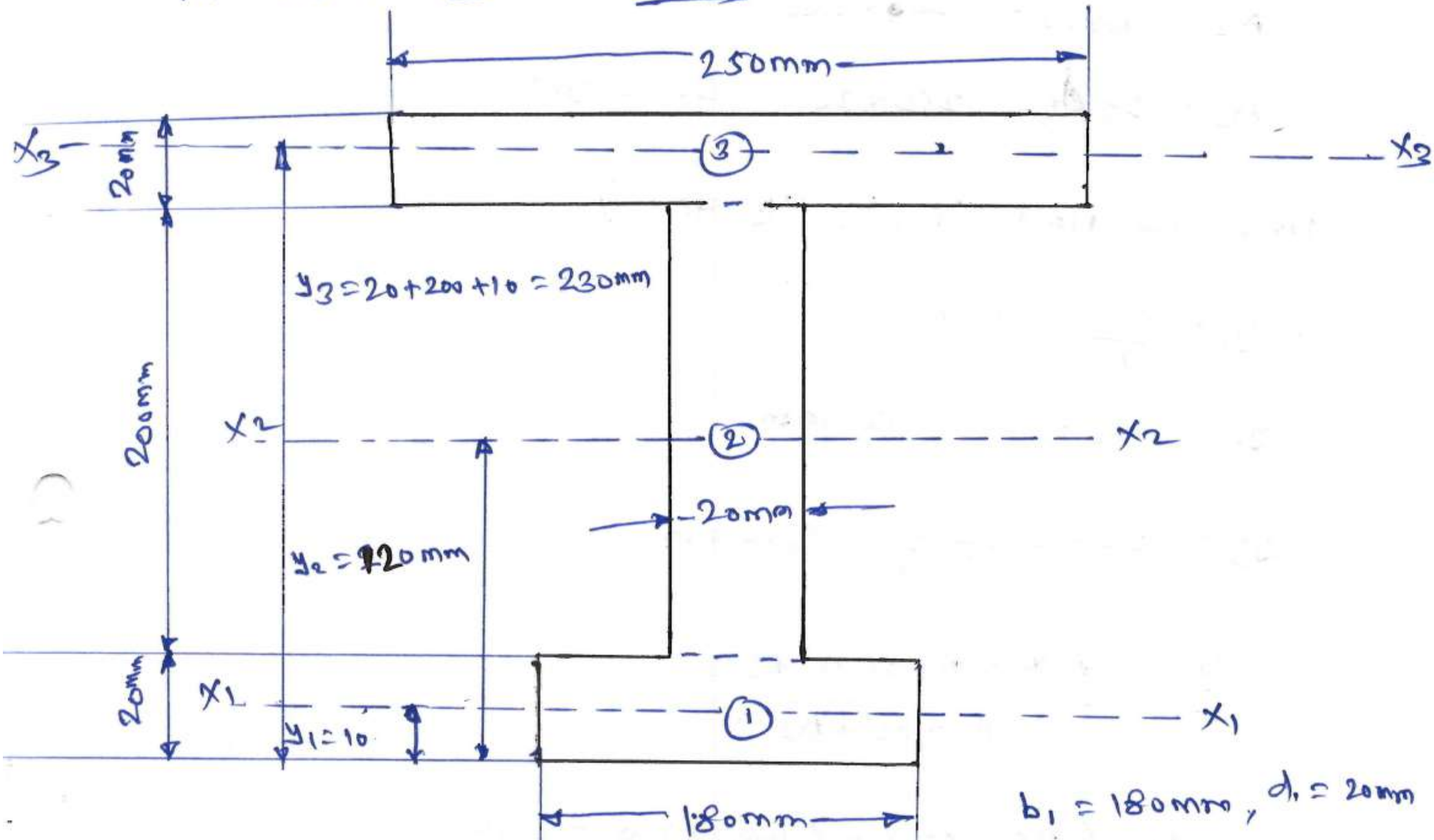
$$= \left[\frac{12 \times (240)^3}{12} \right] + \left[\frac{200 \times (10)^3}{12} \right] + \left[\frac{12 \times (160)^3}{12} \right]$$

