R.C.Patel College Of Engineering & Polytechnic, Shirpur

Department of Civil Engineering

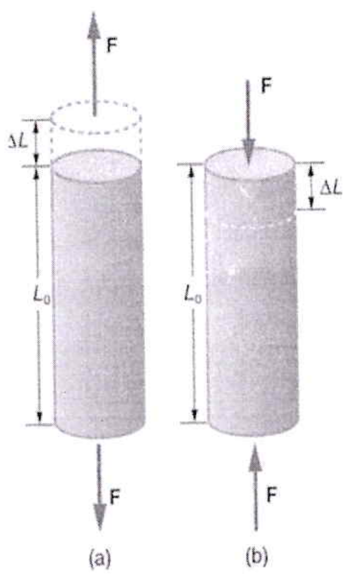
Course Title- Strength of Material

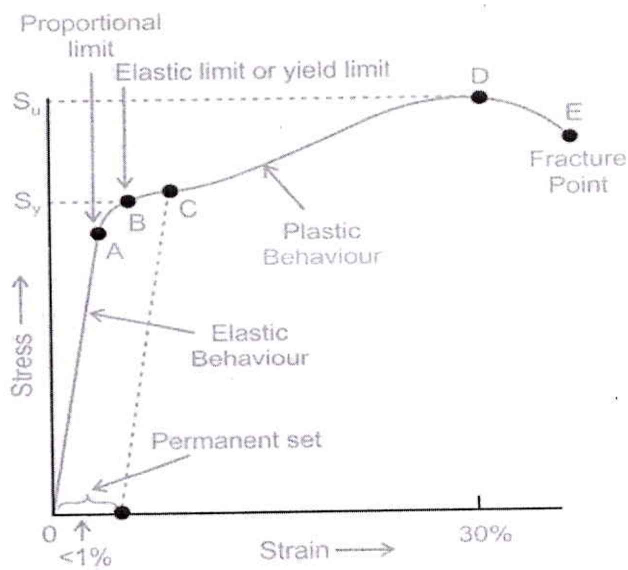
Programme Name -Civil Engineering

Course Code -313308

Semester-Third

Unit	Title	COs	Learning hours	R Level	U Level	A Level	Total Marks
II	Simple Stresses, Strains & Elastic Constants	CO2	16	6	8	4	18





Subject Teacher

Mr.R.B.Patil

PROPERTIES OF MATERIALS :-

1. ELASTICITY :- It is the property of material by virtue of which regains it's original shape and size after removal forces.

2. PLASTICITY :- It is the property of material by virtue of which it does not regains it's original shape and size after removal of forces.

3. DUCTILITY :- It is the property of material due to which it can be drawn into thin wires after applying tensile force.

4. MALLEABILITY :- It is the property of material due to which it can be beaten up into thin sheets without cracking when loading.

5. BRITTLENESS :- It is the tendancy of material to fracture or fail upon the application of small ammount of force.

6. FATIGUE :- Failure of material under cycling loading

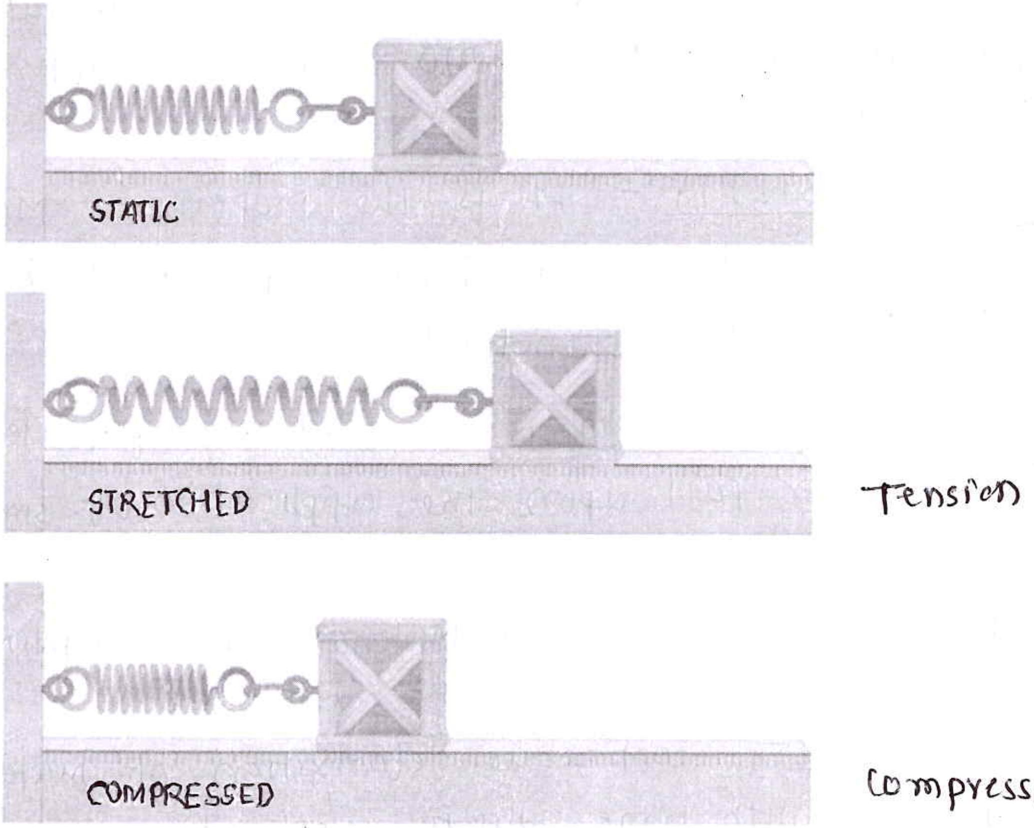
7. CREEP :- contineous deformation of material which undergoes with time due to application of external steady load [e.g. Boilers, steam turbine]

8. TOUGHNESS :- Tendancy of body it does not break after sudden load.

ELASTIC BODY :-

After applying external load it gets deform or change it's dimensions But the body regain it's original dimensions after complete removal of Applied force within it.

e.g. Al, mild steel, copper.



ELASTIC LIMIT - The maximum stress upto which body behaves elastically is called elastic limit.

STRESS:– stress is internal resistance or counter-force of material against deformation is called as stress.

i.e. $$\text{stress} = \frac{\text{Applied load}}{\text{Area of c/s.}}$$

$$\boxed{\sigma = \frac{P}{A}}$$

Here,
σ = stress
P = Applied load.
A = Area of c/s.

Unit = N/m^2 or Mpa.

