R.C.Patel College Of Engineering & Polytechnic, Shirpur

Department of Civil Engineering

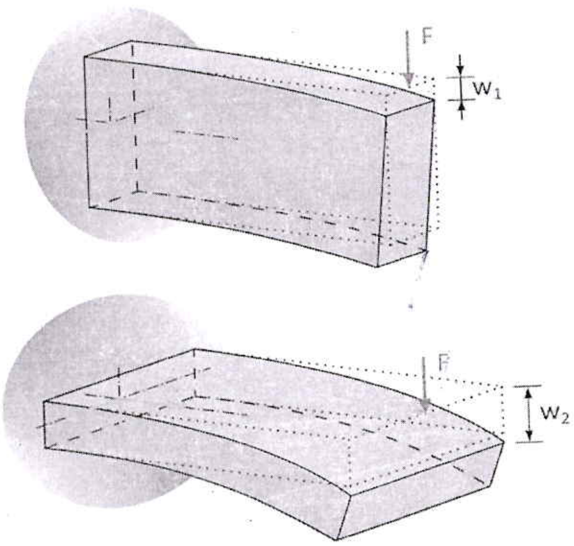
Course Title- Strength of Material

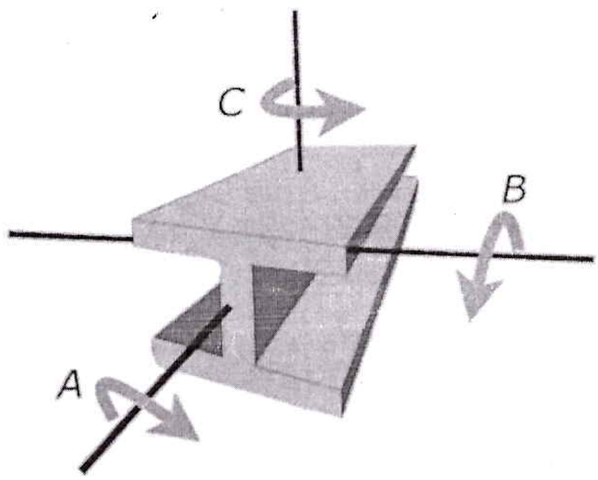
Programme Name -Civil Engineering

Course Code -313308

Semester-Third

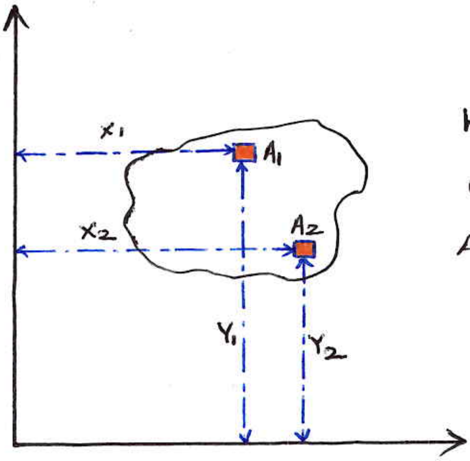
Unit	Title	COs	Learning hours	R Level	U Level	A Level	Total Marks
I	Moment of Inertia	CO1	10	2	4	6	12





MOMENT OF INERTIA :

The product of Elemental area and square of the perpediculas distance betⁿ the centroid of area and axis of reference is called as moment of Inertia.



consider a plane lamina having area A. Now split the area into small areas such as $A_1, A_2, A_3, \ldots$

Here x_1, x_2 = Distances From centroid of A_1 and A_2 to reference axis (Y)

and Y_1, Y_2 = Distances From centroid of A_1 and A_2 to reference axis (X)

We know that,

Moment of Inertia about x-axis.

$$\therefore I_{xx} = A_1Y_1^2 + A_2Y_2^2 + A_3Y_3^2 + \ldots + A_nY_n^2$$

$$\therefore \boxed{I_{xx} = \Sigma AY^2}$$

Similarly,

Moment of Inertia about Y-axis.

$$I_{YY} = A_1x_1^2 + A_2x_2^2 + A_3x_3^2 + A_nx_n^2$$

$$\therefore \boxed{I_{YY} = \Sigma Ax^2}$$

Why we calculate moment of Inertia –

To determine resistance against bending and deflection

RADIUS OF GYRATION :

Radius of gyration is the distance from an axis at which the whole mass of the body supposed to be concentrated without changing the moment of Inertia

We know that

Moment of Inertia = $I = A \cdot K^2$

$$\therefore K^2 = \frac{I}{A}$$

$$\therefore \boxed{K = \sqrt{\frac{I}{A}}}$$

Similliarly,

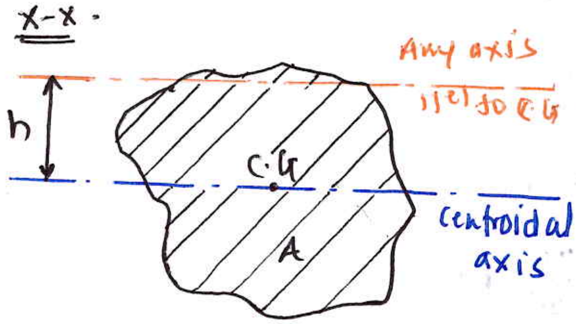
$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} \qquad K_{yy} = \sqrt{\frac{I_{yy}}{A}}$$

Application :– To determine Buckling load on long. column.

PARALLEL AXIS THEOREM:-

MI of plane section about any axis parallel to the centroidal axis is equal to the MI of section about the centroidal axis plus the product of the area of the section and the square of the distance betn two axis.

X-X.



A plane lamina having area A.

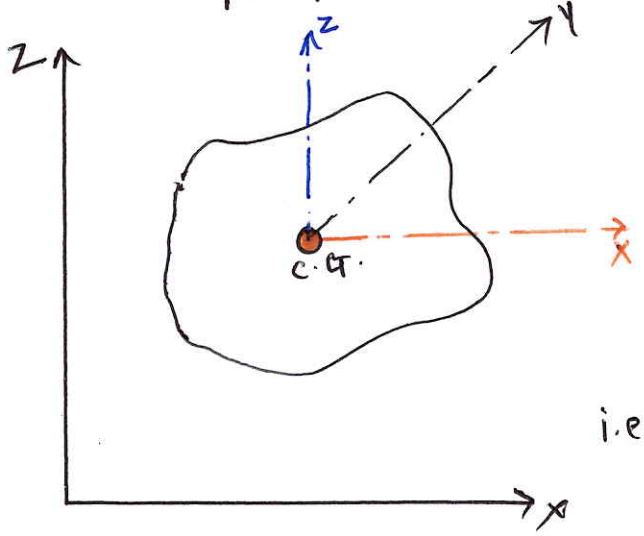
I_{xx} - MI @ cross section passing through C.G.

h - perpendicular distance betn the two axis.

$$\therefore \boxed{I = I_{xx} + Ah^2} \qquad \boxed{I = I_{yy} + Ah^2}$$

PERPENDICULAR AXIS THEOREM:-

For a plane lamina, the MI about an axis perpendicular to it's plane and passing through a point is equal to the Addition of the MI about two mutually perpendicular axes through the same point.



$$\therefore \boxed{I_{zz} = I_{xx} + I_{yy}}$$

i.e M.I of plane about polar axis.