$$= (13.824 \times 10^6 + 0.01667 \times 10^6 + 4.096 \times 10^6)$$

$$\boxed{I_{yy} = 17.94 \times 10^6 \text{ mm}^4}$$

06 marks Questions

Q.② Determine M.I. about X-X and Y-Y axis as shown in Fig. No. ⑤ S-26



$b_1 = 180mm$, $d_1 = 20mm$

$b_2 = 20mm$, $d_2 = 200mm$

$b_3 = 250mm$, $d_3 = 20mm$

To find : ① $I_{xx} = ?$

② $I_{yy} = ?$

⇒ Sol^n. To find I_{xx}

Now Here Applying Parallel axis theorm

$$I_{xx} = \left[I_{xx_1} + A_1h_1^2\right] + \left[I_{xx_2} + A_2h_2^2\right] + \left[I_{xx_3} + A_3h_3^2\right] \quad -①$$

Now,

$$I_{xx_1} = \frac{b_1d_1^3}{12} = \frac{180 \times (20)^3}{12} = 0.12 \times 10^6 \text{ mm}^4$$

$$I_{xx_2} = \frac{b_2d_2^3}{12} = \frac{20 \times (200)^3}{12} = 13.33 \times 10^6 \text{ mm}^4$$

$$I_{xx_3} = \frac{b_3d_3^3}{12} = \frac{250 \times (20)^3}{12} = 0.166 \times 10^6 \text{ mm}^4$$

Step-(2)

$A_1 = b_1 d_1 = 180 \times 20 = 3600 mm^2$

$A_2 = b_2 d_2 = 20 \times 200 = 4000 mm^2$

$A_3 = b_3 d_3 = 250 \times 20 = 5000 mm^2$

Now, we find y_1, y_2, y_3 and $\bar{y}$

$\therefore y_1 = \frac{20}{2} = 10 mm$

$y_2 = 20 + \frac{200}{2} = 120 mm$

$y_3 = 20 + 200 + \frac{20}{2} = 230 mm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(3600 \times 10) + (4000 \times 120) + (5000 \times 230)}{3600 + 4000 + 5000}$$

$$\bar{y} = 132.22 mm$$

Also,

$\therefore h_1 = \bar{y} - y_1 = 132.22 - 10 = 122.22 mm$

$h_2 = \bar{y} - y_2 = 132.22 - 120 = 12.22 mm$

$h_3 = y_3 - \bar{y} = 230 - 132.22 = 97.78 mm$

Put all values in eqn (1) becomes.

$$I_{xx} = \left[(0.12 \times 10^6 + 3600 \times (122.22)^2\right]_1 + \left[(13.33 \times 10^6 + 4000 \times (12.22)^2\right]_2$$
$$+ \left[(0.166 \times 10^6 + 5000 \times (97.78)^2\right]$$

$$I_{xx} = (53.895 \times 10^6)_1 + (13.927 \times 10^6)_2 + (47.97 \times 10^6)_3$$

$$\therefore \boxed{I_{xx} = \underline{115.792 \times 10^6}\ mm^4}$$

Step ③:– As fig. ⑤ is symmetrical about y-y axis so there is no need to apply the parallel axis ~~theorem~~, theorem.

$$\therefore I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3} \quad \text{— (B)}$$

$$\therefore I_{yy_1} = \frac{d_1 b_1^3}{12} = \frac{20 \times (180)^3}{12} = 9.72 \times 10^6\ mm^4$$

$$I_{yy_2} = \frac{d_2 b_2^3}{12} = \frac{200 \times (20)^3}{12} = 0.133 \times 10^6\ mm^4$$

$$I_{yy_3} = \frac{d_3 b_3^3}{12} = \frac{20 \times (280)^3}{12} = 26.04 \times 10^6\ mm^4$$

put in eqn (B) becomes.

$$\therefore I_{yy} = (9.72 \times 10^6)_1 + (0.133 \times 10^6)_2 + (26.04 \times 10^6)_3$$

$$\boxed{I_{yy} = \underline{35.89 \times 10^6}\ mm^4}$$