Important units :–

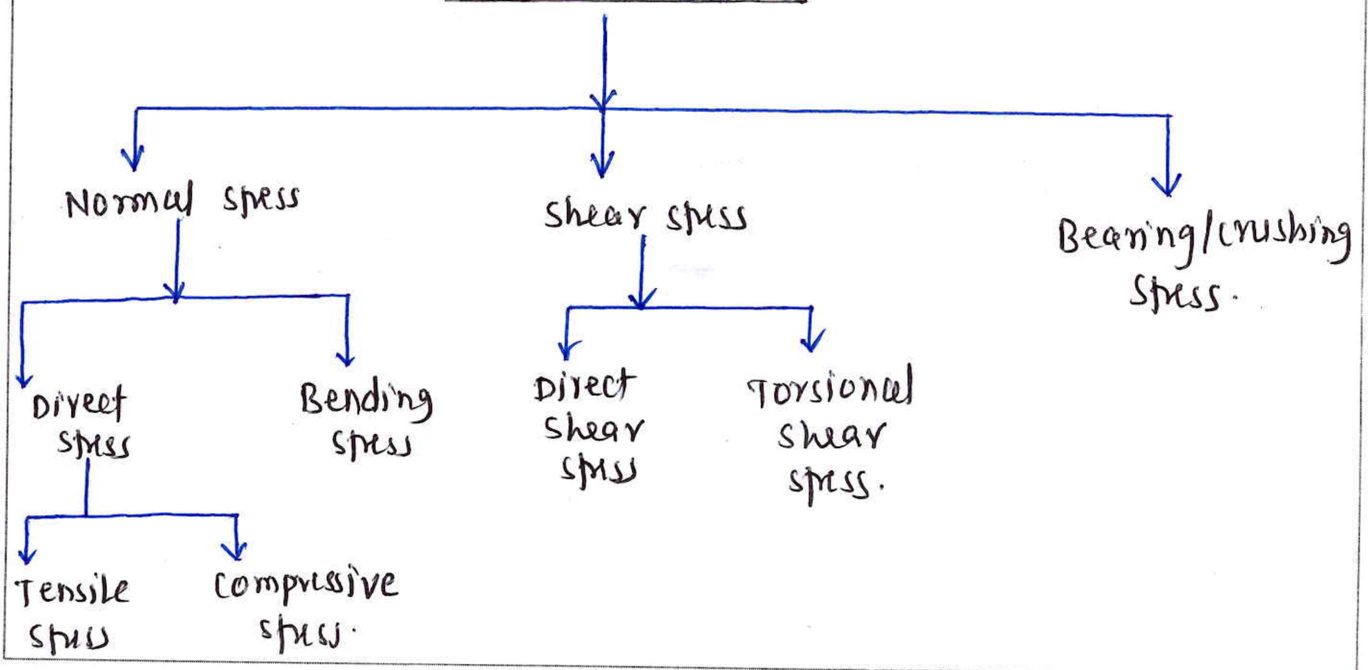
pascal

$1\,pa = 1\,N/m^2$
$1\,kpa = 10^3\,pa.$
$1\,Mpa = 10^6\,pa.$
$1\,Gpa = 10^9\,pa.$

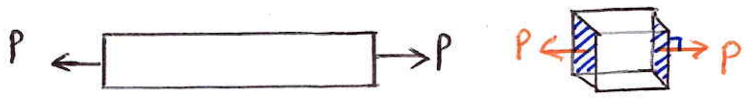
Important Relation :–

$1\,N/mm^2 = 1\,Mpa.$

CLASSIFICATION.



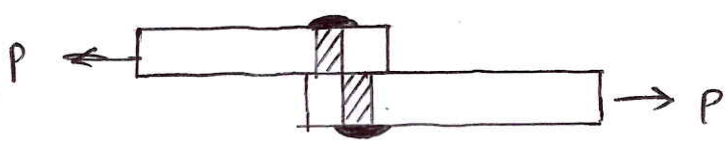
1. **NORMAL STRESS :-** Normal stress induced into the body when an external force acts Normal to the area of cross section of body.



2. **DIRECT STRESS :-** Direct stress induced into the body due to axial load acting directly on the body.

3. **BENDING STRESS :-** When the body possess bending moment due to external loads, ~~Body~~ then Bending stress induced into body.

4. **SHEAR STRESS :-** When the body is subjected to two equal and opposite forces acting tangentially across the resisting section, then the body tends to shear off the section, then the stress induced in the body is called shear stress.



$$\text{Shear stress} = \frac{\text{tangential force}}{\text{c/s area.}}$$

HOOKE'S LAW :-

When a material is loaded, within it's elastic limit, the stress is proportional to the strain.

i.e $\boxed{\dfrac{\text{stress}}{\text{strain}} = E \text{ (constant)}}$

E = Modulus of elasticity or young's modulus.

POISSON'S RATIO :-

If a body stressed within it's elastic limit the lateral strain bears a constant ratio to the linear strain.

i.e. $\boxed{\text{poisson's ratio} = \dfrac{\text{Lateral strain}}{\text{Linear strain}} = \text{constant}}$

- It is denoted by $\frac{1}{m}$ or μ

For steel - 0.27 - 0.3
CI - 0.2 - 0.3

[poisson is a french mathematician]

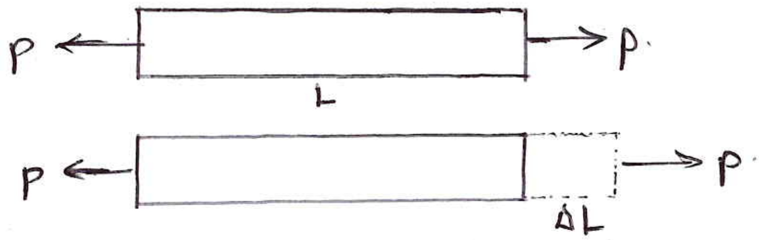
LINEAR STRAIN :-

When an elastic body subjected to axial tensile force, then the deformation per unit length in the direction of the force.

i.e $\dfrac{\delta l}{l}$

STRAIN :- Whenever a forces acts on a body, it undergoes some deformation, this deformation per unit length is known as strain.

i.e strain = $\dfrac{\text{change in length}}{\text{original length.}}$



$\therefore$ strain = $\dfrac{\Delta L}{L}$

$\therefore e = \dfrac{\Delta L}{L}$

$\therefore$ strain has no unit.

LATERAL STRAIN :-

When an elastic body is subjected to axial load then their will be change in dimensions perpendicular to applied force.

$$e_{La} = \frac{\delta b}{b} \text{ or } \frac{\delta t}{t}$$

[For rectangular]

$$e_{La} = \frac{\delta d}{d}.$$

[for circular]

YOUNG'S MODULUS :- [MODULUS OF ELASTICITY]

Whenever a material is loaded within it's elastic limit stress and strain are proportional to each other.

$$\therefore \sigma \propto e$$

$$\sigma = E \cdot e.$$

$$\therefore \boxed{E = \frac{\sigma}{e}.}$$

SHEAR MODULUS :- [MODULUS OF RIGIDITY]

It is the ratio of shear stress to shear strain is called shear modulus.

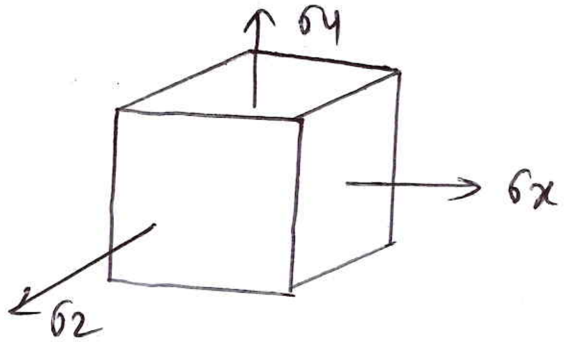
- Which is denoted by G

$$\therefore \boxed{G = \frac{\text{shear stress}}{\text{shear strain}}}$$

Relation betn E and G = $\underline{E = 2G(1+\mu)}$

BULK MODULUS :-

It is the ratio of direct stress to volumetric strain is called as bulk modulus.