POLAR MOMENT OF INERTIA :-

The polar moment of inertia of a plane lamina area about a pole is equal to the sum of MI of that area about two mutually perpendicular axes rest in the plane and passing through the pole.

$$\therefore \boxed{I_{zz} = I_{xx} + I_{yy}.}$$

For ex. - For circular section.

$$M.I @ xx = I_{xx} = \frac{\pi}{64} D^4$$

$$MI @ yy = I_{yy} = \frac{\pi}{64} D^4$$

$$\therefore \quad I_{zz} = \frac{\pi}{64} D^4 + \frac{\pi}{64} D^4$$

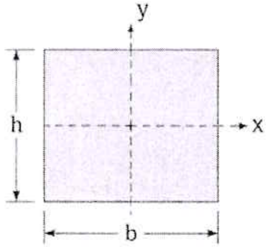
$$I_{zz} = 2\left[\frac{\pi}{64} D^4\right]$$

$$I_{zz} = \frac{\pi}{32} D^4$$

MOMENT OF INERTIA OF COMMON SHAPES

I = Moment of Inertia (Unit: m^4)

RECTANGLE
(b × h)



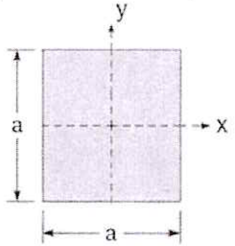
About centroidal x-axis:

$$I_x = \frac{bh^3}{12}$$

About centroidal y-axis:

$$I_y = \frac{hb^3}{12}$$

SQUARE
(a × a)



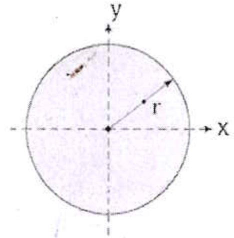
About centroidal x-axis:

$$I_x = \frac{a^4}{12}$$

About centroidal y-axis:

$$I_y = \frac{a^4}{12}$$

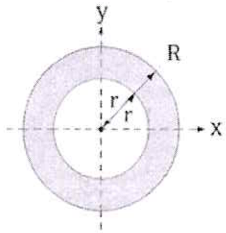
CIRCLE
(radius r)



About any centroidal axis:

$$I_x = I_y = \frac{\pi r^4}{4}$$

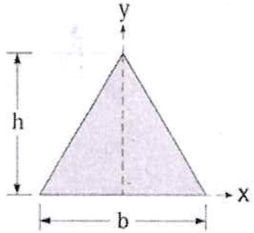
HOLLOW CIRCLE
(outer radius R, inner radius r)



About any centroidal axis:

$$I_x = I_y = \frac{\pi (R^4 - r^4)}{4}$$

TRIANGLE
(base b, height h)



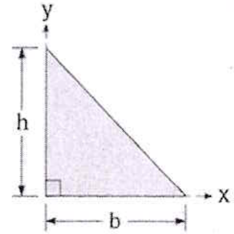
About centroidal x-axis (parallel to base):

$$I_x = \frac{bh^3}{36}$$

About centroidal y-axis (parallel to height):

$$I_y = \frac{hb^3}{48}$$

RIGHT TRIANGLE
(base b, height h)



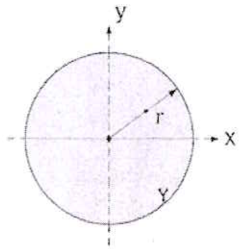
About centroidal x-axis (parallel to base):

$$I_x = \frac{bh^3}{36}$$

About centroidal y-axis (parallel to height):

$$I_y = \frac{b^3h}{36}$$

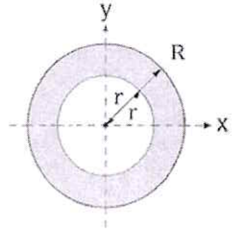
CIRCLE (about diameter)



About any diameter (axis through center):

$$I = \frac{\pi r^4}{4}$$

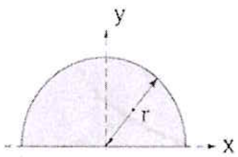
HOLLOW CIRCLE
(about diameter)



About any diameter (axis through center):

$$I = \frac{\pi (R^4 - r^4)}{4}$$

SEMICIRCLE
(radius r)



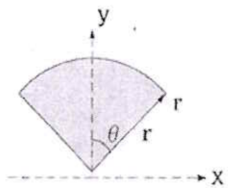
About centroidal x-axis (along the base):

$$I_x = \frac{\pi r^4}{8}$$

About centroidal y-axis (through center):

$$I_y = \frac{\pi r^4}{8}$$

CIRCULAR SECTOR
(radius r, angle θ in radians)



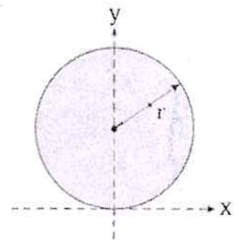
About centroidal x-axis (along one side):

$$I_x = \frac{r^4}{4}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

About centroidal y-axis (along the other side):

$$I_y = \frac{r^4}{4}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

CIRCLE (about tangent)



About tangent to circle:

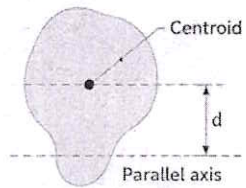
$$I = \frac{3\pi r^4}{4}$$

NOTES

- All centroidal axes pass through the center of area.
- For axes not shown, use the parallel axis theorem:

$$\boxed{I = I_c + Ad^2}$$

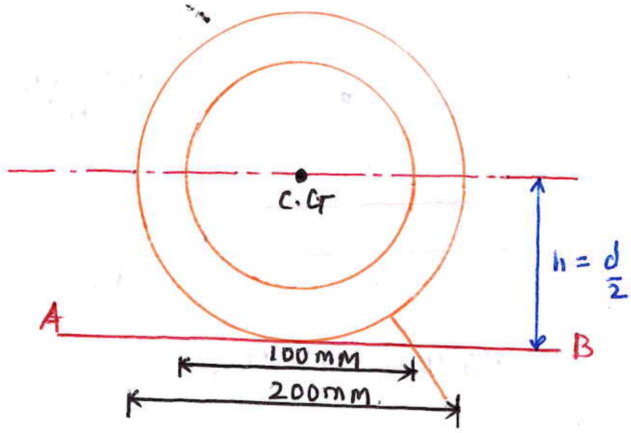
where I_c = centroidal MOI
A = area
d = distance between axes



NUMERICALS :

1. A Hollow circular section having 200mm external Diameter and 100 mm internal dia. calculate MI about any tangent. Also find polar moment of Inertia.

— Winter-25



Ans:- Given Data:

section is circular

Internal dia = 100mm

External dia = 200mm

Tangent AB

$h = \frac{d}{2} = \frac{200}{2} = 100mm.$

We know that,

Moment of Inertia of Hollow circular section

$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$= \frac{\pi}{64}(200^4 - 100^4)$$

$$I_G = 73631077.8 \ mm^4$$

Now, To calculate MI about tangent AB using parallel axis theorem

$$I = I_G + Ah^2$$

$$= 73631077.8 + [23561.9 \times 100^2]$$

Here $A = \frac{\pi}{4}(D^2 - d^2)$

$A = 23561.9 \ mm^2$

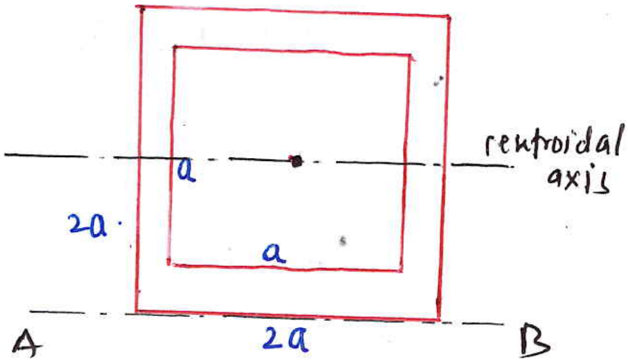
$$\underline{I = 309250077.8 mm^4}$$

$\therefore$ Polar moment of Inertia $= 2 \times 73631077.8 = \underline{1.4726 \times 10^8 mm^4}$

(2). A Hollow square has inner dimentions $a \times a$ and outer dimentions $2a \times 2a$. Find moment of Inertia about the outer side.