Which is denoted by K.

$$\therefore \boxed{K = \frac{\text{Direct stress}}{\text{volumetric strain}}}$$

Relation between Young's modulus and bulk modulus = $\underline{E = 3K(1-2\mu)}$

RELATION BETWEEN E, G, K :-

As we know that, Relation betn E and K.

$$E = 3K(1-2\mu) \quad \text{--- (1)}$$

Relation betn E and G.

$$E = 2G(1+\mu)$$

$$\therefore \frac{E}{2G} = \cancel{2G}(1+\mu)$$

$$\frac{E}{2G} - 1 = \mu$$

$$\therefore \mu = \frac{E}{2G} - 1$$

Put this value into eqn (1).

$$\therefore E = 3K\left[\left(1 - 2 \times \left(\frac{E}{2G} - 1\right)\right)\right]$$

$$= 3K\left[1 - \frac{E}{G} + 2\right]$$

$$= 3K\left[3 - \frac{E}{G}\right]$$

$$= 3K\left[\frac{3G-E}{G}\right]$$

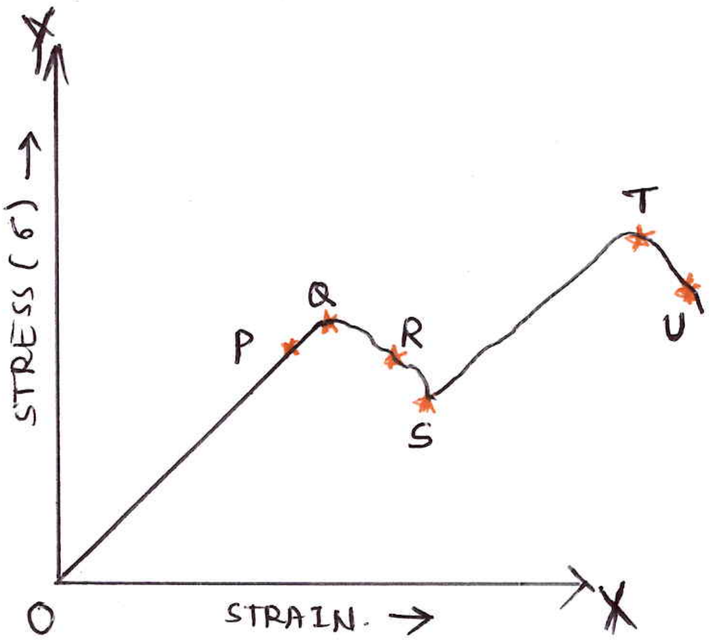
$$EG = 3K[3G-E]$$

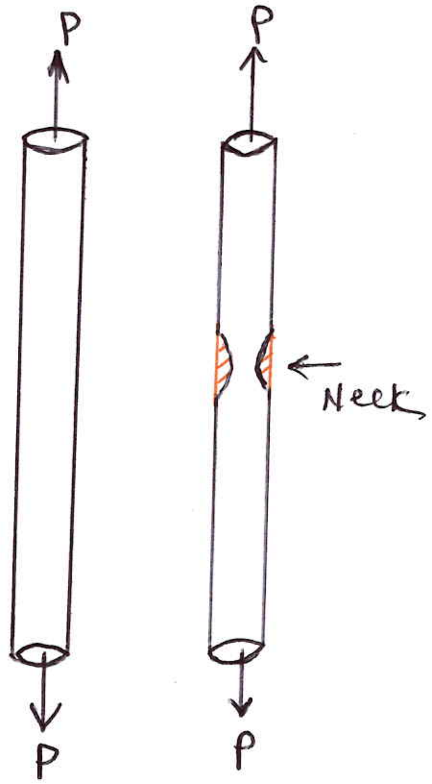
$$= 9KG - 3KE$$

$$EG = 3K(3G-E)$$

$$\boxed{\therefore E = \frac{3K(3G-E)}{G}}$$

STRESS - STRAIN CURVE FOR MILD STEEL UNDER TENSILE LOADING, AND BEHAVIOUR OF MATERIAL WITH RESPECT TO SALIENT POINTS ON GRAPH.





Location of points.

P - proportional limit.

Q - Elastic limit.

R - upper yield point.

S - Lower yield point.

T - Ultimate load point.

U - Breaking point

Here, OP - limit of proportionality

PQ - limit of elasticity

① P- (proportional limit)

- If tensile load is applied to mild steel it will have some deformation.

- If the force is in small ammount gradually applied then the stress-strain graph will increase in straight line.

② Q - (Elastic limit)

- Applied force increased then material experience elastic deformation up to point Q.

③ R-S (Yield stress point)

- The stress after which material deformation occures more quickely with no increase in load.
- plasticity will be occure beyond the point Q.
- R- upper yield stress.
  S - lower yield stress.

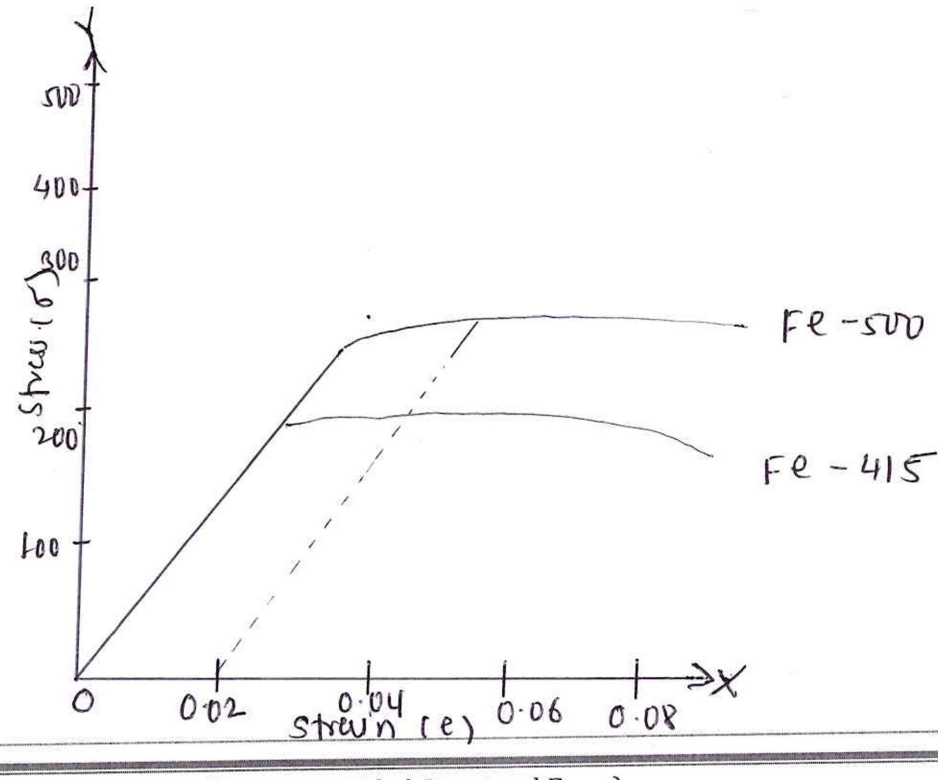
④ T- (ultimate stress)

- it will give maximum value of stress
- it is the point at which material gain maximum strength before Breaking or failure

⑤ U- (Breaking stress)

- After ultimate stress point material reach to neck deformation at which c/s area will be reduced.

STRESS - STRAIN CURVE FOR STEEL BAR UNDER AXIAL LOAD



HYSD :- High yield strength deformed bars.

- HYSD bar contain high percentage of carbon as compared to mild steel so the strength of HYSD bar is greater than mild steel.
- their are two types of HYSD bars.

i) Hot rolled high yield strength bars.

ii) cold twisted high yield strength bars. (Tor steel)

Fe - 500 and Fe - 415.

specification :-

- It has 50% more yield stress than mild steel.
- It has corrugations on surface of the bar to increase the bond and prevent sliping.
- This bar does not show any yield point.
- For HYSD bars yield stress point is considered as 0.02 i.e. 0.2% of proof stress.
- The point where this lines cuts the stress strain curve is taken as yield stress.

i.e - 0.2% of proof stress.

WORKING STRESS :-

It is the ratio of direct axial load to the origional c/s area as called as working stress.

$$\therefore \text{working stress} = \frac{\text{direct load}}{\text{original c/s area.}}$$

safe working stress = maximum allowable stress.

safe working stress < Elastic limit.

PROOF STRESS :-

proof stress is defined as the stress at where material goes to permanent deformations i.e plastic deformation.

FACTOR OF SAFETY :-

It is the ratio of maximum stress to working stress.

$$FOS = \frac{\text{maximum stress}}{\text{working stress.}}$$

$$\text{FOS for ductile material} = \frac{\text{Yield stress}}{\text{working stress.}}$$

$$\text{FOS for Brittle Material} = \frac{\text{Ultimate stress}}{\text{working stress.}}$$

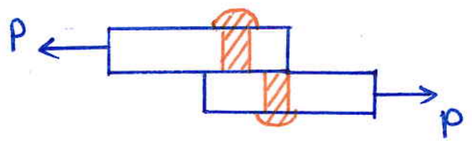
PERCENTAGE OF ELONGATION :-

$$\% \text{ of elongation} = \frac{\text{Final length} - \text{Initial length}}{\text{Initial length}} \times 100$$

SHEAR STRAIN :-

It is the ratio of length of deformation to the perpendicular length in the section of force applied.

SINGLE SHEAR :-



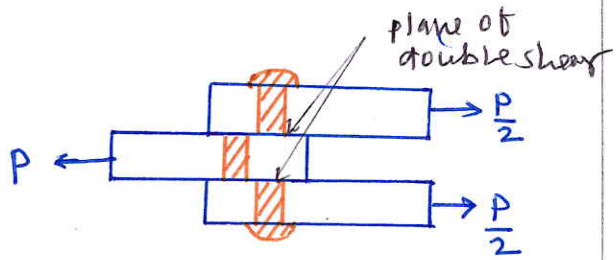
∴ shear stress = $\frac{\text{shear force}}{\text{Area}}$

$$q = \frac{P}{A}$$

$$\therefore q = \frac{P}{\frac{\pi}{4}d^2}$$

d = dia. of rivet.

DOUBLE SHEAR :-

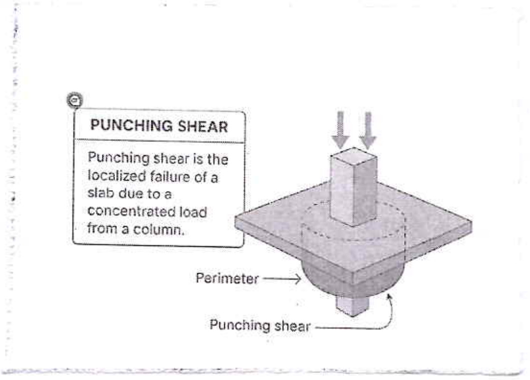


$$q = \frac{P}{2A}$$

$$\therefore q = \frac{P}{\frac{2\pi}{4}d^2}$$

PUNCHING SHEAR :-

When a plate is subjected to some punch into the plate, then circumference of the punch faces some shear resistance from that plate is called as punching shear.



$\therefore$ shear stress $= \frac{F}{A}$

Here, shear area = circumference x thickness

$= \pi d \times t$

$\therefore q = \frac{F}{\pi d t}$.

DEFORMATION OF BODY SUBJECTED TO AXIAL FORCE FOR UNIFORMED AND STEPPED SECTION :-

⇒ consider a body subjected to axial tensile stress.

let P = Force acting on body

l = length of body

A = Area of c/s.

σ = stress

E = Young's modulus.

e = strain

δl = deformation

As we know that,

$\sigma = \frac{P}{A}$ and strain $e = \frac{\sigma}{E}$

$\therefore e = \frac{P}{AE}$ —(I)

we know,

$e = \frac{\delta L}{L}$ —(II)

$\therefore$ equating eqn (I) and (II)

$\frac{P}{AE} = \frac{\delta L}{L}$

$$\boxed{\therefore \delta L = \frac{PL}{AE}}$$

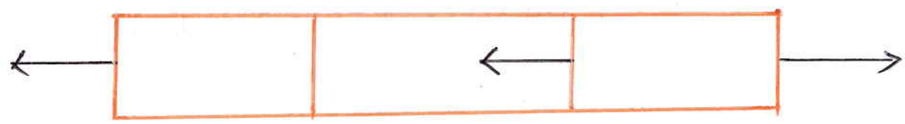
DEFORMATION OF BODY SUBJECTED TO FORCES AT IT'S INTERMIDIATE SECTIONS :- [UNIFORM BODY]

- sometimes the sections subjected a number of forces along the length of the body. In such a case Forces are split up and their effects are considered on Indivisual sections.
- The resulting deformation, is equal to algebric sum of the deformations of the indivisual sections.

$$\therefore \quad \delta L = \frac{PL}{AE}$$

$$\boxed{\delta L = \frac{1}{AE}\left(P_1L_1 + P_2L_2 + P_3L_3 + \cdots P_nL_n\right)}$$

$\therefore$ It is also known as principle of superposition.



NUMERICALS :-

① For a certain material, modulus of Elasticity is 169 mpa, If poisson's ratio is 0.32 calculate the values of modulus of rigidity and bulk modulus.

— WINTER - 25

Ans: Given data:-

Modulus of elasticity E = 169 mpa.

poisson's ratio (μ) = 0.32

To Find :-

Modulus of rigidity (G)

Bulk modulus - (K)

Soln :-

As we know that

Relation betn youngs modulus and modulus of rigidity.

$$E = 2G(1+\mu)$$

$$\therefore 169 = 2G(1+0.32)$$

$$\therefore 2G = \frac{169}{1+0.32}$$

$$2G = \frac{169}{1.32} =$$

$$G = 64.01$$

As we know that,

$$E = 3K(1-2\mu)$$

$$169 = 3K[1-(2\times0.32)]$$

$$169 = 3K[1-0.64]$$

$$\therefore K = 156.48$$

② A bar of 500mm long and 22mm dia is elongated by 1.2mm under the effect of axial pull of 105KN. calculate Intensities of stress, strain and the modulus of elasticity. — WINTER-25

Ans:- Given data:

length of bar (l) = 500 mm

dia = 22mm.

Elongation (δL) = 1.2mm.

Axial pull = 105 KN.

= 105×10^3 N.

To Find :-

i) stress (σ)

ii) strain (e)

iii) modulus of elasticity (E)

soln

i) stress $= \frac{P}{A}$

$\therefore A = \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times 22^2$

$A = 379.94 \text{ mm}^2$

$\therefore \sigma = \frac{105 \times 10^3}{379.94}$

$\sigma = 276.35 \text{ N/mm}^2$

ii) strain

$\therefore e = \frac{\delta L}{L}$

$= \frac{1.2}{500}$

$e = 0.0024$

iii) Modulus of elasticity (E)

$\therefore E = \frac{\sigma}{e}$

$= \frac{276.35}{0.0024}$

$E = 115145.83$

i.e $E = 1.15 \times 10^5 \text{ N/mm}^2$

③ calculate minimum dia. of steel wire to lift a load of 8.2 KN, If permissible stress in wire is 120 MPa.

— SUMMER-25

Ans:- Given data:-

load (P) = 8.2 KN.
$= 8.2 \times 10^3$ N.

permissible stress (σ) = 120 MPa.

To Find:-

Diameter of steel bar (d)

Soln

Area of steel bar A

$\therefore A = \frac{\pi}{4} d^2$

As we know,

$\text{stress} = \frac{P}{A}$

$\therefore 120 = \frac{8.2 \times 10^3}{A}$

$\therefore A = \frac{8.2 \times 10^3}{120}$

$\therefore \frac{\pi}{4} d^2 = 68.33$

$\therefore d^2 = \frac{68.33}{\pi/4} =$

$d^2 = 87.04$

$\therefore d = 9.33$ mm

④ A mild steel flat 120 mm wide, 12 mm thick and 5 m long carries an axial load of 25 KN, Find stress, strain and change in length of bar ($E = 2 \times 10^5$ N/mm²)

— WINTER-24.

Ans:- Given data:-

Width = 120 mm.

Thickness = 12 mm.

length = 5 m.

Load = 25 KN = 25×10^3 N.