— SUMMER-25

Ans:



Hollow square

MI of outer square @ outer side

Here, $b = 2a$
$h = 2a$.

We know that,

$$MI = \frac{(2a)^4}{12}$$

$$I_{outer} = \frac{16a^4}{12}$$ — (about centroidal axis)

Now MI @ Inner side about centroidal axis.

$$I_{inner} = \frac{a^4}{12}$$

using ||el axis thm,

$$I_{inner} = I_G + Ah^2$$

$$= \frac{a^4}{12} + a^2 \cdot a^2$$

$$I_{inner} = \frac{a^4}{12} + a^4$$

$$= \frac{13a^4}{12}$$

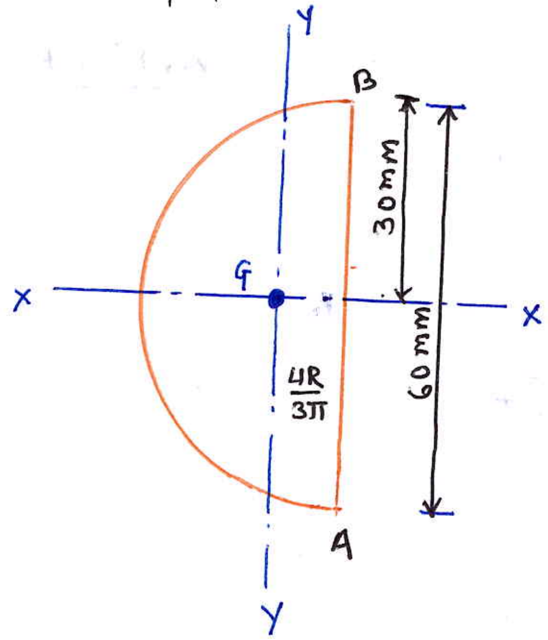
Now

$$I_{AB} = \frac{16a^4}{12} - \frac{13a^4}{12}$$

$$\boxed{I_{AB} = \frac{51a^4}{12}}$$

③ calculate polar MI of semi circle having 60 mm dia Also calculate minimum radius of gyration. Diameter is parallel to y-y-axis.

SUMMER-26



Ans :- Given,

$d = 60mm$.

$\therefore R = 30mm$.

MI about x-x-axis.

$$I_{xx} = \frac{\pi R^4}{8}$$

$$= \frac{\pi \times 30^4}{8}$$

$$I_{xx} = 318.08 \times 10^3 \, mm^4$$

MI about y-y-axis

$$I_{yy} = 0.11 R^4$$

$$= 0.11 \times 30^4$$

$$I_{yy} = 89.1 \times 10^3 \, mm^4$$

Now to calculate polar moment of Inertia.

$$I_p = I_{xx} + I_{yy}$$

$$= 318.08 \times 10^3 + 89.1 \times 10^3$$

$$I_p = 407.18 \times 10^3 \, mm^4$$

To calculate Radius of gyration

$$\therefore K_{min} = \sqrt{\frac{I}{A}}$$

$$\therefore A = \frac{\pi r^2}{2} = \frac{\pi \times 30^2}{2}$$

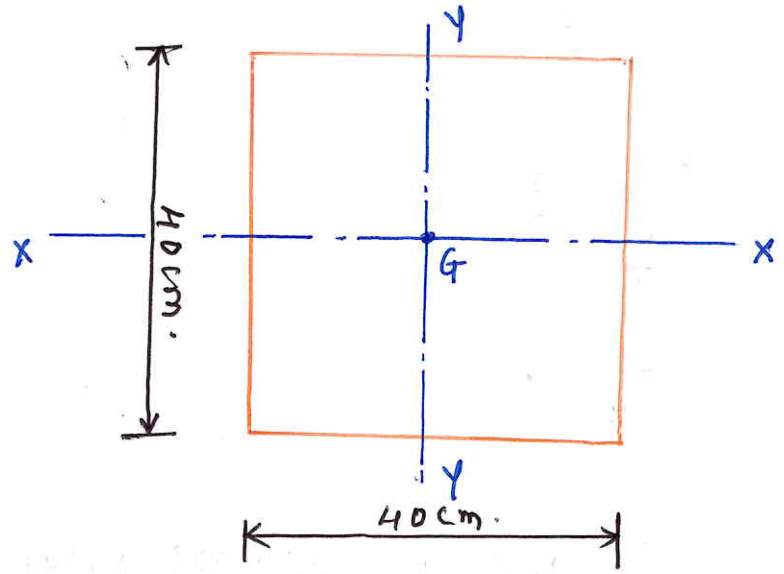
$$= 1413.77 \, mm^2$$

$$\therefore K_{min} = \sqrt{\frac{407.18 \times 10^3}{1413.77}}$$

$$K_{min} = 7.94 \, mm.$$

(4) Define 'polar moment of inertia'. Calculate polar moment of inertia for square lamina of side 40cm.

WINTER- 24



A square lamina having 40cm side

∴ We know that moment of inertia about centroidal x and y-axis is

$$I_{xx} = \frac{a^4}{12} = \frac{40^4}{12}$$

$$I_{xx} = 213.33 \times 10^3 \, mm^4$$

$$I_{xx} = I_{yy} = 213.33 \times 10^3 \, mm^4$$

∴ polar moment of Inertia

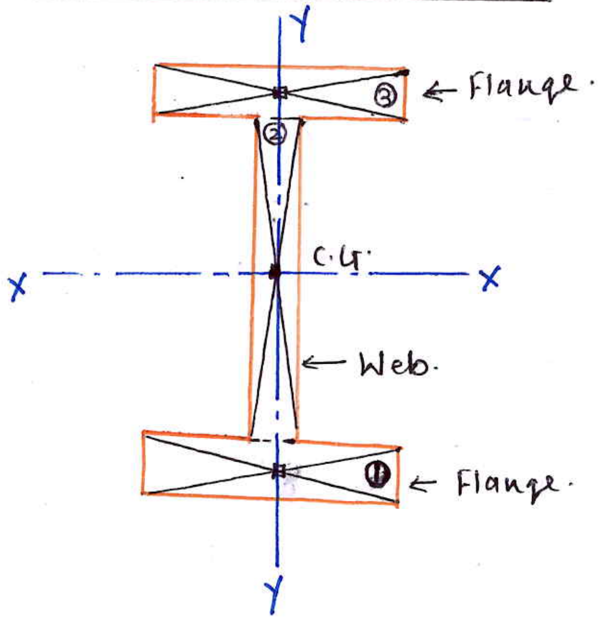
$$I_p = I_{xx} + I_{yy}$$

$$= 213.33 \times 10^3 + 213.33 \times 10^3$$

$$\underline{I_p = 426.67 \times 10^3 \, mm^4}$$

MOMENT OF INERTIA OF COMPOSITE PLANE FIG.

1. SYMMETRICAL I-SECTION.



MI about centroidal axis –

MI about xx- axis.