$E = 2 \times 10^5$ N/mm²

To Find :-

Stress (σ)

Strain (e)

Change in length (δL)

Soln. -

i) Stress

$$\sigma = \frac{P}{A}$$

$$\therefore A = b \times t = 120 \times 12$$
$$= 1440 \text{ mm}^2$$

$$\therefore \sigma = \frac{25 \times 10^3}{1440}$$

$$\sigma = 17.36 \text{ N/mm}^2$$

ii) Strain

$$e = \frac{\delta L}{L}$$

But,

$$E = \frac{\sigma}{e}$$

$$\therefore e = \frac{\sigma}{E}$$

$$= \frac{17.36}{2 \times 10^5}$$

$$\therefore e = 8.68 \times 10^{-5}$$

iii) δL

$$\therefore \delta L = L \times e$$

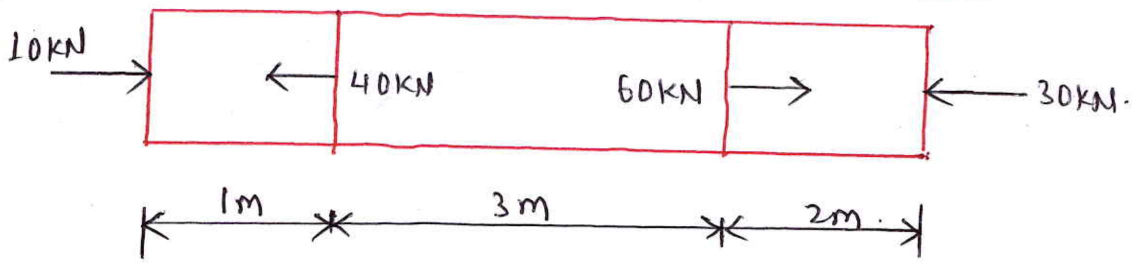
$$= 5000 \times 8.68 \times 10^{-5}$$

$$\delta L = 0.434 \text{ mm}$$

NUMERICALS ON DEFORMATION OF UNIFORM BARS:-

① A brass bar having c/s area of $1000\,mm^2$ is subjected to axial force as shown in fig. Find net deformation in the bar. Take [$E = 1.05\times10^5\,N/mm^2$

SUMMER-25



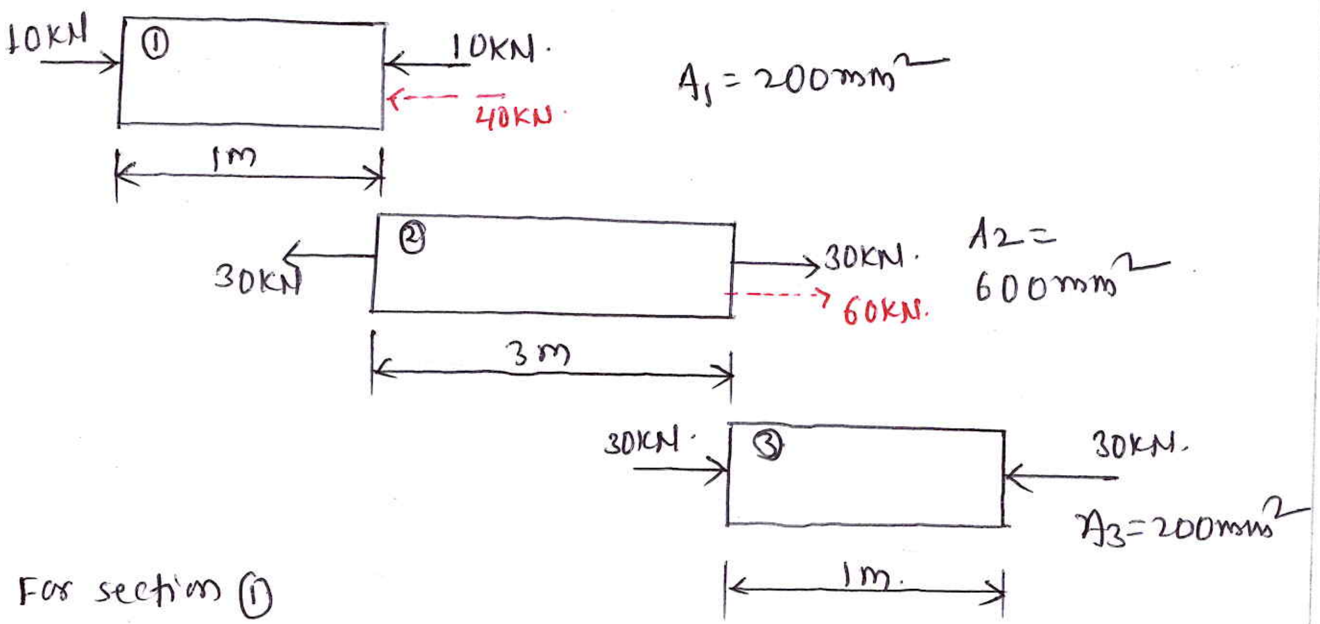
Ans:- Given data:-

$A = 1000\,mm^2$

$E = 1.05\times10^5\,N/mm^2$

Soln :-

Using principle of superposition, we have split the bars into various section.



For section ①

$$\delta L_1 = \frac{P_1 L_1}{A_1 E} = \frac{10\times10^3\times1000}{200\times1.05\times10^5}$$

$\delta L_1 = 0.476\,mm.$

For section ②

$$\delta L_2 = \frac{P_2 L_2}{A_2 E} = \frac{30\times10^3 \times 3000}{600\times1.05\times10^5} = 1.43\,mm.$$

For section ③

$$\delta L_3 = \frac{P_3 L_3}{A_3 E} = \frac{30\times10^3\times2000}{200\times1.05\times10^5} = \cancel{1.43\,mm}\ 2.85\,mm$$

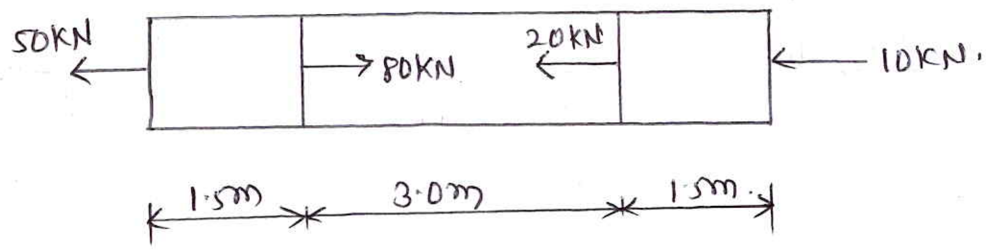
$\therefore$ Total change in length $= \delta L_1 + \delta L_2 + \delta L_3$

$= 0.48 + 1.43 + 2.85$

$\delta L = 3.34\ mm$

② A steel bar $800\,mm^2$ c/s area is subjected to axial forces as shown in fig. Find total change in length of the bar ($E = 2\times10^5\,N/mm^2$)

Ans:-



Given data:-

$A = 800\,mm^2$

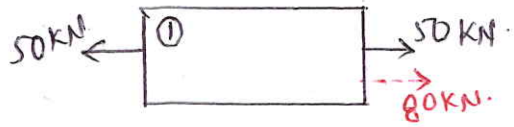
$E = 2\times10^5\,N/mm^2$

Check:-

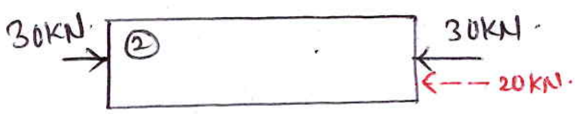
80KN →

50+20+10 = 80KN ←

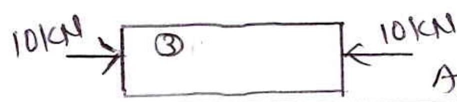
Soln:-



$A_1 = 200\,mm^2$



$A_2 = 400\,mm^2$



$A_3 = 200\,mm^2$

For section (f)

using principle of superposition.