For area A_1

$I_{xx_1} = \cancel{I_1 + A_1 h_1^2}\ \dfrac{b_1 d_1^3}{12}$

~~We used~~

$I_{xx_2} = \dfrac{b_2 d_2^3}{12}$

$I_{xx_3} = \dfrac{b_3 d_3^3}{12}$

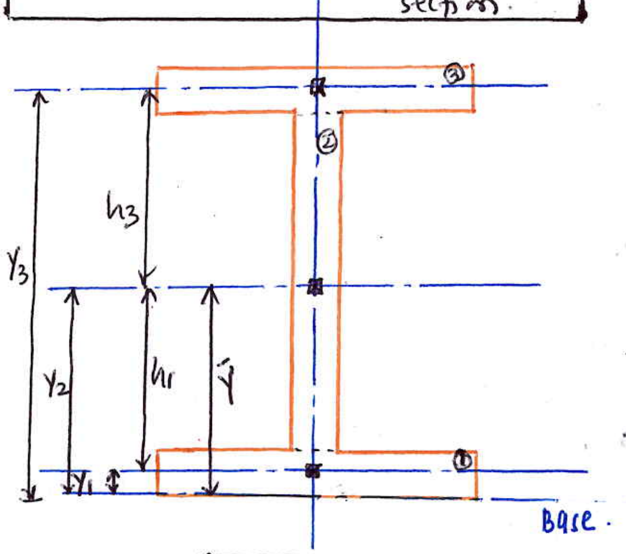
$$\boxed{I_{xx} = \frac{b_1 d_1^3}{12} + \frac{b_2 d_2^3}{12} + \frac{b_3 d_3^3}{12}}$$

Y_1, Y_2, Y_3 – Are the vertical distances from the axis passing through the C.G. of respective section to the base.

h_1, h_2 – Are the distances from the axis passing through C.G. to the C.G. of whole section.

Similarly y-y- axis.

$$\boxed{I_{yy} = \frac{d_1 b_1^3}{12} + \frac{d_2 b_2^3}{12} + \frac{d_3 b_3^3}{12}}$$



$$\bar{y} = \boxed{\frac{Y_1 A_1 + Y_2 A_2 + Y_3 A_3}{A_1 + A_2 + A_3}}$$

MI about the base –

$I_{xx_1} = I_1 + A_1 h_1^2$

$I_{xx_2} = I_2 + A_2 h_2^2$

$I_{xx_3} = I_3 + A_3 h_3^2$

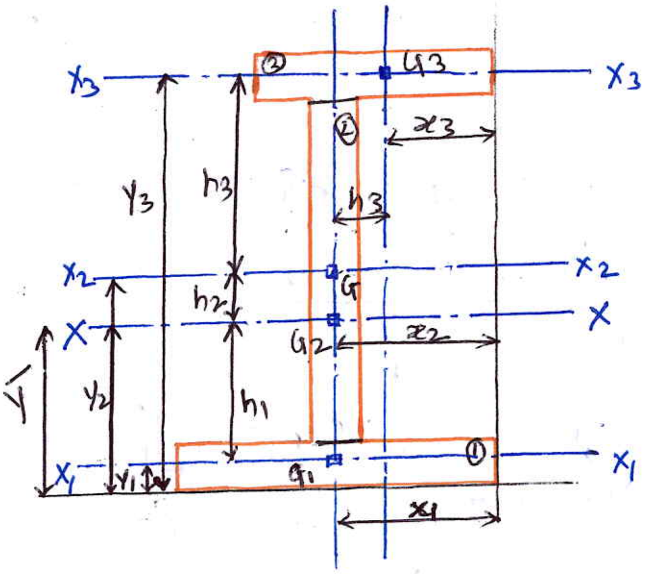
$\therefore \boxed{\text{Total MI} = I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}}$

$I_{yy_1} = \cancel{I_1 + A_1}\ \dfrac{d_1 b_1^3}{12}$

$I_{yy_2} = \dfrac{d_2 b_2^3}{12}$, $I_{yy_3} = \dfrac{d_3 b_3^3}{12}$

$\therefore \boxed{I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3}}$

2. UNSYMMETRICAL I-section -



Moment of Inertia about X-axis

$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$

$I_{xx_1} = I_1 + A_1 h_1^2$ — using ‖^{el} axis thm.

$I_{xx_2} = I_2 + A_2 h_2^2$

$I_{xx_3} = I_3 + A_3 h_3^2$

Here,

$$\bar{y} = \frac{y_1 A_1 + y_2 A_2 + y_3 A_3}{A_1 + A_2 + A_3}$$

$h_1 = \bar{y} - y_1$

$h_2 = y_2 - \bar{y}$

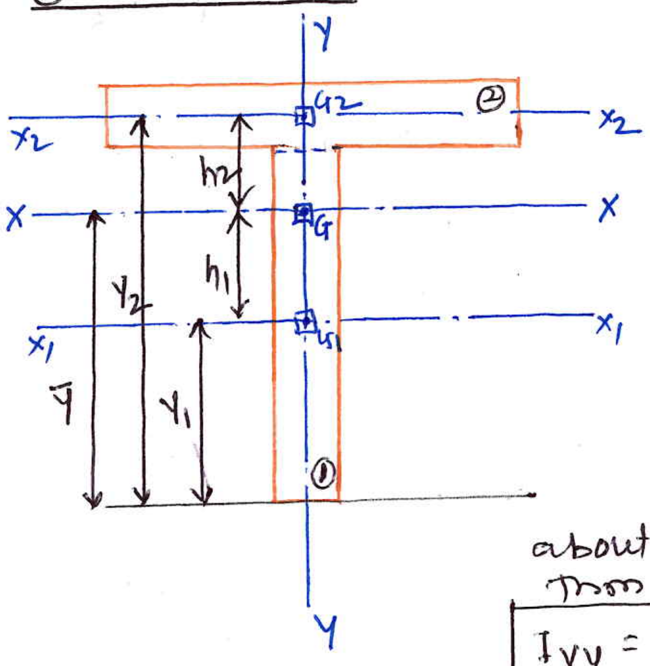
$h_3 = y_3 - y_2$

$$\bar{x} = \frac{x_1 A_1 + x_2 A_2 + x_3 A_3}{A_1 + A_2 + A_3}.$$

Here, $h_1 = h_2 = 0$

But $h_3 = x_2 - x_3$

③ T- section



Moment of Inertia about centroidal x-x-axis.

$I_{xx_1} = I_1 + A_1 h_1^2$

$I_{xx_2} = I_2 + A_2 h_2^2$

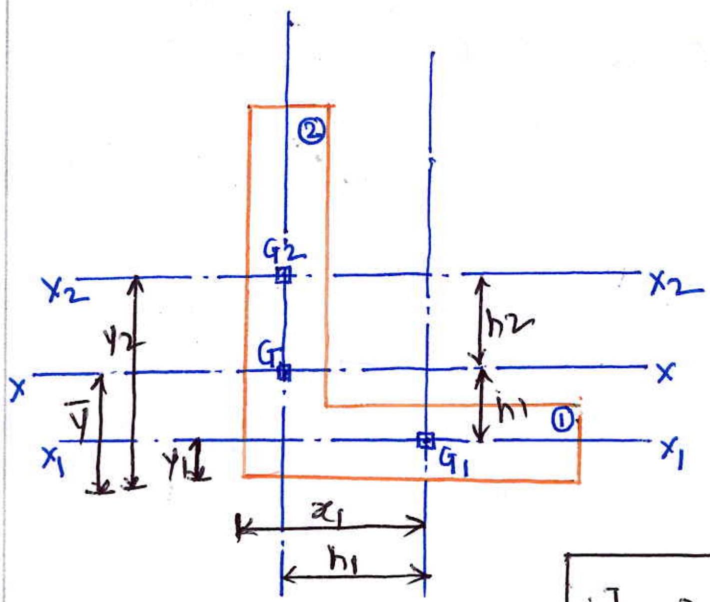
$I_{xy} = I_{xx_1} + I_{xx_2}$

Moment of Inertia about centroidal yy- axis.