$$\delta L = \frac{1}{AE}\left[P_1L_1 + P_2L_2 + P_3L_3\right]$$

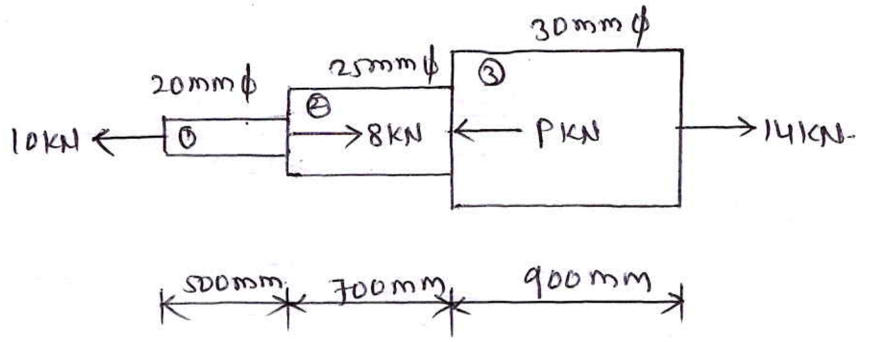
$$= \frac{1}{800\times2\times10^5}\left[(50\times10^3\times1500) - (30\times10^3\times3000) - (10\times10^3\times1500)\right]$$

$$= \frac{1}{160\times10^6}\left[75\times10^6 - 90\times10^6 - 15\times10^6\right]$$

$$= \frac{1}{160\times10^6}\times(-30\times10^6)$$

$$\underline{\delta L = -0.1875\,mm.}\ \text{(compression)}$$

(3) Determine the magnitude of 'P' for equilibrium and total elongation of the bar take E = 210 GPa. Also calculate minimum stress induced.



Ans:-

$10 + P = 8 + 14$

$P = 22 - 10$

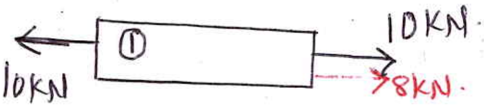
$\therefore \underline{P = 12\,KN.}$

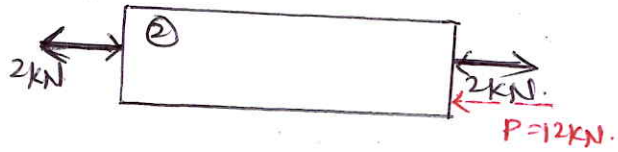
$E = 210\,GPa$

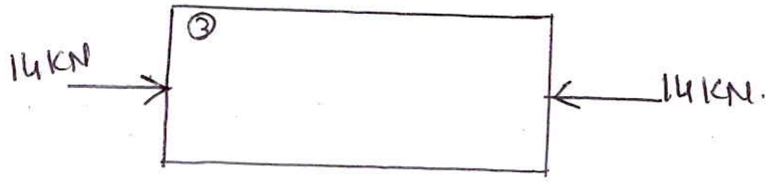
i.e. ~~$E = 2.1\times10^5\,N/m$~~

$E = 210\times10^3$

Now split the given stepped section into various section and analyze individual.







Now, $A_1 = \frac{\pi}{4} d^2$ $\quad A_2 = \frac{\pi}{4} \times 25^2$ $\quad A_3 = \frac{\pi}{4} \times 30^2$

$= \frac{\pi}{4} \times 20^2$ $\quad A_2 = 490.87 \, mm^2$ $\quad A_3 = 706.86 \, mm^2$

$A_1 = 314.16 \, mm^2$

$\therefore \delta L_1 = \frac{P_1 L_1}{A_1 E} = \frac{10 \times 10^3 \times 500}{314.16 \times 210 \times 10^3} = 0.075$ (Tensile)

$\delta L_2 = \frac{P_2 L_2}{A_2 E} = \frac{2 \times 10^3 \times 700}{490.87 \times 210 \times 10^3} = 0.0136$ (tensile)

$\delta L_3 = \frac{P_3 L_3}{A_3 E} = \frac{-14 \times 10^3 \times 900}{706.86 \times 210 \times 10^3} = -0.0848$

$\therefore$ Total change in length

$\delta L = \delta L_1 + \delta L_2 + \delta L_3 = 0.075 + 0.0136 - 0.0848$

$$\boxed{\delta L = 0.038 \, mm}$$

Tensile)

Now total ~~minimum~~ stress

$\sigma = E \times e = 210 \times 10^3 \times 0.038 = 798 \, N/mm^2$

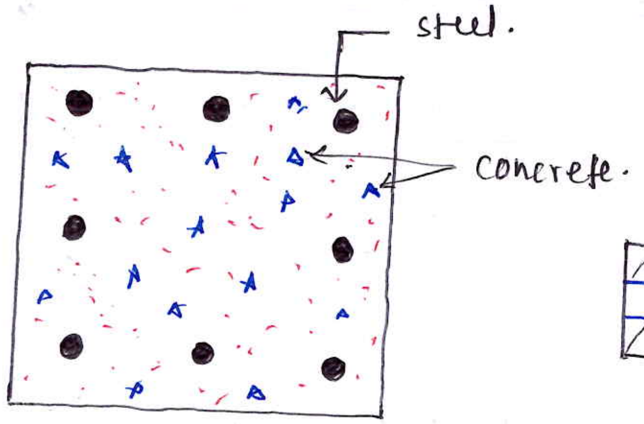
COMPOSITE SECTION :-

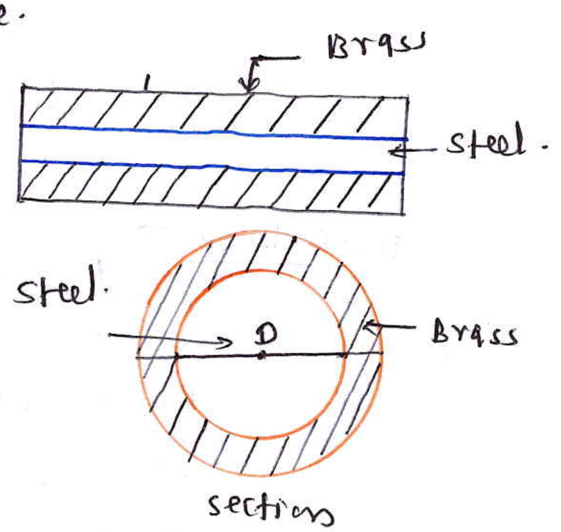
A structure which is built by two or more different material and acts as a single structural member to withstand the forces acting on it.

- There are various types of composite section. such as concrete and steel, A composite bar of different metal.

- For calculation of stresses induced into the composite section we have required modular ratio.

- each material has different Young's Modulus.





MODULAR RATIO :-

It is the ratio Young's Modulus first material to youngs modulus of second material.

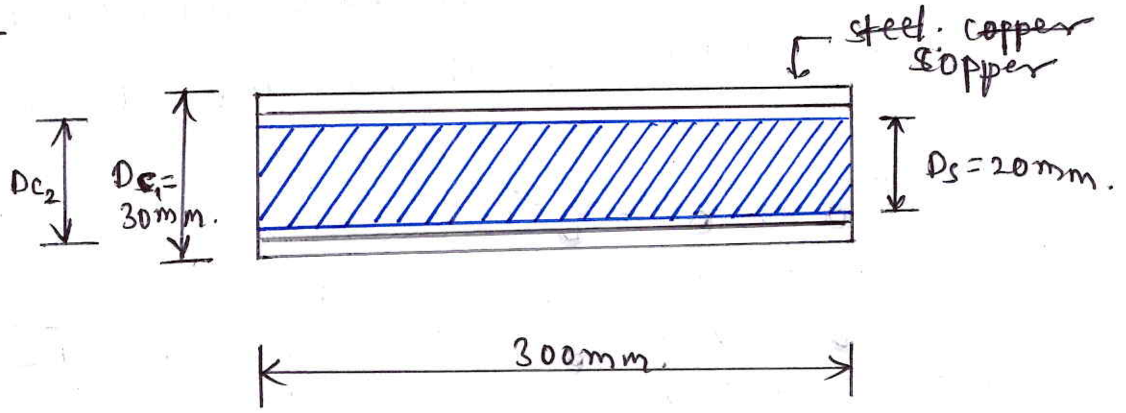
$$\therefore \text{Modular ratio} = \frac{E_1}{E_2}$$

NUMERICALS ON COMPOSITE SECTION:—

SUMMER – 2026

① A mild steel rod of 20mm dia. and 300mm long is enclosed centrally inside a hollow copper tube of external dia. 30mm and internal dia. 25mm the composite bar is subjected to an axial pull of 40kN. If E for steel and copper is 200GPa and 100GPa respectively find stresses developed in the rod and tube, also find extension of the rod.

Ans:—



Given data:—

D_s = dia. of steel = 20 mm.

$L_c = L_s = 300$ mm. $P = 40kN = 40 \times 10^3 N$.

$D_{c_1} = 30mm$ (external), $D_{c_2} = 25mm$ (Internal)

$E_s = 200GPa = 2 \times 10^5 N/mm^2$

$E_c = 100GPa = 1 \times 10^5 N/mm^2$

$\therefore A_s = \frac{\pi}{4} d^2$ $\qquad A_c = \frac{\pi}{4}(D_{c_1}^2 - D_{c_2}^2)$

$= \frac{\pi}{4} \times 20^2$ $\qquad = \frac{\pi}{4}(30^2 - 25^2)$

$A_s = 314.15 mm^2$ $\qquad A_c = 215.98 mm^2$

As we know that,

$$e_s = \frac{\sigma_s}{E_s} \quad \text{and} \quad e_c = \frac{\sigma_c}{E_c}$$ { strain or elongation is equal

$$\therefore \frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\therefore \frac{\sigma_s}{2\times10^5} = \frac{\sigma_c}{1\times10^5}$$

$$\sigma_s \times 1\times10^5 = 2\times10^5 \sigma_c$$

$$\therefore \underline{\sigma_s = 2\sigma_c}$$

$\therefore \sigma_s = 94.76\ N/mm^2$

Now,

Total load $= P = P_s + P_c$

$$40\times10^3 = P_s + P_c$$
$$= \sigma_s A_s + \sigma_c A_c$$

{ Here $\sigma = \frac{P}{A}$ $\therefore P = \sigma A$.

$$= \cancel{2\sigma_c +}$$
$$= (2\sigma_c \times 314.16) + (\sigma_c \times 215.98)$$

$$40\times10^3 = 844.30\sigma_c$$

$$\therefore \sigma_c = \frac{40\times10^3}{844.30} = 47.38\ N/mm^2$$

Now, to calculate total elongation

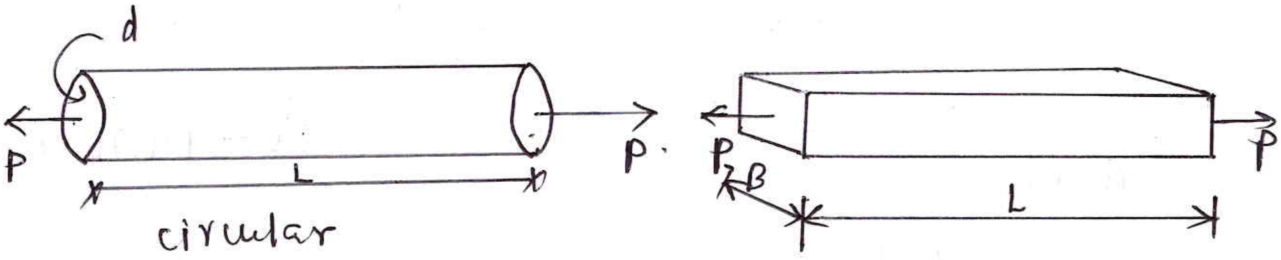
$$\therefore \delta L_s = \delta L_c = \frac{\sigma_s L}{E_s} = \frac{94.76\times300}{2\times10^5}$$
$$= \underline{\underline{0.142\,mm.}}$$

UNIAXIAL STRESS SYSTEM :-

In the uniaxial stress system stress is induced in one direction only when the axial load is applied i.e in x, y, and z axis indivisually.

let us, assume,

stress is occures in x- Axis so other two stresses in y and z axis should be zero.



circular

Rectangular

As we know that.

$\sigma_x = \frac{P}{A}$ and $e_x = \frac{\sigma_x}{E_x}$

$\therefore E_x = \frac{\sigma_x}{e_x}$

Here, e_x - linear strain

But strain along y and z axis is lateral strain.

$\therefore e_y = e_z =$ lateral strain.

i.e the diameter of rod should be reduced w.r.to x-axis.

$\mu = -\frac{e_z}{e_x}$

$\therefore e_z = -\mu e_x$

$\therefore e_z = \frac{-\mu \sigma_x}{E}$

similarly,

$\mu = -\frac{e_y}{e_x}$

$e_y = -\mu e_x$

$e_y = -\frac{\mu \sigma_x}{E}$

We know that

volumetric strain $= \frac{\delta V}{V}$

$\frac{\delta V}{V} = e_x + e_y + e_z$

$= \frac{\sigma_x}{E} + \frac{\mu \sigma_x}{E} - \frac{\mu \sigma_x}{E}$

$\frac{\delta V}{V} = \frac{\sigma_x}{E}(1 - \mu - \mu)$

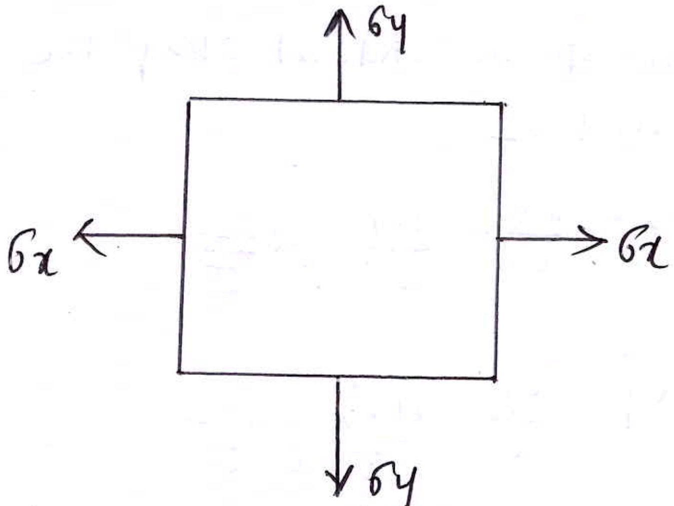
$$\therefore \frac{\delta v}{V} = \frac{\sigma_x}{E} \times (1-2\mu)$$

$$\frac{\delta v}{V} = e(1-2\mu)$$

$$\boxed{\delta v = v\,[e(1-2\mu)]}$$

BI-AXIAL STRESS SYSTEM :-

When two forces are acted upon two mutually perpendicular plane then the stresses induced into the body is called Bi-axial stress system.