Given section is symmetrical about y-y axis. therefor using ⊥ axis thm.

$$\boxed{I_{yy} = \left[\frac{db^3}{12}\right]_1 + \left[\frac{db^3}{12}\right]_2} \cancel{+ \frac{db^3}{1}}$$

(7) ANGLE SECTION :-



Moment of Inertia about centroidal x-x axis.

$$I_{xx} = I_{xx_1} + I_{xx_2}$$

$$I_{xx_1} = I_1 + A_1 h_1^2$$

$$I_{xx_1} = \frac{b_1 d_1^3}{12} + A_1 h_1^2$$

$$I_{xx_2} = \frac{b_2 d_2^3}{12} + A_2 h_2^2$$

$$\boxed{\therefore I_{xx} = \left[\frac{b_1 d_1^3}{12} + A_1 h_1^2\right] + \left[\frac{b_2 d_2^3}{12} + A_2 h_2^2\right]}$$

$$I_{yy_1} = \frac{d_1 b_1^3}{12} + A_1 h_1^2$$

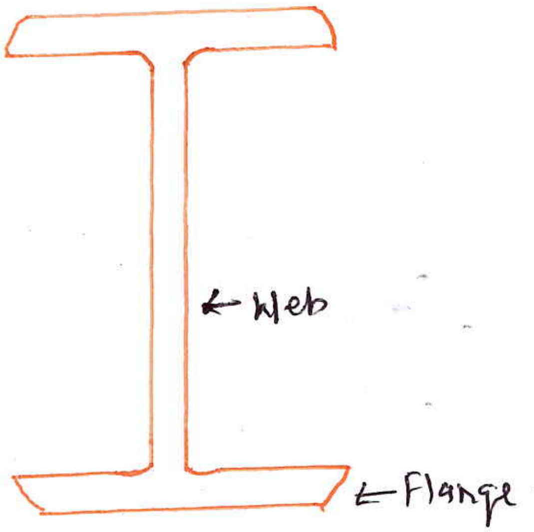
$$I_{yy_2} = \frac{d_2 b_2^3}{12}$$

$$\boxed{I_{yy} = \left[\frac{d_1 b_1^3}{12} + A_1 h_1^2\right] + \left[\frac{d_2 b_2^3}{12}\right]}$$

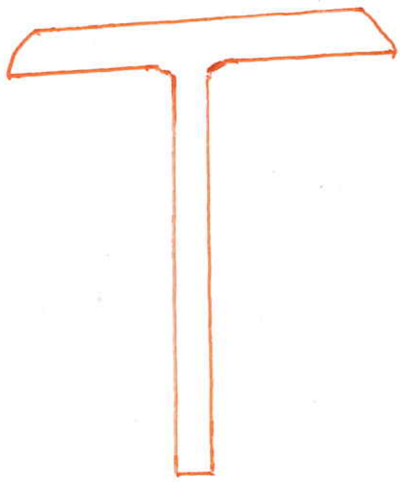
MOMENT OF INERTIA OF BUILT-UP-SECTION :—

Built-up section consist of number of different section such as Angel section, channel section, I-section, rectangular sections. etc.

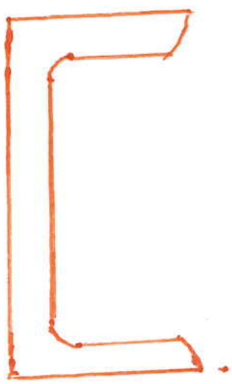
- Buit up section required larger MI.
- It has greater load carrying capacity
- Built up section used in construction of Bridges, Industrial sheds, multistoreyd building Electrical transmission towers, Railway, Heavy cranes. etc.



I- section.



T- section.



– c-section.
(chanel)

MI OF COMPOSITE PLANE SECTION : [NUMERICALS]

I - Section

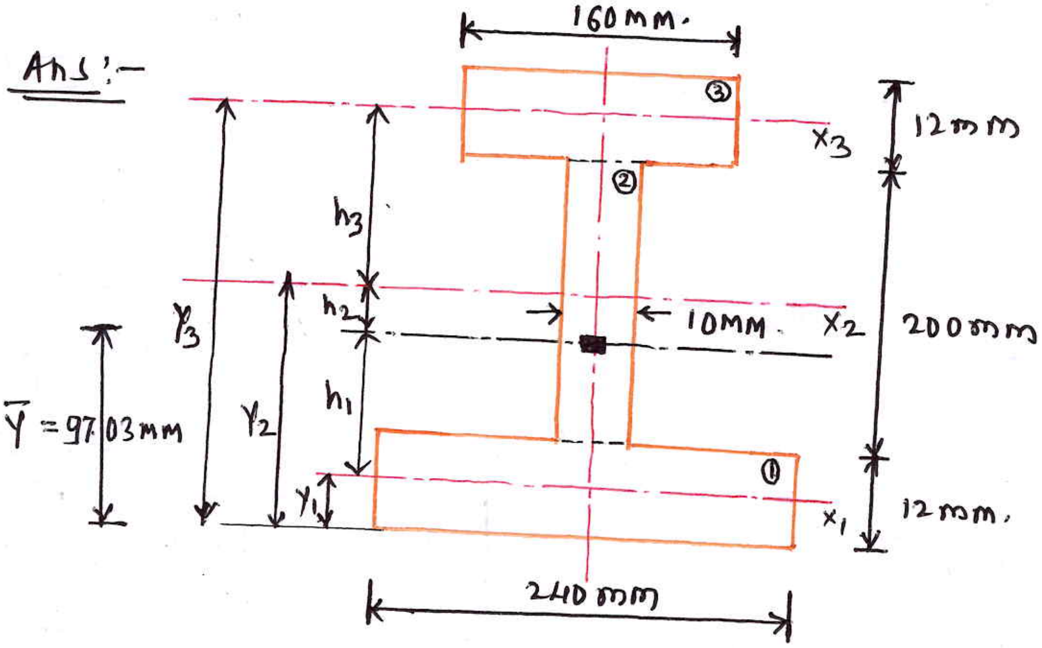
(5) Determine the MI of unsymmetrical I-section having following details — SUMMER-25

Top flange – 160mm X 12mm

Bottom flange – 240mm X 12mm.

Web – 200mm X 10mm.

Ans :-



A_1	240X12 = 2880	mm^2
A_2	200X10 = 2000	mm^2
A_3	160X12 = 1920	mm^2

Y_1	6mm.
Y_2	112mm
Y_3	218mm

$h_1 = Y_2 - Y_1 = 112 - 6 = 106\,mm$

$h_2 = Y_2 - \bar{Y} = 112 - 97.03 = 14.97\,mm$

$h_3 = Y_3 - Y_2 = 218 - 112 = 106\,mm.$

$$\therefore \bar{Y} = \frac{A_1Y_1 + A_2Y_2 + A_3Y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(2880 \times 6) + (2000 \times 112) + (1920 \times 218)}{2880 + 2000 + 1920}$$

$\bar{Y} = 97.03\,mm$

using parallel axis thm.