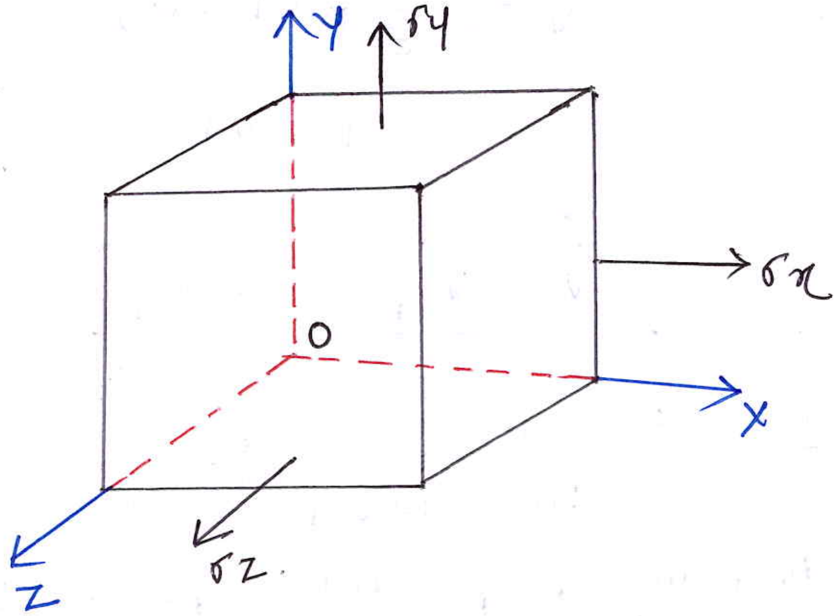
Now, loads are applied along x and y-axis (Mutually perpendicular), then stresses produced into the planes are σ_x and σ_y

$$\therefore e_x = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E}$$

and

$$e_y = \frac{\sigma_y}{E} - \frac{\mu \sigma_x}{E}$$

TRI-AXIAL STRESS SYSTEM :-



Here, Tensile forces are applied along X, Y, and Z direction then the stresses induced along the respective axis are σ_x, σ_y and σ_z

$$\therefore e_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

$$e_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E}$$

$$e_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$\therefore$ volumetric strain $= \frac{\delta V}{V}$.

$$\therefore \frac{\delta V}{V} = e_x + e_y + e_z$$

$$\frac{\delta V}{V} = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

NUMERICALS ON UNI, BI AND TRI-AXIAL STRESS SYSTEM:-

(1) A cube of 50 mm side is subjected to force of 6 KN (Tensile), 8 KN (compressive) and 4 KN (Tensile) along, x, y and z axis respectively. Determine change in volume [E = 200 GPa and $\mu = 10/3$]

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Ans:- Given Data:-

Side of cube = 50 mm.

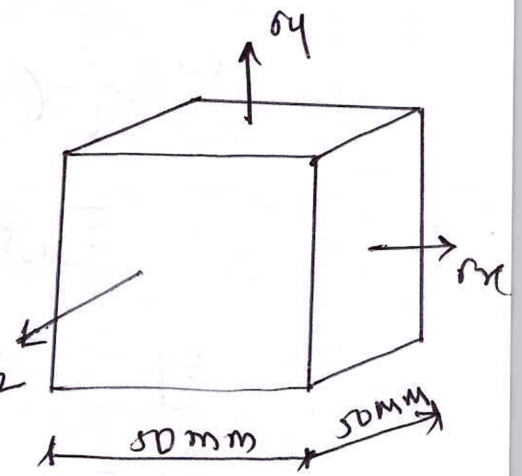
$P_x = 6\,KN = 6 \times 10^3\,N \cdot (T)$

$P_y = 8\,KN = 8 \times 10^3\,N\,(C)$

$P_z = 4\,KN = 4 \times 10^3\,N\,(T)$

$E = 200\,GPa = 2 \times 10^5\,N/mm^2$

$\mu = 10/3 = 0.33$



To find

Change in volume δV

Soln:-

Area of cube $= 50^2 = 2500\,mm^2$

$\therefore \sigma_x = \frac{P_x}{A} = \frac{6 \times 10^3}{2500} = 2.4\,N/mm^2$ — (T)

$\sigma_y = \frac{P_y}{A} = \frac{8 \times 10^3}{2500} = 3.2\,N/mm^2$ — (C)

$\sigma_z = \frac{P_z}{A} = \frac{4 \times 10^3}{2500} = 1.6\,N/mm^2$ — (T)

Now, to find strain in each direction:-

$$\therefore e_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

$$= \frac{1}{E}(\sigma_x - \mu\sigma_y - \mu\sigma_z)$$

$$= \frac{1}{2\times10^5}\left[2.4 + (0.33\times3.2) - (0.33\times1.6)\right]$$

$$e_x = 1.46\times10^{-5}$$

$$e_y = \frac{1}{E}(\sigma_y - \mu\sigma_x - \mu\sigma_z)$$

$$= \frac{1}{2\times10^5}\left[3.2 - (0.33\times2.4) - (0.33\times1.6)\right]$$

$$e_y = -2.26\times10^{-5}$$

$$e_z = \frac{1}{E}(\sigma_z - \mu\sigma_x - \mu\sigma_y)$$

$$= \frac{1}{2\times10^5}\left[1.6 - (0.33\times2.4) + (0.33\times3.2)\right]$$

$$e_z = 9.32\times10^{-6}$$

As we know that,

$$\frac{\delta V}{V} = e_x + e_y + e_z$$

$$\therefore \delta V = V(e_x + e_y + e_z)$$

$$= 50^3(1.46\times10^{-5} - 2.26\times10^{-5} + 9.32\times10^{-6})$$

$$\delta V = 0.165\ mm^3$$

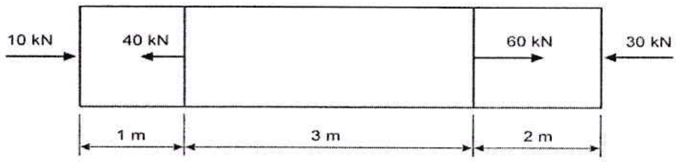
Patil 28/1/22

Unit - II

Simple Stresses, Strains & Elastic Constants

Question Bank

1. Define shear strain and modulus of elasticity.
2. Define Young's Modulus, Shear Modulus
3. Define section modulus and neutral axis
4. Differentiate between single shear and double shear.
5. State Hooke's Law.
6. For a certain material, modulus of elasticity is 169 MPa. If poisson's ratio is 0.32. Calculate the values of modulus of rigidity and bulk modulus
7. A bar 500 mm long and 22 mm in diameter is elongated by 1.2 mm under the effect of axial pull of 105 kN. Calculate the intensities of stress, strain and the modulus of elasticity of the bar
8. Draw stress-strain curve for ductile material and explain the following terms :– i) Elastic limit ii) Upper yield point iii) Ultimate load point iv) Breaking load point
9. A m.s. bar is subjected to a load of 80 kN. The diameter of the bar is 16 mm and its length is 320 mm. Calculate elongation, if E = 196 kN/mm2. Also calculate change in diameter if μ = 0.28.
10. A brass bar having cross sectional area of 1000 mm2 is subjected to axial force as shown in Fig. No. 2 find the net deformation in the bar. Take E = 1.05 × 105 N/mm2.



11. A cube of 50mm side is subjected to a force of 6kN (Tensile), 8 kN (compressive) and 4 kN (Tensile) along X, Y, Z respectively. Determine change in volume. Take E = 200 GPa and m as 10/ 3.
12. Define :– i) Normal stress ii) Direct stress iii) Bending stress iv) Shear stress
13. State the relation between E, G, K.
14. A cube of 200 mm side is subjected to a compressive force of 3500 kN. on all its faces. The change in volume of the cube is 5000 mm3. Calculate the bulk modulus and modulus of elasticity if Poisson's ratio is 0.28
15. A steel bar 800 mm2 cross-sectional area is subjected to axial forces as shown in Figure No. 2. Find total change in length of the bar if E = 2 × 105 N/mm2.