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3} \quad \text{—— (1)}$$

Now,

$$I_{xx_1} = I_1 + A_1 h_1^2$$

$$= \frac{bd^3}{12} + A_1 h_1^2$$

$$= \frac{240 \times 12^3}{12} + [2880 \times 106^2]$$

$$= 34560 + 32.35 \times 10^6$$

$$I_{xx_1} = 32.39 \times 10^6 \text{ mm}^4$$

$$I_{xx_2} = I_2 + A_2 h_2^2$$

$$= \frac{10 \times 200^3}{12} + [2000 \times 14.97^2]$$

$$I_{xx_2} = 7.11 \times 10^6 \text{ mm}^4$$

$$I_{xx_3} = I_3 + A_3 h_3^2$$

$$= \frac{160 \times 12^3}{12} + [1920 \times 106^2]$$

$$I_{xx_3} = 21.59 \times 10^6 \text{ mm}^4$$

$$\therefore \boxed{I_{xx} = 61.09 \text{ mm}^4} \times 10^6$$

Now to find I_{yy}

section is symmetrical about y-y-axis.

$$\therefore I_{yy} = I_{yy_1} + I_{yy_2} + I_{yy_3} \quad \text{—— (2)}$$

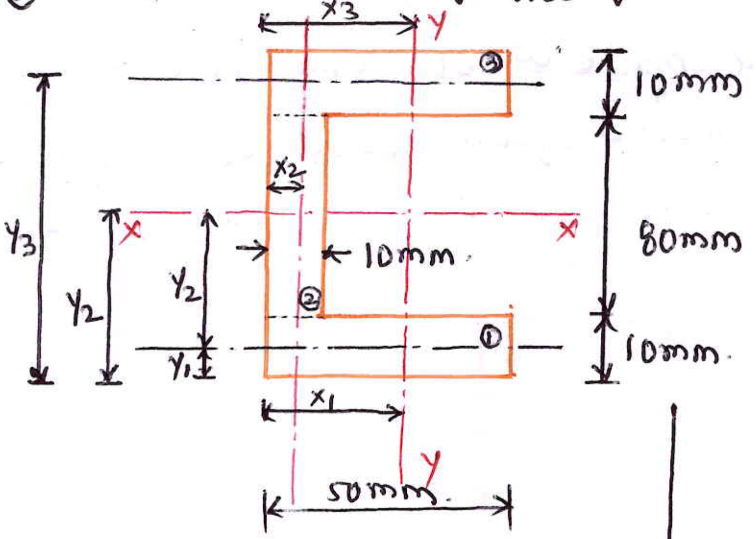
$$\therefore I_{yy} = \left[\frac{db^3}{12}\right]_1 + \left[\frac{db^3}{12}\right]_2 + \left[\frac{db^3}{12}\right]$$

$$= \left[\frac{12 \times 240^3}{12} + \frac{200 \times 10^3}{12} + \frac{12 \times 160^3}{12}\right]$$

$$= \left[13.82 \times 10^6 + 16.67 \times 10^3 + 4.096 \times 10^6\right]$$

$$\boxed{I_{yy} = 17.93 \times 10^6 \text{ mm}^4}$$

⑥ calculate MI of the following given section — WINTER-24.



$x_1 = 25mm$	$y_1 = 5mm$	$A_1 = 50\times10 = 500mm^2$
$x_2 = 5mm$	$y_2 = 50mm$	$A_2 = 80\times10 = 800mm^2$
$x_3 = 25mm$	$y_3 = 95mm$	$A_3 = 50\times10 = 500mm^2$

$$\bar{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$= \frac{[500\times25 + 800\times5 + 500\times25]}{500+800+500}$$

$$\bar{X} = 16.11\ mm.$$

$$\bar{Y} = \frac{A_1Y_1 + A_2Y_2 + A_3Y_3}{A_1 + A_2 + A_3}$$

$$= \frac{[500\times5 + 800\times50 + 500\times95]}{500+800+500}$$

$$\bar{Y} = 50mm.$$

$h_1 = Y_2 - Y_1 = 50 - 5 = 45mm$

$h_2 = 0$

$h_3 = Y_3 - Y_2 = 95 - 50 = 45mm$

Using parallel axis thm.

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

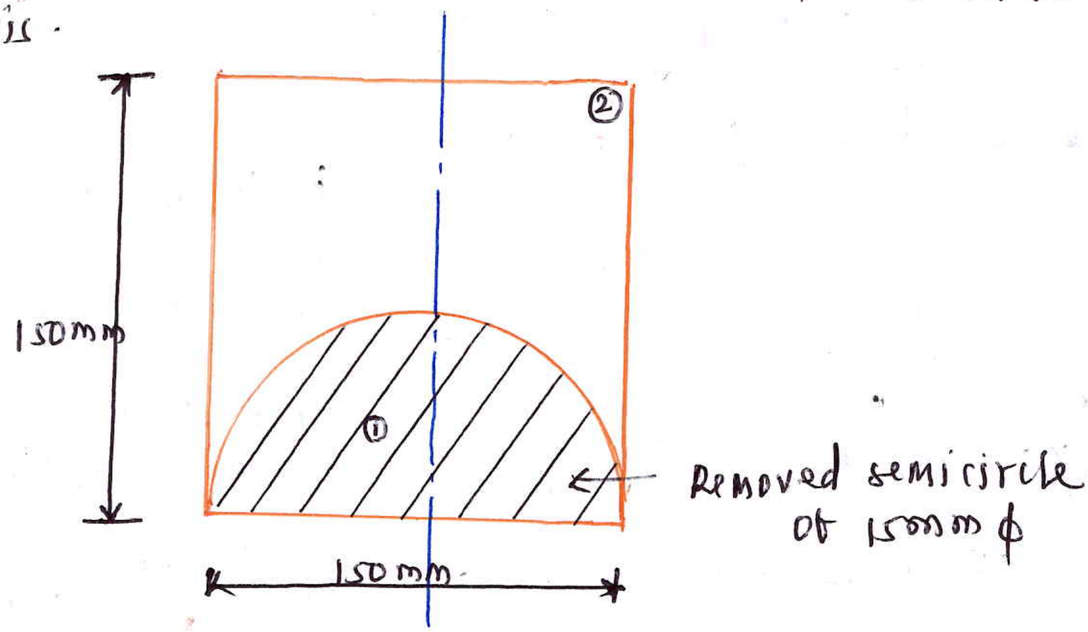
$$= \left[\frac{bd^3}{12} + A_1h_1^2\right]_1 + \left[\frac{bd^3}{12} + A_2h_2^2\right]_2 + \left[\frac{bd^3}{12} + A_3h_3^2\right]_3$$

$$= \left[\frac{50\times10^3}{12} + 500\times45^2\right] + \left[\frac{10\times80^3}{12} + 800\times0^2\right] + \left[\frac{50\times10^3}{12} + 500\times45^2\right]$$

$$I_{xx} = [1.016\times10^6 + 4.26\times10^5 + 1.016\times10^6]$$

$$I_{xx} = 2.458\times10^6\ mm^4$$

④ calculate the moment of Inertia about the base of composite lamina made up of semicircle of 150mm base diameter is removed from base of Rectangle 150 mm x 150 mm such that lamina is symmetrical to Y - axis.



Ans :- Give section is composite (i.e semicircle and Rectangle)

∴ Rectangle = 150 mm x 150 mm.

∴ MI of Rectangle $= \frac{bd^3}{03}$

$= \frac{150 \times 150^3}{03}$

$I_2 = 168.75 \times 10^6 \text{ mm}^4$

Now, MI of Semicircle

$I_1 = \frac{\pi r^4}{8}$

$= \frac{\pi \times 75^4}{8}$

$I_1 = 12.42 \times 10^6 \text{ mm}^4$

∴ ~~$I_{xx} = I_1 + I_2$~~

~~$= 168.75 \times 10^6 + 12.42 \times 10^6$~~

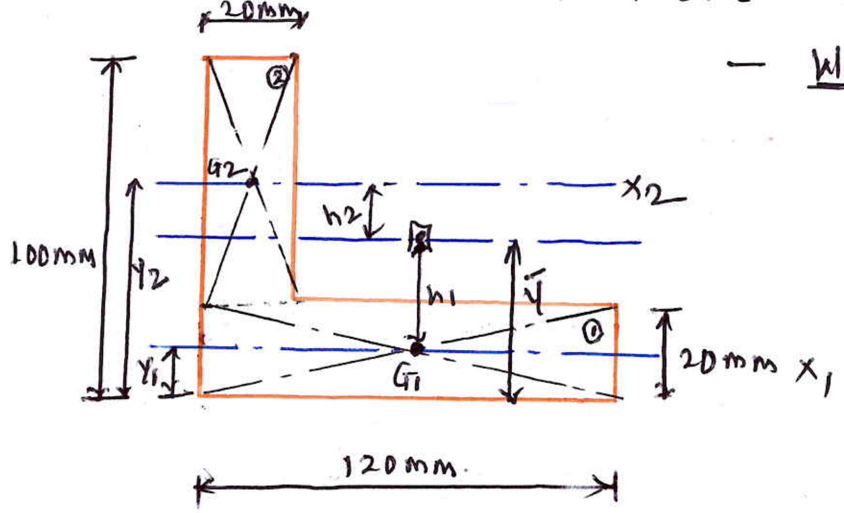
$I_{xx} = I_2 - I_1$

$= 168.75 \times 10^6 - 12.42 \times 10^6$

$\boxed{I_{xx} = 156.33 \times 10^6 \text{ mm}^4}$

(8) An angle section 120mm × 100mm × 20mm is placed such as it's longer leg is horizontal. calculate MI about centroidal Horizontal axis (I_{xx} Only)

— WINTER -25



section is unsymmetrical about x-x axis.

∴ used parallel axis theorem.

i.e $I_{xx} = I_{xx_1} + I_{xx_2}$ —— ①

Here, $A_1 = 120 \times 20 = 2400\ mm^2$

$A_2 = 80 \times 20 = 1600\ mm^2$

$Y_1 = 10\ mm$, $Y_2 = \frac{80}{2} + 20 = 60\ mm$.

~~$h_1 = Y_2 - Y_1 = 60 - 10 = 50\ mm$~~ ~~$h_2 = Y_2 - (h_1 + Y_1) \to 60 - ($~~

$$\bar{Y} = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2} = \frac{(2400 \times 10) + (1600 \times 60)}{2400 + 1600}$$

$\bar{Y} = 30\ mm$

∴ $h_1 = \bar{Y} - Y_1 = 30 - 10 = 20\ mm$

$h_2 = Y_2 - \bar{Y} = 60 - 30 = 30\ mm$

∴ $I_{xx_1} = I_1 + A_1 h_1^2$

$= \frac{bd^3}{12} + A_1 h_1^2$

$= \left[\frac{120 \times 20^3}{12} + 2400 \times 20^2\right]$

$I_{xx_1} = 1.04 \times 10^6\ mm^4$

Now, $I_{xx_2} = I_2 + A_2 h_2^2$

$= \frac{bd^3}{12} + A_2 h_2^2$

$= \left[\frac{20 \times 80^3}{12}\right] + \left[1600 \times 30^2\right]$

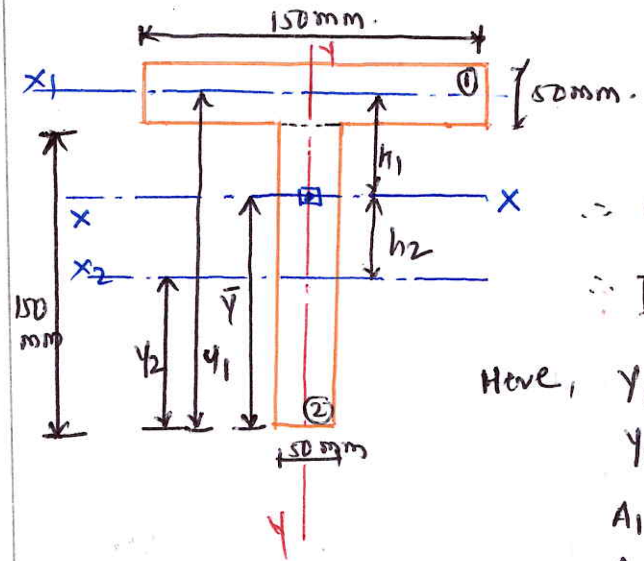
$I_{xx_2} = 2.29 \times 10^6\ mm^4$

∴ eqn ① becomes.

$I_{xx} = 1.04 \times 10^6 + 2.29 \times 10^6$

$$\boxed{\therefore I_{xx} = 3.33 \times 10^6\ mm^4}$$

(9) Find Moment of Inertia of T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about XX and YY-axis through the center of gravity of the section.



Ans - given section is unsymmetrical about xx axis

$\therefore$ using parallel axis theorem.

$$\therefore I_{xx} = I_{xx_1} + I_{xx_2} \quad \text{--- (1)}$$

Here, $y_1 = 150 + \frac{50}{2} = 175\,mm$

$y_2 = \frac{150}{2} = 75\,mm$

$A_1 = 150 \times 50 = 7500\,mm^2$

$A_2 = 50 \times 150 = 7500\,mm^2$

$$\therefore \bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500}$$

$$\bar{y} = 125\,mm$$

$\therefore h_1 = y_1 - \bar{y} = 175 - 125$, $h_1 = 50\,mm$

$h_2 = \bar{y} - y_2 = 125 - 75$, $h_2 = 50\,mm$

Now,

$$I_{xx_1} = I_1 + A_1 h_1^2$$

$$= \frac{bd^3}{12} + A_1 h_1^2$$

$$= \frac{150 \times 50^3}{12} + [7500 \times 50^2]$$

$$I_{xx_1} = 20.31 \times 10^6\,mm^4$$

$$I_{xx_2} = I_2 + A_2 h_2^2$$

$$= \frac{50 \times 150^3}{12} + [7500 \times 50^2]$$

$$I_{xx_2} = 32.81 \times 10^6\,mm^4$$

$$\therefore I_{xx} = 20.31 \times 10^6 + 32.81 \times 10^6$$

$$\boxed{I_{xx} = 53.12 \times 10^6\,mm^4}$$

Now MI about YY-axis.

$$I_{yy} = I_{yy_1} + I_{yy_2} \quad \text{--- (2)}$$

$$I_{yy_1} = \frac{d_1 b_1^3}{12} = \frac{50 \times 150^3}{12} = 14.06 \times 10^6\,mm^4$$

$$I_{yy_2} = \frac{d_2 b_2^3}{12} = \frac{150 \times 50^3}{12} = 1.56 \times 10^6\,mm^4$$

eqn (2) becomes.

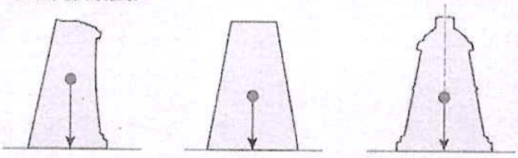
$$\boxed{I_{yy} = 15.625 \times 10^6\,mm^4}$$

CONCEPT OF CENTRE OF GRAVITY (C.G.) & MOMENT OF INERTIA (M.I.) USED IN ANCIENT CONSTRUCTIONS

Ancient Indian Engineers used scientific principles in construction.
The concepts of C.G. and M.I. were applied practically for strength, stability and durability.

1. CONCEPT OF CENTRE OF GRAVITY (C.G.)

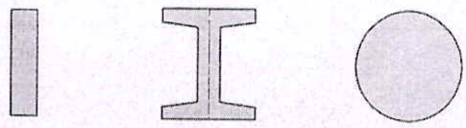
The point through which the total weight of a body acts. Keeping the C.G. low and at the center gives stability to the structure.



UNSTABLE (High C.G.) | STABLE (Low C.G.) | SYMMETRICAL (C.G. at Center)

2. CONCEPT OF MOMENT OF INERTIA (M.I.)

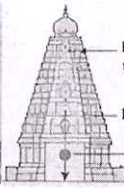
M.I. is the resistance offered by a section against bending or deformation. Ancient builders used suitable shapes and dimensions to increase M.I. and strength.



LOW M.I. (Weak Section) | HIGH M.I. (Strong Section) | HIGH M.I. (Strong Section)

APPLICATIONS IN ANCIENT CONSTRUCTIONS

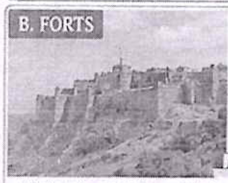
A. TEMPLES

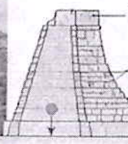


- Wide and heavy base, gradually lighter towards the top.
- Symmetrical structure keeps C.G. at the center and low.

Name : Brihadeeswarar Temple, Thanjavur
Era : 1010 AD
King : Raja Raja Chola I
Dynasty : Chola Dynasty

B. FORTS

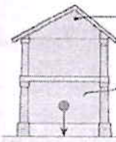




- Fort walls are thicker at the base and narrower at the top.
- Heavy construction keeps C.G. low, preventing overturning.

Name : Raigad Fort, Maharashtra
Era : 17th Century (1674 AD)
King : Chhatrapati Shivaji Maharaj
Dynasty : Maratha Empire

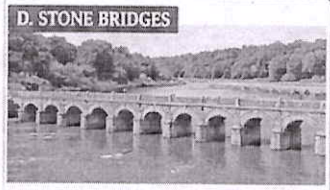
C. HOUSES (TRADITIONAL)

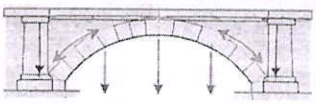


- Heavy walls and columns, light roof.
- Low C.G. gives stability and safety against wind and earthquakes.

Name : Traditional Wada House, Maharashtra
Era : 18th Century
King : Peshwa Madhavrao I
Dynasty : Maratha Empire (Peshwa Period)

D. STONE BRIDGES

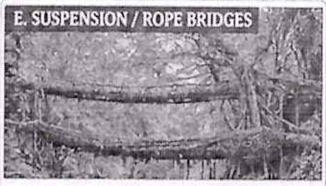


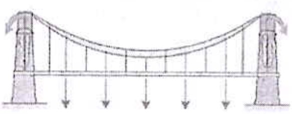


- Arch shape distributes loads uniformly.
- C.G. acts downwards at the center, compressive forces increase stability.

Name : Kallanai (Grand Anicut), Tamil Nadu
Era : ~200 BC
King : Karikala Chola
Dynasty : Early Chola Dynasty

E. SUSPENSION / ROPE BRIDGES



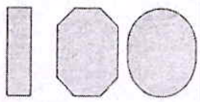


- The load acts at the center and distributes along the curve.
- Proper balance of C.G. and tension in ropes provides stability.

Name : Living Root Bridges, Meghalaya
Era : 15th – 19th Century
Community : Khasi Tribe (Traditional Builders)

APPLICATIONS IN ANCIENT CONSTRUCTIONS

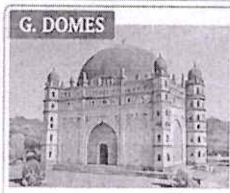
F. PILLARS & COLUMNS

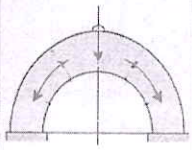


- Large cross-sectional area increases M.I.
- Circular and polygonal columns resist bending better.

Name : Ashoka Pillar, Sarnath
Era : 250 BC
King : Emperor Ashoka
Dynasty : Maurya Dynasty

G. DOMES

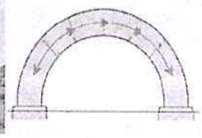




- Curved shape increases M.I.
- Loads are transferred as compression, reducing bending stress.

Name : Gol Gumbaz, Bijapur
Era : 1626–1656 AD
King : Sultan Muhammad Adil Shah
Dynasty : Adil Shahi Dynasty

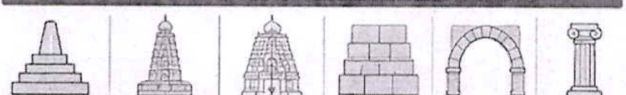
H. ARCHAES & VAULTS



- Arch shape has high M.I. and strength.
- Load is transferred to the supports, improving stability.

Name : Sanchi Stupa (Gates & Arches)
Era : 3rd Century BC – 1st Century BC
King : Emperor Ashoka
Dynasty : Maurya Dynasty

KEY ENGINEERING PRACTICES IN ANCIENT INDIA



Wide Foundations | Tapering Structures | Symmetrical Design | Stone Interlocking | Load Distribution | Strong Sections

CONCLUSION

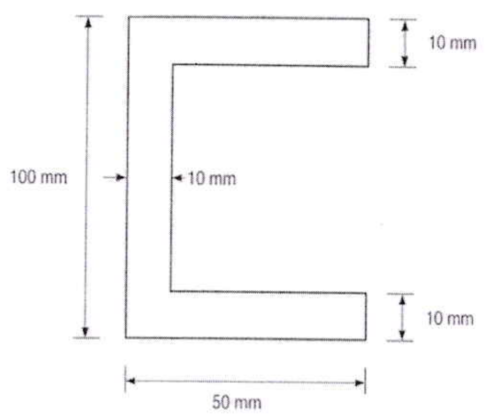
Ancient Indian architects and engineers had deep understanding of structural behaviour. By applying the concepts of Centre of Gravity and Moment of Inertia through experience and observation, they built temples, bridges, forts and houses that are still standing strong even today.

Unit - I

Moment of Inertia

Question Bank

1. Define – i) Moment of inertia ii) Radius of gyration
2. Explain polar moment of inertia
3. State the parallel axis theorem & perpendicular axis theorem with mathematical formula.
4. An Angle section 120 mm × 100 mm × 20 mm is placed such as its longer leg is horizontal. Calculate M.I. about centroidal horizontal axis only. (i.e. Ixx only)
5. A hollow circular section having 200 mm external diameter and 100 mm internal diameter. Calculate the moment of the section about any of the tangent. Also find polar moment of inertia.
6. A hollow square has inner dimensions a × a and outer dimensions 2a × 2a. Find moment of inertia about the outer side.
7. A circular disc has diameter of 80mm. Calculate M.I. about its any one tangent.
8. Determine the MI of unsymmetrical I-section having following details : Top flange 160mm × 12mm Bottom flange 240mm × 12mm Web 200mm × 10mm
9. Define 'Polar moment of Inertia'. Calculate Polar moment of Inertia for square lamina of side 40 cm
10. Calculate the M.I. for the following given section.



11. Calculate the moment of inertia about the base of composite lamina made up of a semicircle of 150 mm base diameter is removed from base of rectangle 150 mm × 150 mm such that lamina is symmetrical to Y–axis.
12. Calculate the M.I. of a T-section about its horizontal centroidal axis. Assume flange size = 200×10mm, vertical web = 180×12mm.
13. Calculate the moment of inertia about both centroidal axis for an equal angle section 180×180×10mm size

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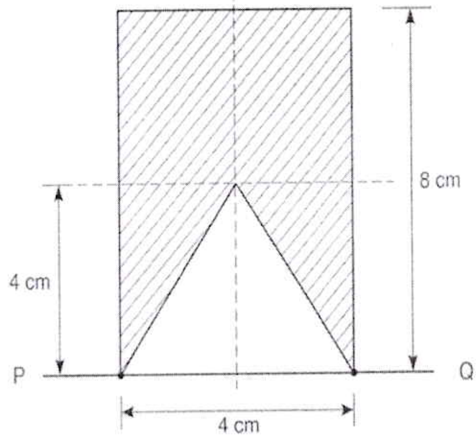
14. From a plate 4 cm × 8 cm a triangular portion is cut-off as shown in Figure No. 1. Calculate MI of remainder about horizontal line PQ passing through base of the lamina



15. A T-section having flange 200 mm × 40 mm and web 40 mm × 200 mm, overall depth 240 mm is used as beam. It is subjected to shear force 75 kN. Calculate shear stress and draw shear stress distribution diagram. N.A. is at 80 mm from top.